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SUBJECT	Advanced Mechanics of Materials
PROGRAMME	M.S (STRUCURE ENGINNERING)
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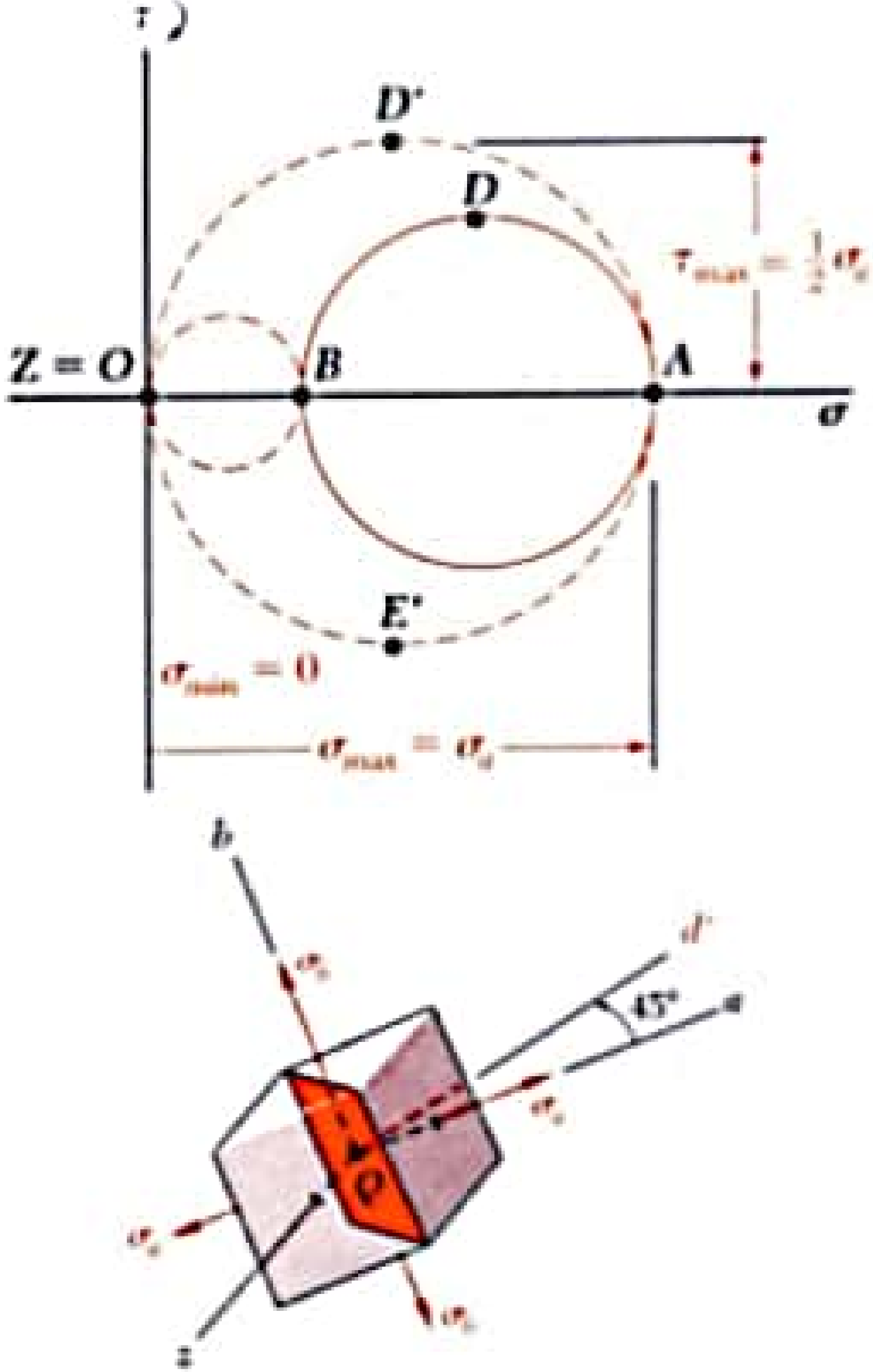
SESSIONAL ASSIGNMENT

