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Subject : SE&S

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Examinations : Final

Date : 25th june, 2020

Question: 1 (a)

Show that with the help of an equation that the differentiation of a function in time ~~dimension~~ domain results in the multiplication by jw in frequency domain.

Answer:

For the prove of the above problem

Let's take,

x(t) be a continous-time signal with a fourier transform of X(jw)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

Diff b/s w.r.t "t"

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \frac{d}{dt} \{e^{j\omega t}\} \, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \{e^{j\omega t} \cdot j\omega\} \, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{j\omega \, X(j\omega)\} \, e^{j\omega t} \, d\omega$$

$$F\left\{\frac{d}{dt} x(t)\right\} = j\omega \, X(j\omega)$$

Conclusion:

From the above result

we've concluded that if a function is differentiated in time domain, it is multiplied by 'jω' in frequency domain.

Question : 1 (b)

if

$$x[n] = 2\,\delta[n] - 4\,\delta[n-2] + 2\,\delta[n-3]$$

$$h[n] = 3\,\delta[n] + \delta[n-1] + 2\,\delta[n-2]$$

Produce $Y(z)$ & $Y[n]$

Answer:

For $Y(z)$:

As we know

$$Y(z) = H(z) \times X(z) \rightarrow (i)$$

So

$$X(z) = 2 - 4^{-2} + 2^{-3}$$
$$H(z) = 3 + 1^{-1} + 2^{-2}$$

putting values in eq (i)

we get,

$$Y(z) = (2 - 4^{-2} + 2^{-3})(3 + 1^{-1} + 2^{-2})$$

Multiplying both with each other

$$Y(z) = 6 + 2z^{-1} - 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-5}$$

By simplification we get.

$$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$$

For $Y[n]$:

To find $Y[n]$, we'll use the delay property

$$Y[n] = 6\,\delta[n] + 2\,\delta[n-1] - 8\,\delta[n-2] + 2\,\delta[n-3] - 6\,\delta[n-4] + 4\,\delta[n-5]$$

Question : 2

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

Retrieve the fourier series for the given function.

Solution:

For a_0:

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} \frac{-\pi}{2}\, dx + \int_{0}^{\pi} \frac{\pi}{2}\, dx \right.$$

$$= \frac{1}{\pi} \left[-\frac{\pi}{2} \int_{-\pi}^{0} 1\, dx + \frac{\pi}{2} \int_{0}^{\pi} 1\, dx \right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2} [x] \Big|_{-\pi}^{0} + \frac{\pi}{2} [x] \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{2} (0) - (-\pi) \right] + \frac{\pi}{2} (\pi) - (0)$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2}{2} + \frac{\pi^2}{2} \right]$$

$$= \frac{1}{\pi}\left[0 \right]$$

$$= 0$$

For a_n:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2} \cos nx \, dx + \int_{0}^{\pi} \frac{\pi}{2} \cos nx \, dx$$

$$= \frac{1}{n\pi}\left(\frac{-\pi}{2}(\sin nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(\sin nx)\Big|_{0}^{\pi}\right)$$

$$= \frac{1}{n\pi}\left(\frac{-\pi}{2}(\sin n(0) - \sin n(-\pi)) + \frac{\pi}{2}(\sin n(\pi) - \sin n(0)\right)$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$= 0$$

For b_n:

As,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} -\frac{\pi}{2} \sin nx \, dx + \int_{0}^{\pi} \frac{\pi}{2} \sin nx \, dx \right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}(-\cos nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(-\cos nx)\Big|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi}\left(-\frac{\pi}{2}\left(-\cos n(0) - (-\cos n(-\pi))\right)\right.$$

$$\left. + \frac{\pi}{2}\left(-\cos n(\pi) - (-\cos n(0))\right)\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(-2) + 2\frac{\pi}{2} \right]$$

$$= \frac{1}{n\pi}\left[\frac{2\pi}{2} + \frac{2\pi}{2} \right]$$

$$= \frac{1}{n\pi} = \left[\frac{4\pi}{2} \right]$$

$$= \frac{{}^{1}\cancel{4}\cancel{\pi}}{n\cancel{2}\cancel{\pi}} \Rightarrow \frac{1}{2n}$$

$$= \frac{1}{2n}$$

$$b_n = \begin{cases} 0 & \text{if "n" is even} \\ 1/2n & \text{if "n" is odd} \end{cases}$$

