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QUESTION - 01

Explain in detail types of stirrups with figures and Also explain ACI Codes for Shear design.

Ans:-

STIRRUP :-

Stirrups are closed-loop bars tied at regular intervals in beam reinforcement to hold the bars in position.

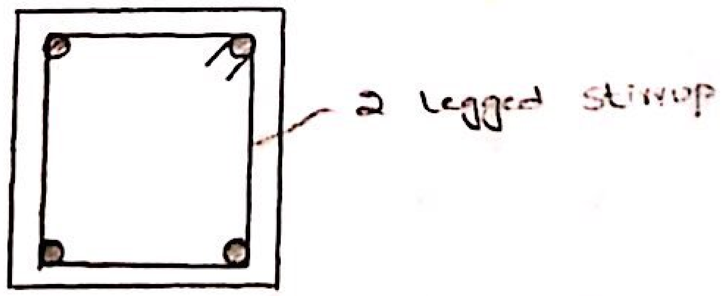
TYPES OF STIRRUPS:-

1- Single Legged Stirrup :-

The Single-leg stirrups have rarely been used because they are mostly used when binding only two rods.

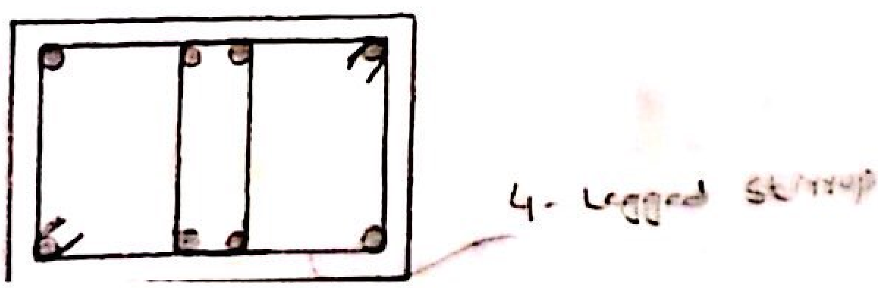
2- Two Legged Stirrup :-

It is most Commonly and widely used stirrup. Minimum 4 bars are required for providing this stirrup.

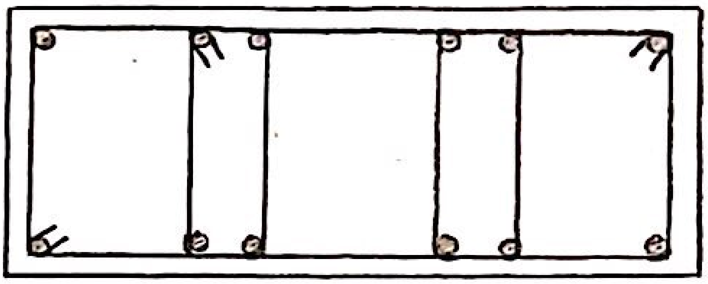


3- Four Legged Stirrup :-

These stirrups are used in Case of web reinforcement.



4- Six Legged Stirrup :-



ACI CODES FOR SHEAR DESIGN OF A BEAM

According to ACI-318, following are the formulas used for the shear design of a Beam.

1- Critical Section:- Critical Section occurs at 45° and is at distance (d)" from the face of support which is equal to effective depth.

2- Shear Strength Capacity of Concrete is

$$V_c = 2 \times \sqrt{f'_c} \times b_w \times d$$

3- Minimum Web Reinforcement:-

If $V_u \leq \phi V_c$, then theoritically no web reinforcement is required. However ACI Code require provision of atleast a minimum area of web reinforcement equal to,

$\phi = 0.75 \longrightarrow$ For shear design

($\because V_u$ = Total factored shear applied at a given section)

=> For Minimum Reinforcement Area:-

$$A_{v_{min}} = \frac{0.75 \times \sqrt{f'_c} \times b_w \times S}{f_y} \quad \text{or} \quad \frac{50 \times b_w \times S}{f_y} \rightarrow \left[\begin{array}{c}\text{Higher} \\ \text{value is} \\ \text{Selected}\end{array}\right]$$

By interchanging the above formulas, we can obtain the formula for maximum Spacing.

$$S_{max} = \frac{A_v \times f_y}{0.75 \times \sqrt{f'_c} \times b_w} \quad \text{or} \quad \frac{A_v \times f_y}{50 \times b_w} \rightarrow \left[\begin{array}{c}\text{Lesser value} \\ \text{is} \\ \text{Selected}\end{array}\right]$$

4- No web-reinforcement is required if

$$V_u < \frac{1}{2}\phi V_c$$

=> Between critical section "V_u" and "ϕV_c", spacing b/w web reinforcement can be find by,

$$S = \frac{\phi \times A_v \times f_y \times d}{V_u - \phi V_c}$$

5- If $V_s \leq 4 \times \sqrt{f'_c} \times b_w \times d$, then max spacing for stirrups will be the smallest of the following.

1- 24"

2- d/2

3- $S_{max} = \dfrac{A_v \times f_y}{0.75 \times \sqrt{f'_c} \times b_w}$

4- $S_{max} = \dfrac{A_v \times f_y}{50 \times b_w}$

∵ (V_s = Shear force carried by web reinforcement)

=> If $V_s > 4 \times \sqrt{f'_c} \times b_w \times d$

↓

Max. spacing will be halved

=> If $V_s > 8 \times \sqrt{f'_c} \times b_w \times d$

↓

Then either increase cross-sectional dimensions or increase f'_c.

QUESTION - 02

A simply supported rectangular beam 14" wide having an effective depth of 22" to carry a lateral load of 6.5 k/ft on a 18' simple span. It is reinforced with 7 in² of tensile steel area, if $f'_c = 4$ ksi and $f_y = 60$ ksi, then design the beam for shear.