1. Application of Mohr's Circle to the three Dimensional Analysis of stress:

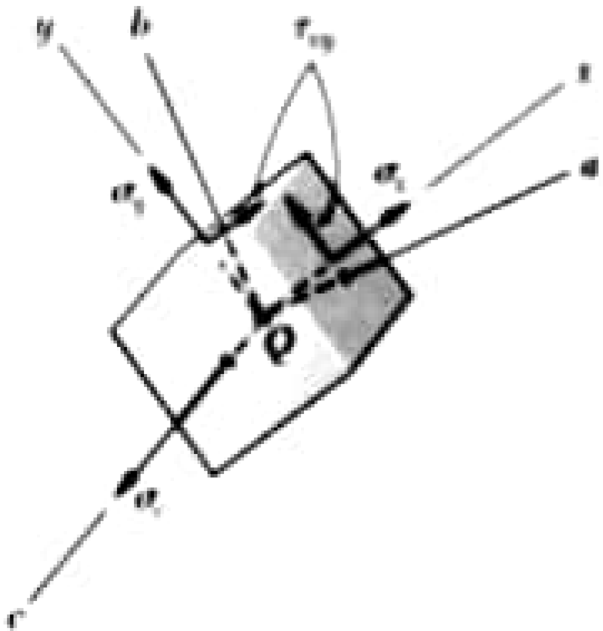


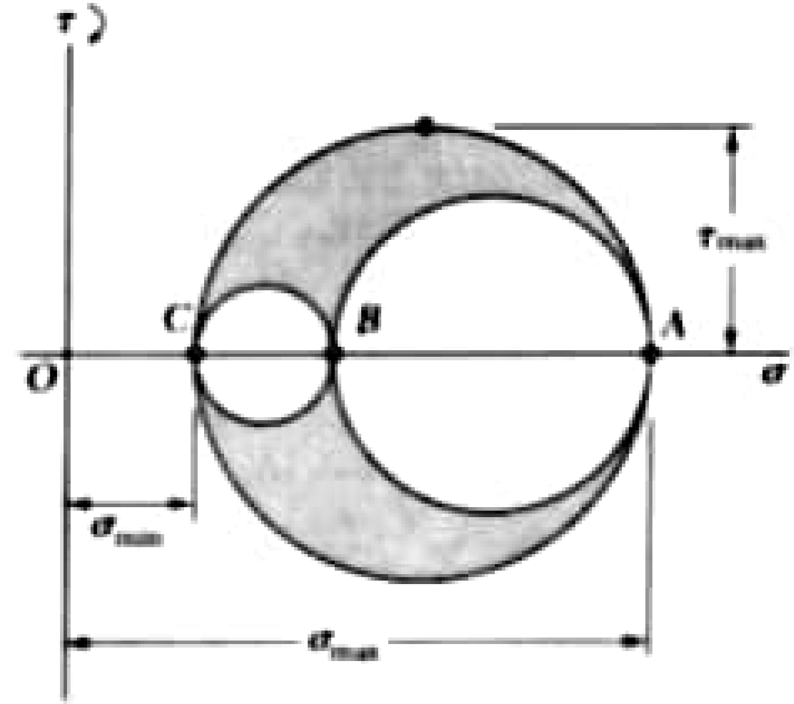
- If A and B are on the same side of the origin (i.e., have the same sign), then

  a) the circle defining σ_{max}, σ_{min}, and τ_{max} for the element is not the circle corresponding to transformations within the plane of stress

  b) maximum shearing stress for the element is equal to half of the maximum stress

  c) planes of maximum shearing stress are at 45 degrees to the plane of stress





- Transformation of stress for an element rotated around a principal axis may be represented by Mohr's circle.
- Points *A*, *B*, and *C* represent the principal stresses on the principal planes (shearing stress is zero)

- The three circles represent the normal and shearing stresses for rotation around each principal axis.
- Radius of the largest circle yields the maximum shearing stress.

