Date: 25/9/2020

ID# 16076 SECTION: 'A'
Submitted to: MA'M Shomaila
Subject: APPlied Calculus.

Q1: Find PQ were P is the point in three dimensional space with co-ordinates (4,1,3) and the point Q with co-ordinates (1,2,4). Find the distance b/w P and Q. Further, find the position vector of the point dividing PQ in the ratio 1:3.

Sol: Given that,

cordintes of $P = (4,1,3)$

OR, $\overrightarrow{OP} = 4i + j + 3k$

and $\overrightarrow{OQ} = i + 2j + 4k$

$PQ = OQ - OP$

$= (i+2j+4k) - (4i+j+3k)$

$= -3i + j + k \longrightarrow (\star)$

Now distance b/w

P and Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9+1+1}$

$\sqrt{11} \longrightarrow (\star\star)$

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Let M be the point which divided PQ in ratio 1:3
Then by ratio theoram.

Position vector of $M = \vec{OM}$

$$= \frac{3(4i + j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad (\star\star\star)$$

Ans.

Hence, eq($\star$), eq($\star\star$) and eq($\star\star\star$) are the required solution.

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Q: Evaluate $\int \frac{4x^3+10x+4}{2x^2+x}$

Sol: Given integration

$$\int \frac{4x^3+10x+4}{2x^2+x} \rightarrow (\star)$$

first we decompose the integrand in eq(★) by Partial fraction.

So,

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \pm \cancel{4x^3} \quad \pm 2x^2 \\ \hline & -\cancel{2x^2}+10x+4 \\ & \pm \cancel{2x^2} \mp x \\ \hline & 11x+4 \end{array}$$

Thus eq(★) becomes,

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int\left(2x-1+\frac{11x+4}{2x^2+x}\right)dx$$

$$= \int 2x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

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$$\frac{\cancel{2}x^2}{\cancel{2}} - x + \int \frac{11x+4}{2x^2+x}\,dx \Rightarrow (\#)$$

Decompose $\frac{11x+4}{2x^2+x}$ in eq (#)

Partialy to get $\frac{4}{x} + \frac{3}{2x+1}$

Putting in eq (#)

$$\int \frac{4x^3+10x+4}{2x^2+x} = x^2 - x + \int \frac{4}{x}dx + \int \frac{3}{2x+1}dx$$

$$\int \frac{4x^3+10x+4}{2x^2+x} = x^2 - x + 4\ln x + \frac{3}{2}\int \frac{2}{2x+1}dx$$

Ans:

$$= x^2 - x + 4\ln x + \frac{3}{2}\ln(2x+1) + c$$

is the required solution.

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Q3a): $\int_0^2 x^2 e^x dx$

Sol: let ist function $= u(x) = x^2$ and Second function $v = e^x$ in the given integral

Then $u'(x) = 2x$ and $\int v = e^x$
Using integration by Parts,
i.e $\int uv = u\int v - \int (u' \int v)$
Thus,

$$\int_0^2 x^2 e^x dx = x^2 e^x \Big|_0^2 - \int_0^2 2xe^x dx$$

$$= (2^2e^2 - (0)^2e^0) - 2\int_0^2 xe^x dx \rightarrow (\star)$$

Again using integration by Parts in eq($\star$), so, that $u(x) = x$, $u'(x) = 1$, $v = e^x$ and $\int v = e^x$

~~So, that u(x) =~~

Hence eq($\star$) becomes,

$$\int_0^2 x^2 e^x dx = 4e^2 - 2\left[xe^x\Big|_0^2 - \int_0^2 (1)e^x dx\right]$$

$$= 4e^2 - 2\left[(2e^2 - (0)e^0) - e^x\Big|_0^2\right]$$

$$= 4e^2 - 2[(2e^2 - (e^2 - e^0)]$$

$$= 4e^2 - 2(2e^2 - e^2 + 1)$$

$$= \cancel{4e^2} - \cancel{4e^2} + 2e^2 + 1 = 2e^2 + 1$$ Ans.