Given :-

Breadth of web of beam (b_w) = 14"

Effective depth (d) = 22"

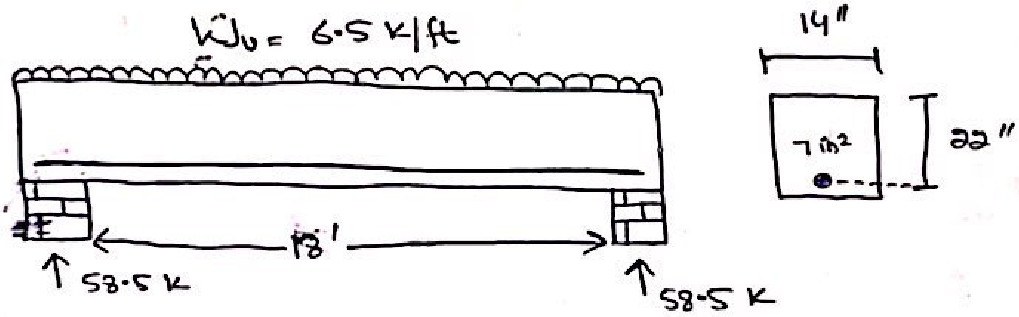
Given load = 6.5 k/ft

Steel Area = 7 in²

$f'_c = 4$ ksi

$f_y = 60$ ksi

Sol:-



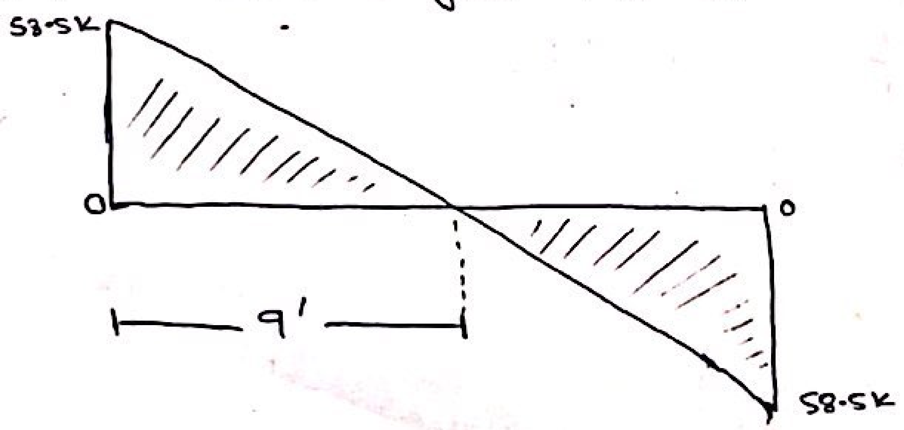
STEP #1 :- (Reactions on Supports)

Finding the reactions due to applied load.

$$\text{Total load} = \frac{6.5 \times 18}{2} = 58.5 \text{ kips}$$

STEP #2 :- (Shear Force Diagram)

The required shear diagram will be.



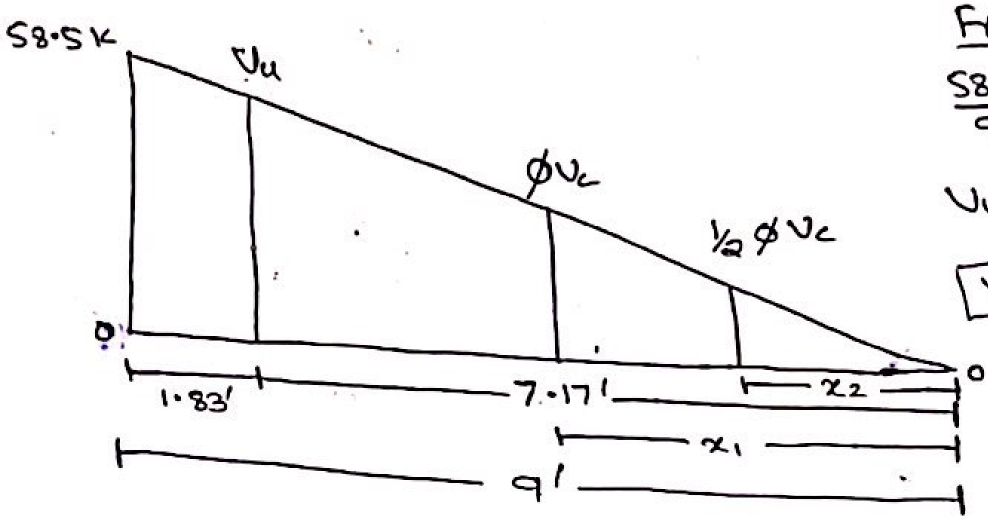
STEP # 3 :-

Finding the value of critical shear 'V_u" and its location.

As,

we know that critical shear is located at distance "d" from face of support (d) = 22" = 1.83'

=> we will find the values of critical shear at distance "d" by use of similar triangles.



From Similar Triangles,

$$\frac{58.5}{9} = \frac{V_u}{8.17}$$

$$V_u = \frac{58.5 \times 7.17}{9}$$

$$\boxed{V_u = 46.61 \text{ kips}}$$

STEP # 4 :-

Finding the value of 'ϕV_c" and "$\frac{1}{2}\phi V_c$" and also its distances from zero shear to right side.