$$\tau_{max} = \frac{1}{2}\left|\sigma_{max} - \sigma_{min}\right|$$

3.SIMPLE BENDING & PURE BENDING

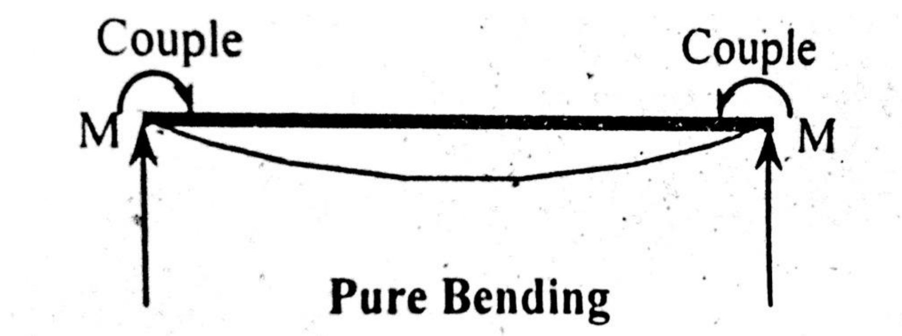
Bending stresses are those that bend the beam because of beam self-load and external load acting on it.

Bending stresses are of two types;

1. Pure Bending
2. Simple Bending

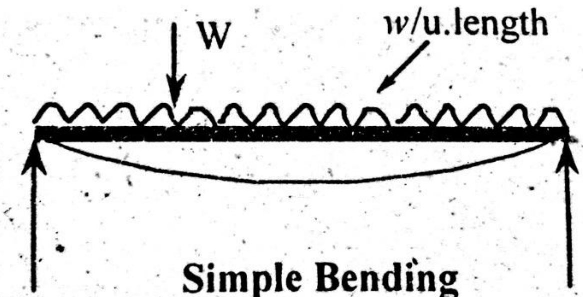
Pure Bending:

Bending will be called as pure bending when it occurs solely because of coupling on its end. In that case there is no chance of shear stress in the beam. But, the stress that will propagate in the beam as a result will be known as normal stress. Normal stress because it not causing any damages to beam. As shown below in the picture.



Simple Bending:

Bending will be called as simple bending when it occurs because of beam self-load and external load. This type of bending is also known as ordinary bending and in this type of bending results both shear stress and normal stress in the beam. As shown below in the figure.



Simple Bending

ASSUMPTIONS MADE IN THEORY OF PURE BENDING:

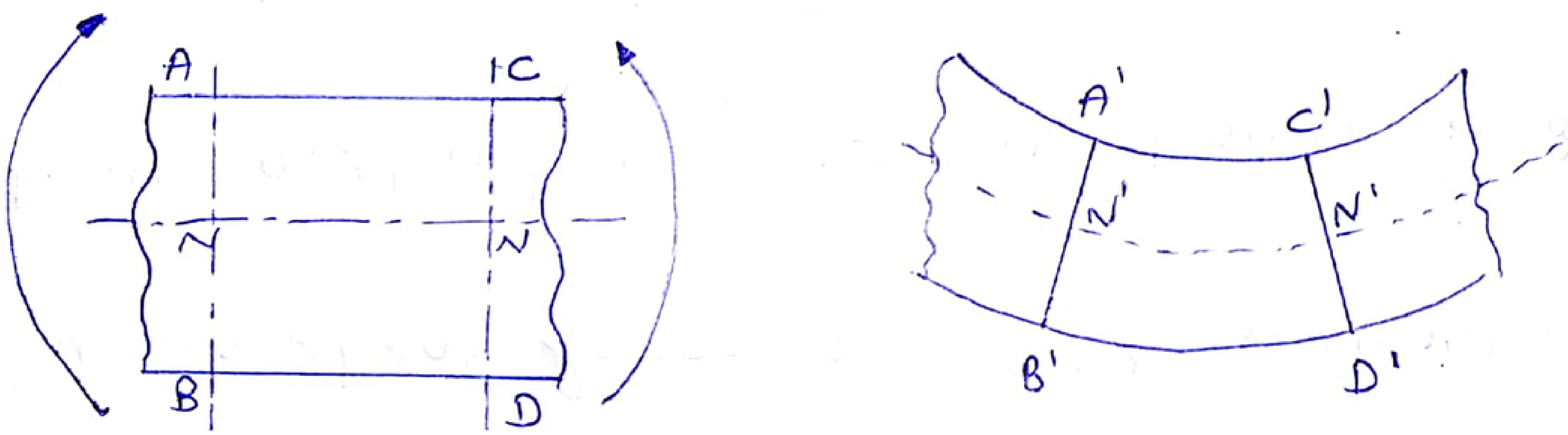
1. The material of the beam is Homogeneous and Isotropic.

Homogeneous → Material is of same kind througout.