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③ b) : $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Sol: Given Integral,

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx \Rightarrow (\#)$$

Let $\sqrt{x} = u \Rightarrow \frac{1}{2\sqrt{x}} = \frac{du}{dx} \Rightarrow \frac{1}{\sqrt{x}}\,dx = 2du$

So eq (#) becomes,

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_{\sqrt{1}}^{\sqrt{2}} \sin u \cdot 2du$$

$$= 2\int_1^{\sqrt{2}} \sin u\,du$$

$$= 2(-\cos u)\Big|_1^{\sqrt{2}}$$

$$= 2(-1)(\cos\sqrt{2} - \cos(1))$$

$$= -2\cos\sqrt{2} + 2\cos 1$$

Ans

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Q4: $U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

Sol:- $U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \rightarrow (\star)$

Three dimension Laplace's equation

$$\frac{\partial^2 (f)}{\partial x^2} + \frac{\partial^2}{\partial y^2}(f) + \frac{\partial^2}{\partial z^2}(f) = 0 \rightarrow (\#)$$

Now from eq $(\star)$,

$$\frac{\partial}{\partial x}(u(x,y,z)) = \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= -x(x^2+y^2+z^2)^{-3/2}$$

Again taking $\frac{\partial}{\partial x}$

$$\frac{\partial^2}{\partial x^2}(u(x,y,z)) = \frac{\partial}{\partial x}\left(-x(x^2+y^2+z^2)^{-3/2}\right)$$

$$= -\left[x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}(1)\right]$$

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$$= \frac{3x^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}} \longrightarrow (A')$$

Taking $\frac{\partial}{\partial y}$ of eq ($\star$),

$$\frac{\partial}{\partial y}(u(x,y,z)) = \frac{\partial}{\partial y}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$= -y(x^2+y^2+z^2)^{-3/2}$$

Again taking $\frac{\partial}{\partial y}$

$$\frac{\partial^2}{\partial y^2}(u(x,y,z)) = \frac{\partial}{\partial y}\left(-y(x^2+y^2+z^2)^{-3/2}\right)$$

$$= -\left[y\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}(1)\right]$$

$$= \frac{3y^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}} \longrightarrow (B)$$

Taking $\frac{\partial}{\partial z}$ of eq ($\star$)

$$\frac{\partial}{\partial z}(u(x,y,z)) = \frac{\partial}{\partial z}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$$

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$$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$= -z(x^2+y^2+z^2)^{-3/2}$$

Again taking $\frac{\partial}{\partial z}$

$$\frac{\partial^2}{\partial z^2}(u(x,y,z)) = \frac{\partial}{\partial z}\left(-z(x^2+y^2+z^2)^{-3/2}\right)$$

$$= -\left[z\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2z) + (x^2+y^2+z^2)^{-3/2}(1)\right]$$

$$= \frac{3z^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}} \longrightarrow (C)$$

Putting (A), (B) and (C) in eq (#),

we get

$$\frac{3x^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)} + \frac{3y^2}{(x^2+y^2+z^2} - \frac{1}{(x^2+y^2+z^2)^{3/2}} + \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}$$

$$- \frac{1}{(x^2+y^2+z^2)^{3/2}}$$

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$$\frac{3x^2+3y^2+3z^2}{(x^2+y^2+z^2)^{5/2}} - \frac{3}{(x^2+y^2+z^2)^{3/2}} = 0$$

$$\frac{3\cancel{(x^2+y^2+z^2)}}{(x^2+y^2+z^2)^{3/2}\cancel{(x^2+y^2+z^2)}} - \frac{3}{(x^2+y^2+z^2)^{3/2}} = 0$$

$$\frac{3}{(x^2+y^2+z^2)^{3/2}} - \frac{3}{(x^2+y^2+z^2)^{3/2}} = 0$$

$$0 = 0$$

True

Hence the given eq (A) satisfie the three dimensional Laplaces Equation.

x —— x —— x —— x

"END"