By formula,

$$\phi V_c = \phi \times 2 \times \sqrt{f'_c} \times b_w \times d$$

$$= 0.75 \times 2 \times \sqrt{4000} \times 14 \times 22. \quad = 29219 \text{ lbs}$$

$$= 29.21 \text{ kips}$$

=> Location of ϕV_c by similar triangles,

$$\frac{58.5}{9} = \frac{\phi V_c}{x_1} \quad \Rightarrow \quad \frac{58.5}{9} = \frac{29.21}{x_1}$$

$$\Rightarrow \boxed{x_1 = 4.49'}$$

=> Similarly,

$$\tfrac{1}{2}\phi V_c = \phi V_c/2 \quad \Rightarrow \quad 29.21/2 \quad = 14.60 \text{ kips}$$

=> Location of $\frac{1}{2}\phi V_c$ will be,

$$\frac{58.5}{9} = \frac{14.60}{x_2} \quad \Rightarrow \quad \boxed{x_2 = 2.24'}$$

STEP #5 :-

Finding the value of ϕV_s

By formula, $V_u = \phi V_s + \phi V_c$

$\Rightarrow \phi V_s = V_u - \phi V_c$

$= 46.61 - 29.21$

$\boxed{\phi V_s = 17.4 \text{ kips}}$

STEP #6 :-

Check on Section adequacy,

By formula,

$= \phi \times 8 \times \sqrt{f'_c} \times b_w \times d$

$= 0.75 \times 8 \times \sqrt{4000} \times 14 \times 22 \quad = 116877 \text{ Lbs}$

$= 116.87 \text{ kips}$

As $\phi \times 8 \times \sqrt{f'_c} \times b_w \times d > \phi V_s$

So Section is Adequate!

STEP #7 :-

Check on Maximum Spacing for Stirrups,

By formula,

$= \phi \times 4 \times \sqrt{f'_c} \times b_w \times d$

$= 0.75 \times 4 \times \sqrt{4000} \times 14 \times 22 \quad = 58438 \text{ Lbs}$

$= 58.43 \text{ kips}$

As $\phi \times 4 \times \sqrt{f'_c} \times b_w \times d > \phi V_s$

So Maximum will be Selected from the following 4 Conditions,

1- $S_{max} = 24''$

2- $d/2 = 22/2 = 11''$

3- $S_{max} = \dfrac{A_v \times f_y}{0.75 \times \sqrt{f'_c} \times b_w}$

Here we are using #3 stirrup,

dia = $(3/8)'' = 0.375''$

So

Area = $\frac{\pi}{4}(0.375)^2 = 0.11 \text{ in}^2$

For 2-legged stirrup

$\Rightarrow$ Area $\times 2$

$\Rightarrow 0.11 \times 2 = 0.22 \text{ in}^2$

3- $S_{max} = \dfrac{0.22 \times 60000}{0.75 \times \sqrt{4000} \times 14} = 19.87''$

4- $S_{max} = \dfrac{A_v \times f_y}{50 \times b_w} = \dfrac{0.22 \times 60000}{50 \times 14} = 18.85''$

From above 4 conditions, Least value of spacing for #3, 2 legged stirrup will be selected as,

$S_{max} = 11''$

STEP # 8 :-

Stirrups spacing from/at critical section will be,

By formula,

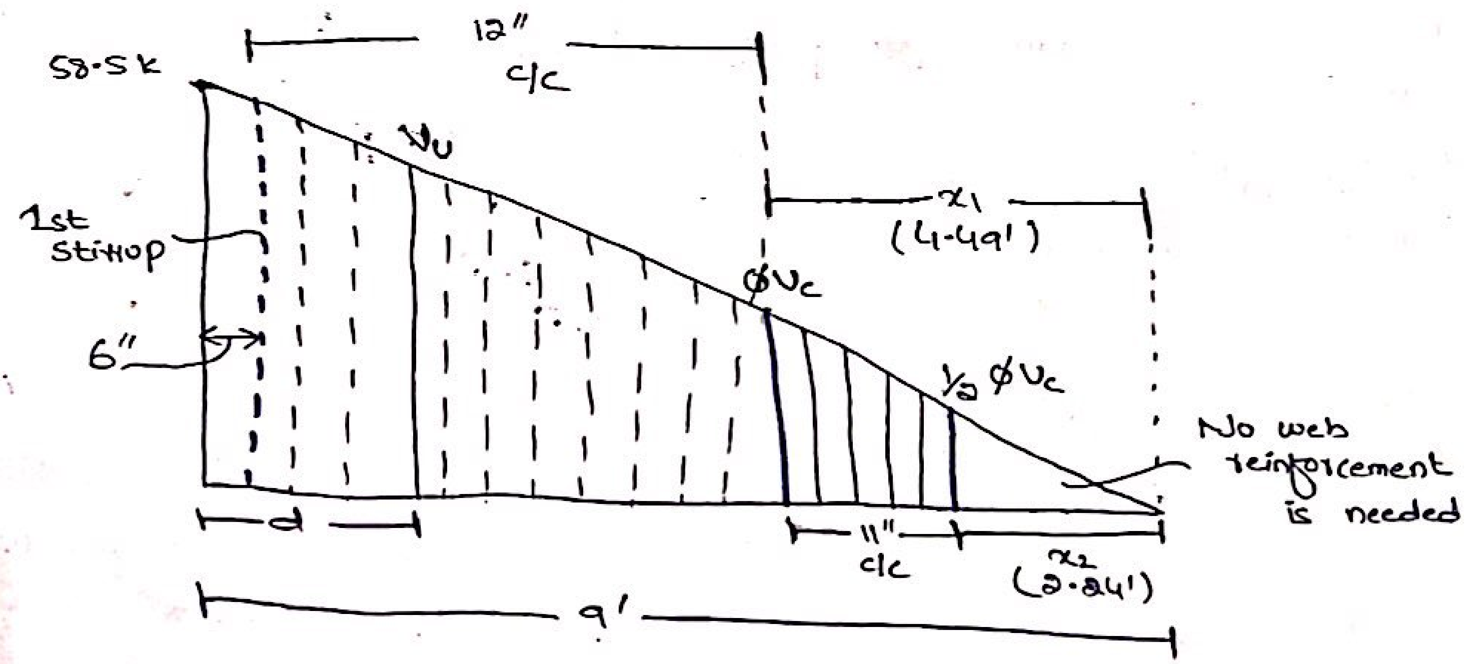
$$S = \frac{\phi \times A_v \times f_y \times d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60000 \times 22}{46.61 - 29.21}$$

$S = 12.5'' \simeq 12''$.