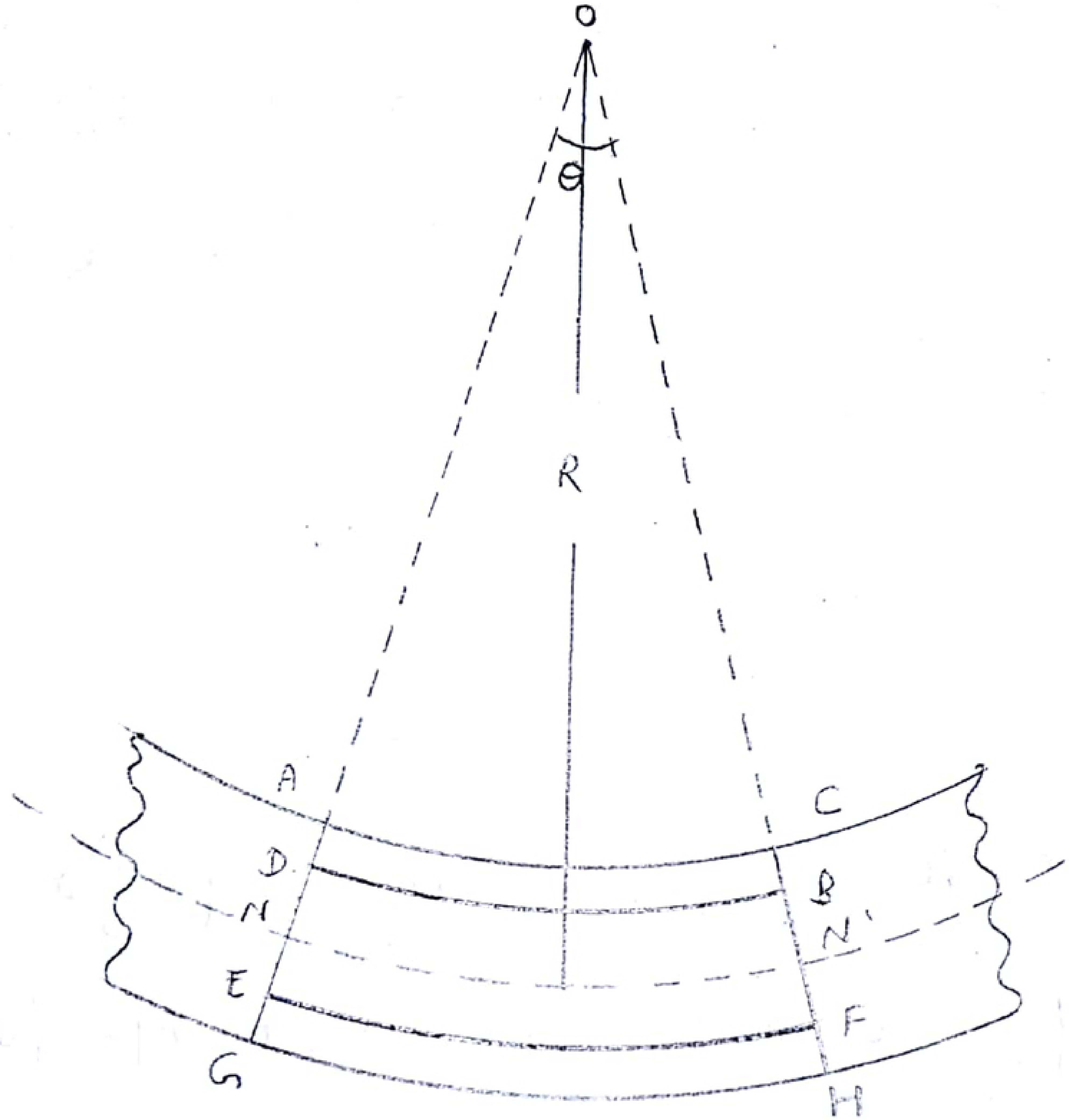
Isotropic → Same Elastic properties in all directions.

2. The value of Young's Modulus of elasticity is same in tension as well as in compression.

3. The transverse section of beam which were plane before bending, remain plane after bending also.



4. The Beam is initially straight and all longitudinal filaments bend into circular arcs with common centre of curvature.