$$= 0 + 0 + 0 + \cdots \cdots + \frac{1}{2} \sin x + \frac{1}{4} \sin 2x$$

$$+ \frac{1}{6} \sin 3x \cdots$$

$$\frac{\pi}{2} = \frac{1}{n\pi} \left[-1 \left[-1 + \cos n(-\pi) \right] + 1 \left[-\cos n\pi + 1 \right] \right.$$

$$= \frac{1}{2n} \left[1 - \cos n\pi - \cos n\pi + 1 \right]$$

$$= \frac{1}{2n} \left[2 - 2\cos n\pi \right]$$

Now:

$$b_n = \begin{cases} 0 & \text{if "n" is even} \\ \frac{4}{2n} & \text{if "n" is odd} \end{cases}$$

$$b_n = \frac{4}{2n}$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \ldots\ldots$$
$$+ b_1 \sin x + b_2 \sin 2x + \ldots\ldots$$

$$f(x) = (0) + (0) \cos x + 0 \cos(2x) + \ldots\ldots$$

$$= \frac{4}{2} \sin x + (0) \sin^2 x + \frac{4}{3(2)} \sin 3x + \ldots\ldots$$

$$\frac{4}{2} \sin x + \frac{4}{6} \sin 3x + \ldots\ldots$$

Question: 5

Apply fourier transform on the signal

$$x(t) = e^{-a|t|} u(t)$$

where $u(t)$ is a unit step function.

Solution:

The fourier transformation of the given function is;

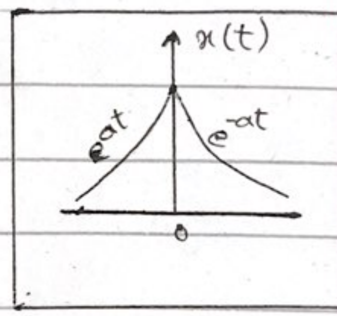
$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega t) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t}\, dt$$

As,

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$$x(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$



$$x(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega t)} dt + \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega} \Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \Bigg|_{0}^{\infty}$$

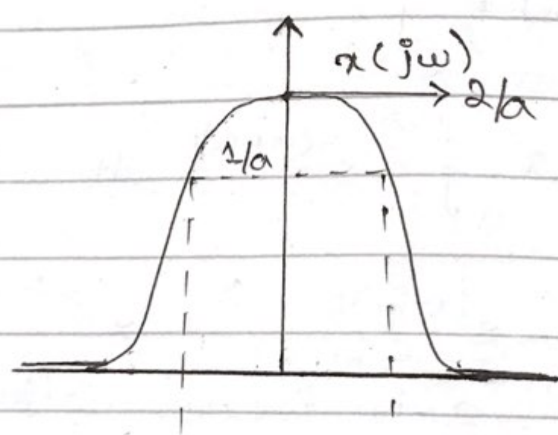
$$= \frac{1}{(a-j\omega)} [e^{0} - e^{-\infty}] - \frac{1}{(a+j\omega)} [e^{-\infty} - e^{0}]$$

$$= \frac{1}{(a-j\omega)}[1-0] - \frac{1}{a+j\omega}[0-1]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+j\omega+a-j\omega}{a^2-(j\omega)^2}$$

$$x(j\omega) = \frac{2a}{a^2+\omega^2}$$



Question : 4 :

Express the transfer function using given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [0]$$

Solution:

As we know that,

$$G(S) = C\,[SI - A]^{-1}\,B + D$$

$$= [1 \;\; 2]\left[\left[S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s(s+2)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2 2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

[num, dem] = ss2tf [A, B, C, D]

[A, B, C, D] = tf2ss [num, dem]

Question: 3

If $x(z) = 2z^2 + \frac{2z}{(z^2+2z-3)}$

Retrieve $x[n]$ using inverse Z-transform method.

Solution:

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$X(z) = \frac{2z\,(z+1)}{z^2+3z-z-3}$$

$$x(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$$

$$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

or

$$\frac{2(z+1)}{z^2+2z-3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

or put

$2(z+1) = A(z-1) + B(z+3) \rightarrow (i)$

for B:

put $z = 1$ in (i)

$2(1+1) = B(1+3)$

$4 = 4B$

$B = 1$

For A:

put $Z = -3$ in (i)

$$2(-3+1) = A(-3-1)$$

$$-4 = -4A$$

$$A = 1$$

Now put values of A & B in eq (i)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{(z-1)}$$

$$x(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

The inverse Z-Transfor is;

$$x[n] = u[3] + 1(-1)^k$$