So $12''$ c/c

STEP # 9 :-

Final sketch will be,



As

First stirrup from face of support,

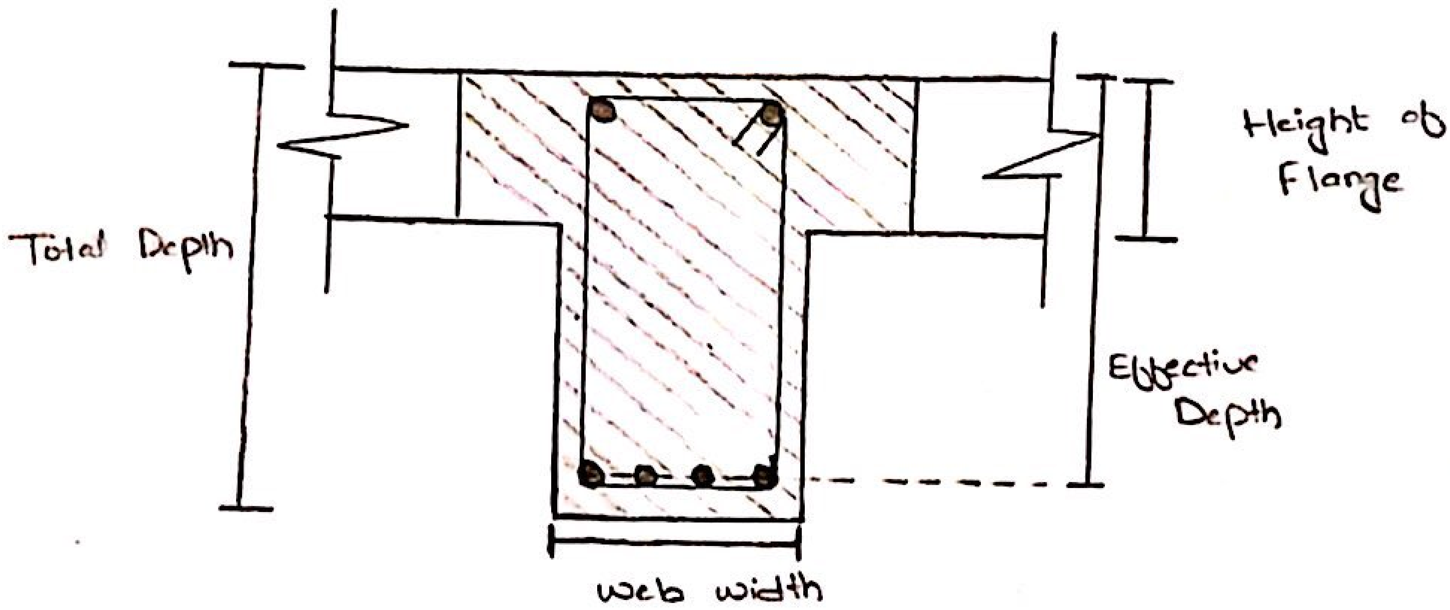
$S/2 = 12/2 = 6''$

Question- 03

Define both the T-Beam and L- Beam with the help of diagram. Also explain flexural analysis of T- Beam.

T- Beam :-

=> In most of the reinforced concrete structures, Concrete slabs are cast monolithically with the slab So, in this case the beam that act as an intermediate beam are called T- Beams.



=> Because of their T- shape, these beams are called T- Beams.

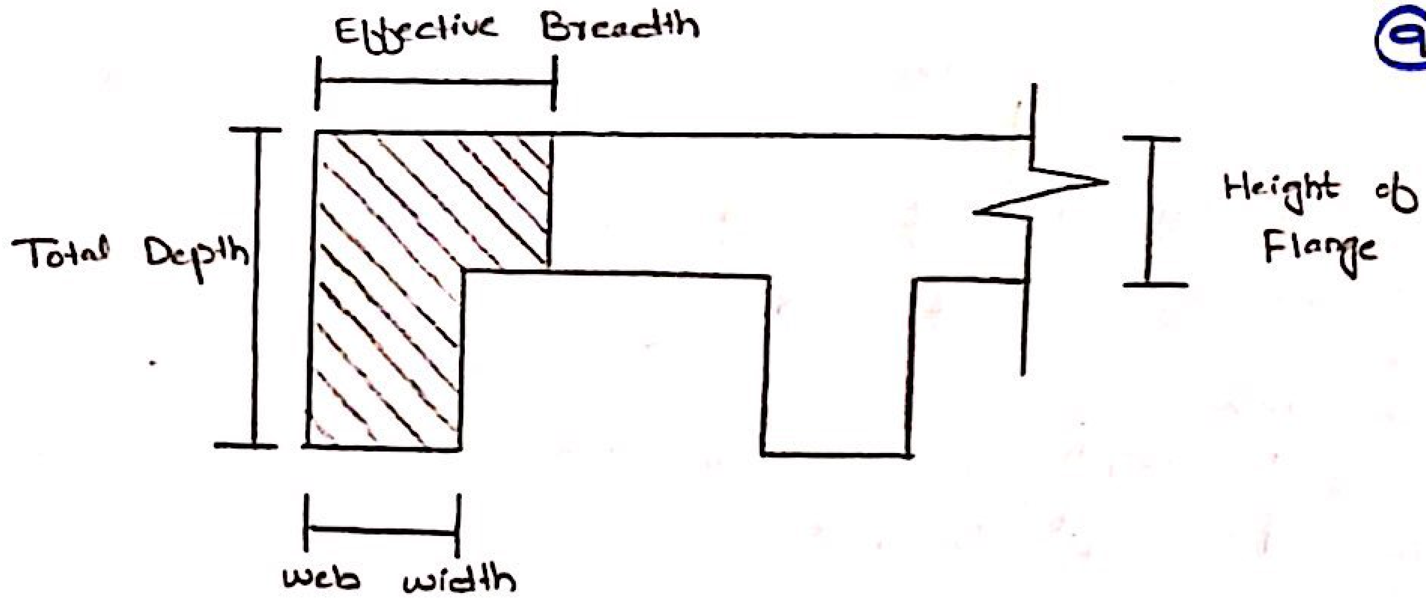
=> It is provided at the center of the slab to resists the loads.