5. The radius of curvature is large as compared with dimension of cross-section.

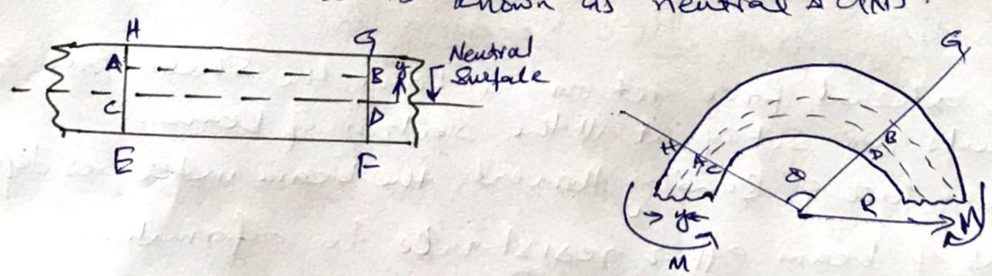
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

5) Classic Flexure Equation.

Following are the assumption made before the derivation of a bending equation

* The beam used is straight with a constant cross-section
* The beam used is of homogeneous material with a symmetrical longitudinal plane.
* The plan of symmetry has all the resultant of applied loads.
* The primary cause of failure is buckling.
* E remains same for tension & compression.
* Cross-section remains the same before & after bending.

Consider an unstressed beam, which is subjected to a constant bending moment such that the beam bends up to radius R. The top fibres are subjected to tension whereas the bottom fibres are subjected to compression the locus of point with zero stress is known as neutral axis.



with the help of above fig the following are the step involve in the derivation of flexure eq

$$\text{Strain in fibre } AB = \frac{A'B' - AB}{AB}$$

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = y/R \quad \frac{\sigma}{E} = y/R$$

Where E is Young's Modulus of Elasticity.

or $\sigma/y = E/R$

$\sigma = E/R\, y$ (eq. 1)

$F = \sigma \delta A = E/R\, y \delta A$ (Force acting on the strip with area dA)

$Fy = E/R\, y^2 \delta A$ (Momentum about Neutral axis).

$M = \sum E/R\, y^2$ (total momentum for entire cross-sectional area)

$\delta A = E/R \sum y^2 \delta A$ $\sum y^2 \delta A$ is known as second moment of area & is represented as I.

$\therefore M = E/R\, I$ (eq 2)

From Eq 1 & 2

$\sigma/y = M/I = E/R$

Therefore the above is the flexure theory Equation

6. SECTION MODULUS:

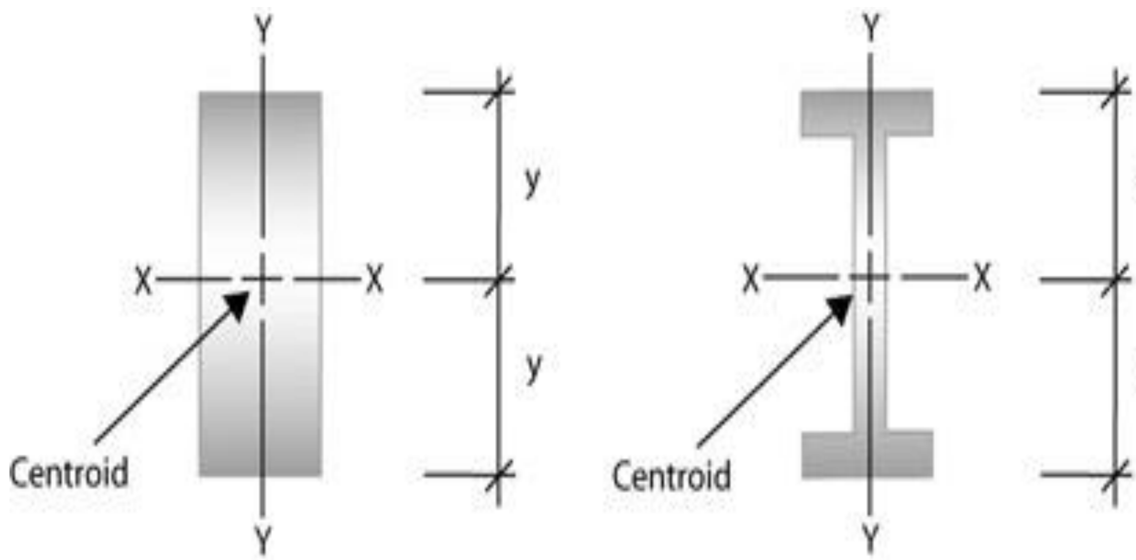
The moment carrying capacity of an object is directly dependent on geometrical property (I) and material property (E) of an object, which is collectively termed as flexural rigidity(EI).Geometry of an object plays an important role in load bearing capacity of an object which is indicated by moment of inertia of a section. Therefore section modulus is the predominant factor which evidences the strength of an object and is defined as the ratio of the moment of inertia of the object about its centroidal axis to the distance of the extreme fibers of the object from the neutral axis.

Section modulus is generally denoted by Z.
Therefore , $Z = I / Y_{max}$
where, I = Moment of inertia of a section.
Y_{max} = Distance of the outer most fiber of the object from the neutral axis.

Section modulus can also be defined by using the simple bending theory as,
we know that, $M / I = \sigma / Y$
Therefore, $Z = M / \sigma$
i.e, section modulus is also expressed as the ratio of bending moment to the bending stress of a given object within the elastic limit.

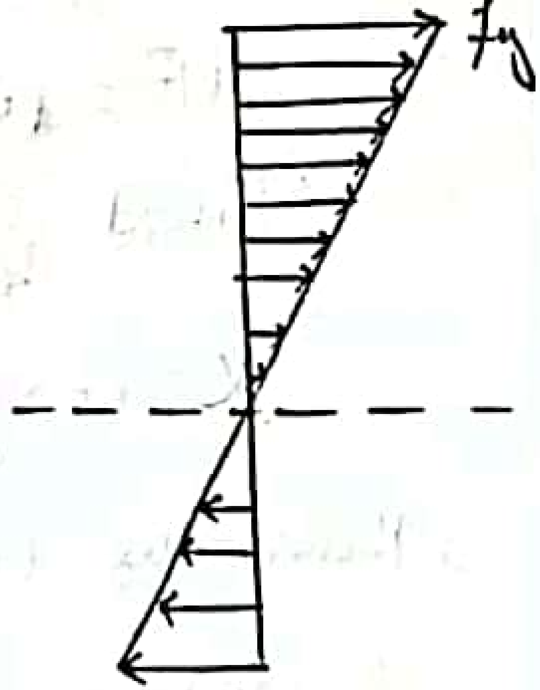
Significance of section modulus

I. Section modulus is the important factor for design of beam and flexural member
II. Higher the value of section modulus, higher will be the resistance of member to bending
III. It is required to calculate stresses in beams.
IV. It is used to calculate strength of the steel structure
V. More the section modulus, it can withstand more load and it is also considered to be more tougher.



7) Application of Bending Equation in any Object.

When you try to break a wood with your foot the breaking occurs due to bending moment. You push the wood from middle point & apply a force perpendicular to allignment of the wood.



Elastic linear stress distribution.

Fig for Question No. 6

Slowly applying the force initially the wood bends. when it bends, wood is Subjected to bending moment due to its inner forces which happen to be compression (closer to your foot), Tension (further to your foots) in this Case. The forces are aligned with the longitudinal axis of the wood. They create the moment in the following way.

Force x radius = Bending Moment.

When the bending moment on the Part Subjected to tension exceeds the Stress of the material, breaking occurs

8. MOMENT OF RESISTANCE:

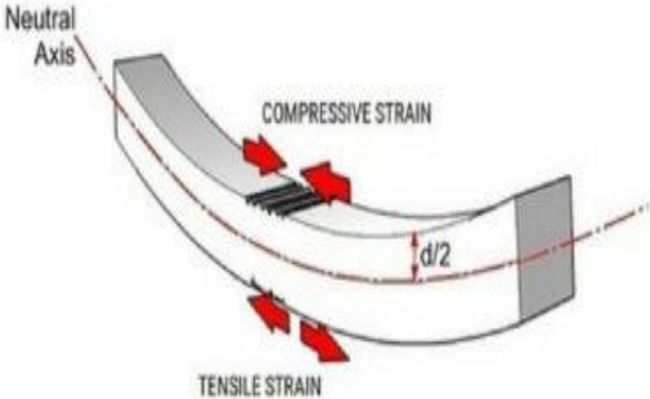
The algebraic sum of moments of the internal forces (compressive and tensile forces developed in the cross-section due to bending) about the neutral axis of the section is called the moment of resistance of the section. For equilibrium condition, the moment of resistance of a section will be equal to the applied bending moment at that section. In equilibrium condition,