=> The upper most area of the the beam attached to the slab is called Flange.

=> The bottom rectangular portion of the beam is called web of the beam.

L- Beam :-

=> L- shaped structure that is in contact with the slab and present at the corner of the floor is called L- Beam.



=> L-Beams are also called Edge Beams.

=> It is always provided at the corners of the slab.

=> L-Beams are typical floor beams because of their reduced overall structural depth, the beams are in Prestressed or reinforced Concrete.

FLEXURAL ANALYSIS OF T-BEAM :-

Flexural Analysis of T-Beam consists of the following steps:-

1 - For Finding the Ultimate Factored moment, we use the following Formula;

$$M_u = \frac{W_u \times L^2}{8}$$

(W_u = Total Factored Load
L = Total span of the beam)

2 - Effective width (b_e) for T-Beam is calculated as:-

1 - $16(h_f) + b_w$

2 - c/c distance

3 - span/4

4 - $\frac{CTS}{2} + b_w$

∴ (h_f = height of flange
CTS = Clear transverse span)

- We have to select the least value from above formulas
- If c/c distance is given, then there is no need of "$\frac{CTS}{2} + b_w$"

3- Checking whether Rectangular or T-Beam Analysis is required

i- If $a > h_f$ → Special Analysis is required

ii- If $a < h_f$ → Rectangular beam Analysis is required

where

$a =$ Depth of Compression block

$h_f =$ Height of flange

4- For Finding Area of Steel, we have to use

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)}$$

where

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b_w}$$

∵ $\phi =$ strength Reduction factor

$d =$ Effective depth

$a =$ Compression block depth

$b_w =$ web width of beam

5- For checking the range of Reinforcement Ratio,

$$\rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \times \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$$\rho_{min} = \frac{200}{f_y}$$

$$\rho = \frac{A_{st}}{b \times d}$$

6- Formula for Finding No. of bars required is,

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

7- For checking Minimum width for bars accomodation,

$$b_{min} = 2(\text{clear cover}) + 2(\text{dia of stirrup}) + \text{No. of bars}(\text{dia of bar}) + \text{Spacing b/w bars}(\text{dia of bar})$$

8- Design Moment is given by,

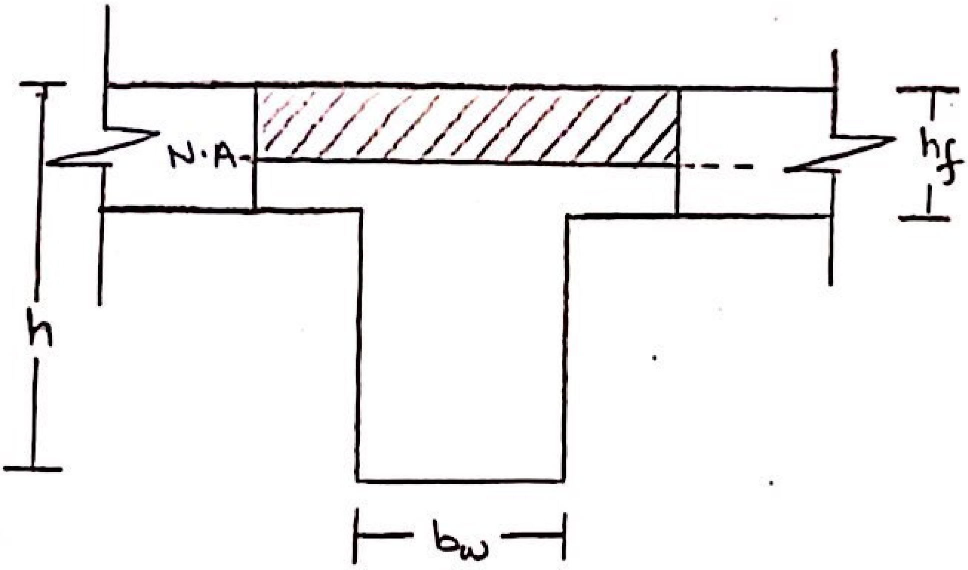
$$M_d = \phi \times f_y \times A_{st} \times (d - a/2) \rightarrow \text{if } a < h_f$$

$$M_d = \phi \times [A_s \times f_y \times (d - h_f/2) + (A_s - A_{st}) + f_y \times (d - a/2)] \rightarrow \text{if } a > h_f$$

QUESTION - 04

What is the difference b/w. CASE-1 and CASE-2 in the design of T-Beam?