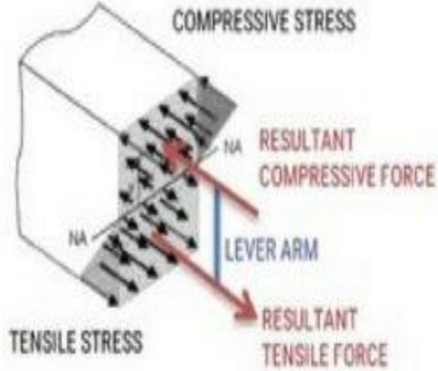
Bending moment = Moment of resistance

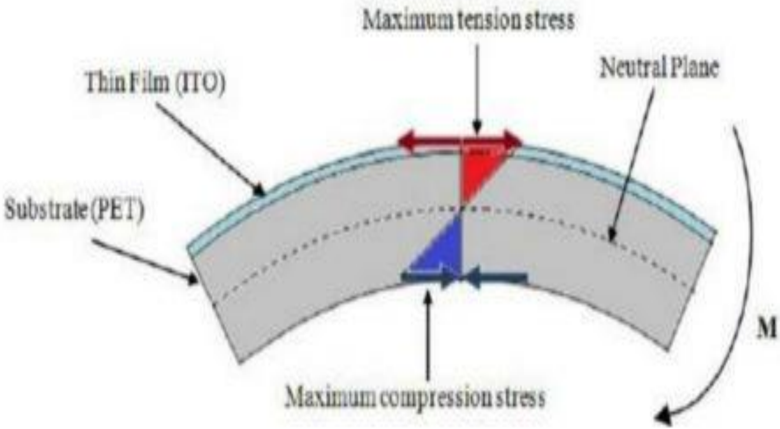
$M/I = \sigma/y$

$M = \sigma Z$ --------------($Z = I/y$)

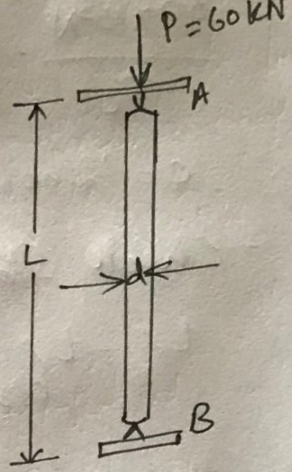
The moment of resistance of a section corresponding to the maximum permissible stresses in the material is called the limiting moment of resistance of the section. This indicates the maximum bending moment that could be resisted by the section without the stresses exceeding the permissible values.


Neutral
Axis
COMPRESSIVE STRAIN
d/2
TENSILE STRAIN


COMPRESSIVE STRESS
NA
RESULTANT
COMPRESSIVE FORCE
NA
LEVER ARM
RESULTANT
TENSILE FORCE
TENSILE STRESS


Maximum tension stress
Neutral Plane
Thin Film (ITO)
Substrate (PET)
M
Maximum compression stress

9) Design of Column under Centric load



Using the aluminum alloy 2014-T6, determine the smallest diameter rod which can be used to support the centric load $P = 60\,kN$ if a) $L = 750\,mm$ b) $L = 300\,mm$

Solution

→ With the diameter unknown, the slenderness ratio can not be evaluated. Must make an assumption on which slenderness ratio regime to utilize.

→ Calculate required diameter for assumed slenderness ratio regime.

→ Evaluate slenderness ratio and verify initial assumption.