CASE-I :-



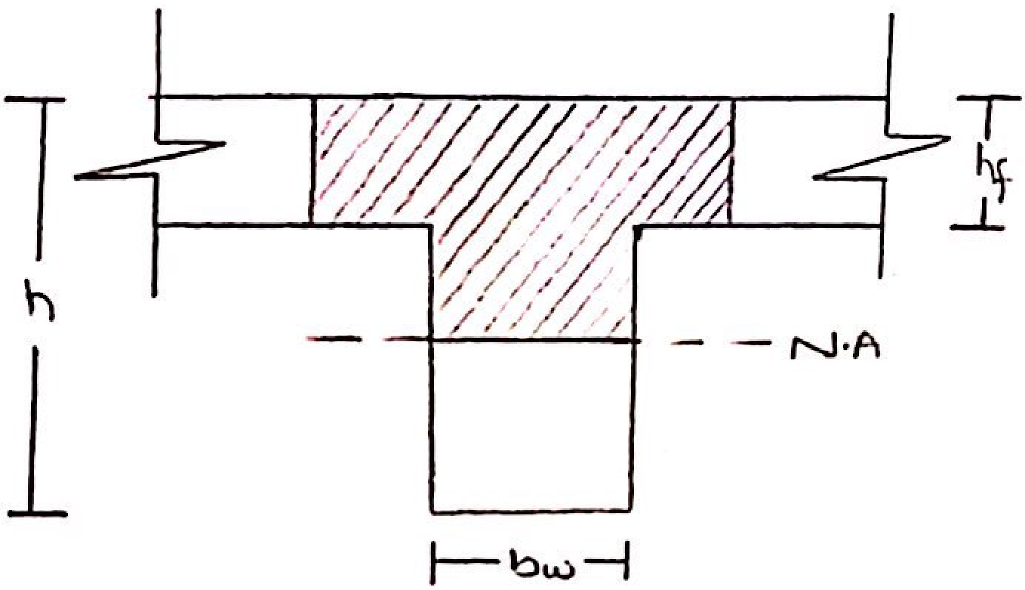
From the figure

$a < h_f$

So in this case,
Rectangular Beam Analysis is Required.
So,
The Design Moment formula will be

$$M_d = \phi \times f_y \times A_{st} \times (d - a/2)$$

CASE - II :-



From the figure,

$a > h_f$

So in this, Special beam analysis i.e, T- Beam Analysis is required.
So
the required Design Moment will be,

$$M_d = \phi \times \left[A_s \times f_y \times \left(d - \frac{h_f}{2}\right) + (A_s - A_{st}) + f_y \times (d - a/2) \right]$$

QUESTION - 05

A floor system consists of 3.5" concrete slab supported by 16' simple span spaced at 9' c/c, the beam having a web width of 10" and effective depth of 18" and total height is 23". Calculate the necessary flexural reinforcement if the factored applied moment is 5800 kip-inch. Use $f'_c = 3$ ksi and $f_y = 60$ ksi

Given:-

Height of flange (h_f) = 3.5"

c/c distance = 9'

Length / span of the beam = 16'

web width (b_w) = 10"

Effective depth (d) = 18"

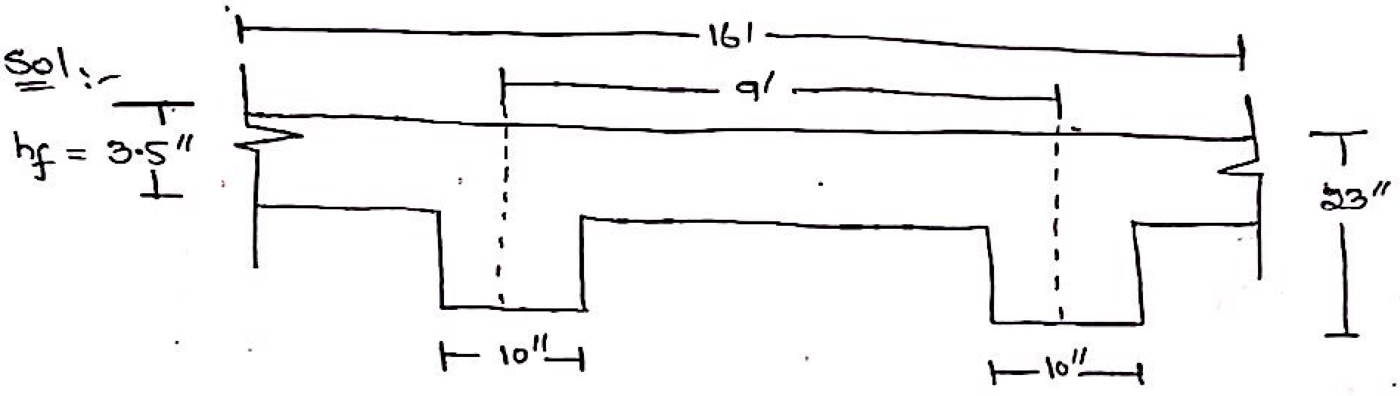
Height (h) = 23"

Total factored moment (M_u) = 5800 kip-inch

$f'_c = 3$ ksi

$f_y = 60$ ksi

Sol:-



STEP #1:-

Calculate the effective width (b_e) for T-beam.

1- $16(h_f) + b_w = 16(3.5) + 10 = 66"$

2- c/c distance $= 9 \times 12 = 108"$

3- Span/4 $= \frac{16}{4} \times 12 = 48"$

Selecting the least value of b_e as,

$$\boxed{b_e = 48"}$$

Step #2 :-

Check whether Rectangular or T-beam Analysis is required.

Trial # 01 :- Let $a = h_f = 3.5''$

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)} = \frac{5800}{0.90 \times 60 \times (18 - 3.5/2)} = 6.61 \text{ in}^2$$

Trial # 02 :-

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b_e}$$

$$a = \frac{6.61 \times 60}{0.85 \times 3 \times 48} = 3.2''$$

$\Rightarrow 3.2'' < 3.5''$

and $\boxed{A_{st} = 6.55 \text{ in}^2}$

So Rectangular Beam Design is Required !

Trial # 03 :- $a = 3.21''$

and $A_{st} = \dfrac{5800}{0.90 \times 60 (18 - \frac{3.21}{2})} = \boxed{6.55 \text{ in}^2}$

So Area of steel is 6.55 in^2.