Repeat if necessary

→ For $L = 750\,mm$, assume $L/r > 55$

c = cylinder radius

r = radius of gyration

$$= \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi c^4/4}{\pi c^2}} = c/2$$

- Determine cylinder radium

$$\sigma_{all} = P/A = \frac{372 \times 10^3\,MPa}{(L/r)^2}$$

$$\frac{60 \times 10^3\,N}{\pi c^2} = \frac{372 \times 10^3\,MPa}{\left(\frac{0.750\,m}{c/2}\right)^2}$$

$c = 18.44\,mm$

- Check slenderness ratio assumption.

$$L/r = L/c/2 = \frac{750\,mm}{(18.44\,mm)} = 81.3 > 55$$

assumption was correct :

$$\boxed{d = 2c = 36.9\,mm}$$

- For $L = 300\,mm$, assume $L/r < 55$
- Determin cylinder radius

$$\sigma_{all} = P/A = \left[212 - 1.585\left(\frac{L}{r}\right)\right] MPa$$

$$\frac{60 \times 10^3 N}{\pi c^2} = \left[212 - 1.585\left(\frac{0.3\,m}{c/2}\right)\right] \times 10^6 Pa$$

$$c = 12.00\,mm$$

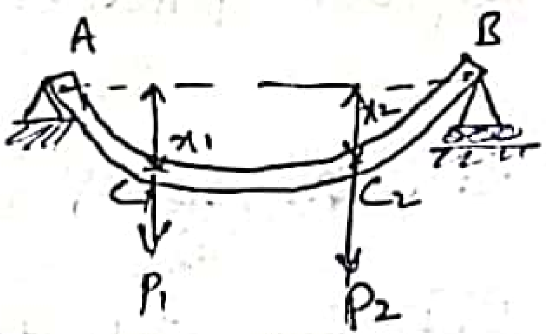
- Check slenderness ratio assumption.

$$L/r = L/_{c/2} = \frac{300\,mm}{(12.00\,mm)} = 50 < 55$$

assumption was correct.

$$\boxed{d = 2c = 24.0\,mm}$$

10) Deflection by Castigliano's Theorem.



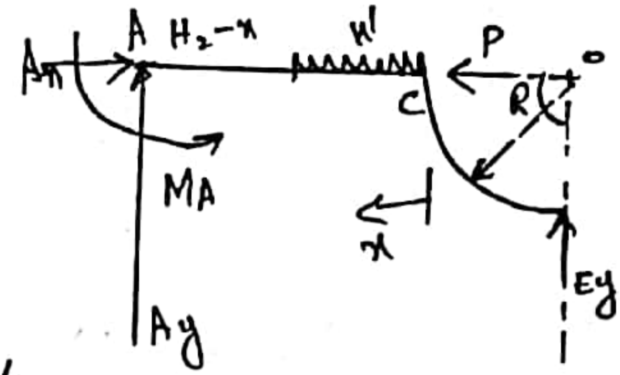
- Application of Castigliano's theorem is simplified if the differentiation with respect to the load P_j is performed before the integration or summation to obtain the strain energy U.

  - In the case of beam.

$$U = \int_0^L \frac{M^2}{2EI}\,dx \qquad x_j = \frac{\partial U}{\partial P_j} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P_j}\,dx.$$

<u>Example</u>: The given beam consist of straight beam AC whe A is fixed. & curved beam CE load P is horizontal. Draw free body diagram & moment equation for AC & CE part, express redundant at E by using Castigliano's theorem. for deflection.

Sol 1. ⓐ PDP



A_y and E_y are vertical reaction

A_x - horizontal & M_A moment of A

ⓑ In EC,
$$M_{EC} = E_y \cdot R \cos\theta \, . \quad 0 \le \theta \le \pi/2$$

at E,

In AC,
$$M_{CA} = E_y (R + x) - \frac{wx^2}{2} \quad \text{for } x \le L/2$$
$$M_{CA} = E_y (R + x) - \frac{wL}{8} \times (x - L/4) \quad \text{for } L/2 \le x \le L$$

ⓒ Considering E_y as redundant, strain energy stored in beam

$$U = U_{EC} + U_{CA} = \int_E^A \frac{\left(E_y (R + x) - \frac{wL}{8}(x - L/4)\right)^2}{2EI} dx + \int_0^{L/2} \frac{\left(E_y (R + x) - \frac{wx^2}{2}\right)^2}{2EI} dx$$

$$+ \int_{\pi/2}^{0} \frac{E_y^2 \cdot R^2 \cos\theta}{2EI} \cdot R\, d\theta = \int_E^A \frac{M^2}{2EI} ds$$

:- Note: only Bending Energy Considered.

once we find U after Integration use the Compatibility eq to find support/redundant of E

Def at E, $\delta_E = \frac{dU}{dE_0}$; But $\delta_E = 0$; so

Put $\frac{\partial U}{\partial E_y} = 0$ to find E_y