Step #3 :-

Check ρ_{max} and ρ_{min}.

$$\Rightarrow \rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}\right)$$

$$= 0.85 \times 0.85 \times \frac{3}{60} \left(\frac{0.003}{0.003 + 0.005}\right) = 0.013$$

$$\Rightarrow \rho_{min} = \frac{200}{f_y} = \frac{200}{60000} = 0.003$$

$$\Rightarrow \rho = \frac{A_{st}}{b \times d} = \frac{6.55}{10 \times 18} = 0.036$$

$$\rho_{min} < \rho < \rho_{max}$$

$$0.003 < 0.036 < 0.013$$

As the value of ρ_{max} is less than ρ, So we have to design it as "Doubly Reinforced Beam."

=> First we have to find the Area of steel against ρ_{max}.

$$\rho_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = \rho_{max} \times (b \times d)$$

$$A_{st} = 0.013 \times (10 \times 18)$$

$$\therefore \boxed{A_{st} = 2.34 \text{ in}^2}$$

STEP # 4 :-

Finding the value of M_{u2}:-

By formula,

$$M_{u2} = \phi \times A_{st} \times f_y \times (d - a/2)$$

First Finding the value of "a"

$$\Rightarrow a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \frac{2.43 \times 60}{0.85 \times 3 \times 10}$$

$$a = 5.72''$$

$$\Rightarrow M_{u2} = 0.90 \times 2.43 \times 60 \times (18 - 5.72/2)$$

$$\boxed{M_{u2} = 1986.67 \text{ kip-inch}}$$

As $M_{u2} < M_u$

$$1986.67 < 5800$$

So we have to design the beam in such away that it can resist more bending moment than the applied external moment.

STEP # 5:-

Finding Difference in moments and Area of steel.

$$Mu_1 = Mu - Mu_2$$
$$= 5800 - 1986.67$$
$$\boxed{Mu_1 = 3813.33 \text{ kip-inch}}$$

By formula,

$$A'_{st} = \frac{Mu}{\phi \times fy \times (d-d')} = \frac{3813.33}{0.90 \times 60 \times (18-2.5)}$$
$$\boxed{A'_{st} = 4.56 \text{ in}^2}$$

STEP #6:-

Finding Total Steel Area.

$$As = A_{st} + A'_{st}$$
$$= 2.43 + 4.56 = 6.99 \text{ in}^2$$

STEP #7:-

Selection of Bar:-

In Tension Zone:-

Let we use #8 bar

dia = $(8/8) = 1''$, Area $= \frac{\pi}{4}(1)^2 = 0.785 \text{ in}^2$

By formula

$$\text{No. of bars} = \frac{\text{Area of Steel}}{\text{Area of single bar}} = \frac{6.99}{0.785} = 8.9 \cong 9$$

So 9 #8 bars

In Compression Zone:-

Let we use #7 bar

dia = $(7/8)''$, Area $= \frac{\pi}{4}(7/8)^2 = 0.601 \text{ in}^2$

By formula.

$$\text{No. of bars} = \frac{\text{Area of Steel}}{\text{Area of Single bar}} = \frac{4.56}{0.601} = 7.5 \Rightarrow 8$$

So 8 #7 bars

STEP # 8 :-

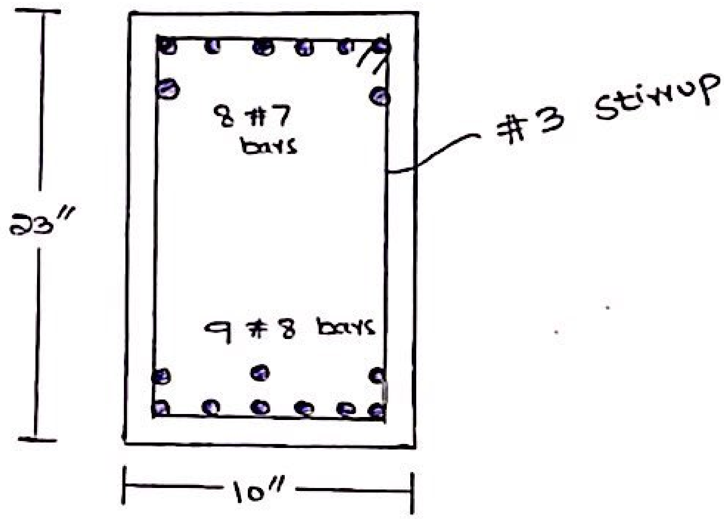
Minimum width for Accomodation of bars.

$$b_{min} = (2 \times 1.5) + (2 \times 3/8) + 9(8/8) + 8(8/8)$$

$$= 20.75''$$

As $20.75'' > 10''$

So, the bars will be placed in multiple Layers.



$$\text{Effective depth } (d) = 23 - 1.5 + \frac{3}{8} + \frac{8}{8} + \frac{1}{2}\left(\frac{8}{8}\right) = 19.6''$$

$$\text{Effective Cover } (d') = 1.5 + \frac{3}{8} + \frac{7}{8} + \frac{1}{2}\left(\frac{7}{8}\right) = 3.18''$$

STEP # 9 :-

Finding the Design Moment.

$$Md = \phi\left[A's \times fy \times (d - d') + (Ast - A'st) \times fy \times (d - a/2)\right]$$

First
$$a = \frac{(As - A's t) \times fy}{0.85 \times f'c \times b} = \frac{(9 \times 0.785 - 8 \times 0.601) \times 60}{0.85 \times 3 \times 10} = 5.31''$$

$$\Rightarrow Md = 0.90\left[(2 \times 0.601) \times 60 \times (19.6 - 3.18) + (9 \times 0.785 - 8 \times 0.601) \times 60 \times \left(19.6 - \frac{5.31}{2}\right)\right]$$

$$\boxed{Md = 6328.38}$$

As $6328.38 > 5800$ → So design is OK!

QUESTION - 06

A beam is revised to developed and Ultimate moment of 6000 kip-inches limited to 14 x 26 inch size, use $f'_c = 4$ ksi and $f_y = 60$ ksi. Determine flexural reinforcement assume two rows of tensile reinforcement and effective depth of beam is 22 inches.

Sol:-

Given :-

Breadth (b) = 14"

Height (h) = 26"

Concrete Compression strength (f'_c) = 4 ksi

Steel Tensile strength (f_y) = 60 ksi

Ultimate Factored Moment (M_u) = 6000 kip-inches

Effective depth of beam (d) = 22"

Assume Effective Cover (d') = 2.5"

STEP #1 (Reinforcement Ratio)

By formula,

$$\rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \times \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$$= 0.85 \times 0.85 \times \frac{4}{60} \times \left(\frac{0.003}{0.003 + 0.005}\right)$$

$$\boxed{\rho_{max} = 0.0180}$$

STEP #2 :- (Area of Steel)

As we know that,

$$\rho_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = \rho_{max} \times (b \times d)$$

$$\Rightarrow A_{st} = 0.0180 \times (14 \times 22) = \boxed{5.54 \text{ in}^2}$$

Step #3 :- (Design Moment) :-

By using formula

$$M_{u_2} = \phi \times A_{st} \times f_y \times (d - a/2)$$

$$\Rightarrow \quad a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \frac{5.54 \times 60}{0.85 \times 4 \times 14} = \boxed{6.98''}$$

So,

$$M_{u_2} = 0.90 \times 5.54 \times 60 \times \left(22 - \frac{6.98}{2}\right)$$

$$= 5537.4 \text{ kip-inch}$$

As,

$$5537.4 < 6000$$

So we have to design a section as doubly reinforced.

Step # 4 :- (Difference In Moments)

$$M_{u_1} = M_u - M_{u_2}$$

$$= 6000 - 5537.4$$

$$\boxed{M_{u_1} = 462.6 \text{ kip-inches}}$$

Step # 5 :- (Area of Steel)

$$M_{u_1} = \phi \times A'_{st} \times f_y \times (d - d')$$

So Area of steel in Compression Zone will be,

$$\Rightarrow \quad A'_{st} = \frac{M_{u_1}}{\phi \times f_y \times (d - d')} = \frac{462.6}{0.90 \times 60 \times (22 - 2.5)}$$

$$\Rightarrow \quad \boxed{A'_{st} = 0.44 \text{ in}^2}$$

Step # 6 :- (Total Steel Area)

$$A_s = A_{st} + A'_{st}$$
$$= 5.54 + 0.44 \quad = 5.98 \text{ in}^2$$

Step # 7 :- (Selection & No. of bars Used)

1 – Steel in Tension Zone :-

we use # 7 bar,

dia = $(7/8)'' = 0.875''$, Area = $\frac{\pi}{4}(0.875)^2$
$= 0.601 \text{ in}^2$

So,

$$\text{No. of bars} = \frac{A_s}{\text{Area of single bar}}$$
$$= \frac{5.98}{0.601} = 9.9 \approx 10 \text{ bars}$$

So 10 # 7 bars.

2 – Steel in Compression Zone :-

we use # 5 bar,

dia = $(5/8)'' = 0.625''$, Area = $\frac{\pi}{4}(0.625)^2$
$= 0.306 \text{ in}^2$

So,

$$\text{No. of bars} = \frac{A'_{st}}{\text{Area of single bar}}$$
$$= \frac{0.44}{0.306} = 1.43 \approx 2 \text{ bars}$$

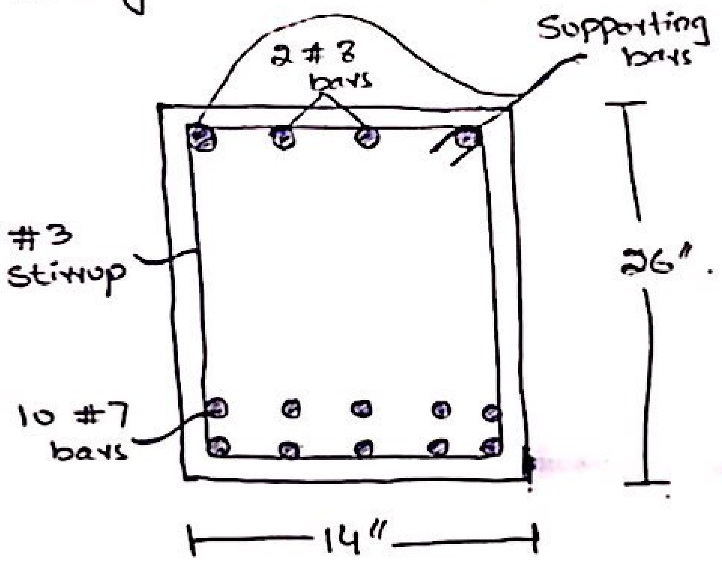
So 2 # 5 bars

STEP # 8 :- (Minimum Width of Beam)

$b_{min} = 2(1.5) + 2(3/8) + 10(7/8) + 9(7/8)$

$b_{min} = 20.37 > 14''$

So not good in one Layer.



Now,

=> Effective depth $(d) = 26 - 1.5 - 3/8 - 7/8 - 1/2(7/8)$

$= 22.82''$

=> Effective Cover $(d') = 1.5 + 3/8 + 1/2(5/8)$

$= 2.18''$

STEP # 9 :- (Design Moment)

$M_d = \phi \times [A'_{st} \times f_y \times (d - d') + (A_{st} - A'_{st}) \times f_y \times (d - a/2)]$

$$a = \frac{(A_{st} - A_{st}') \times f_y}{0.85 \times f'_c \times b}$$

$$= \frac{(10 \times 0.601 - 2 \times 0.306) \times 60}{0.85 \times 4 \times 14} = 6.80''.$$

$M_d = 0.90[(2 \times 0.306) \times 60 \times (22.82 - 2.18) + (10 \times 0.601 - 2 \times 0.306) \times 60 \times (22.82 - 6.80/2)]$

$M_d = 7047.6$ kip - inches

As $7047.6 > 6000$

Design is OK!