Q. 1:-

The function $g(t)$ is defined by:

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4. \end{cases}$$

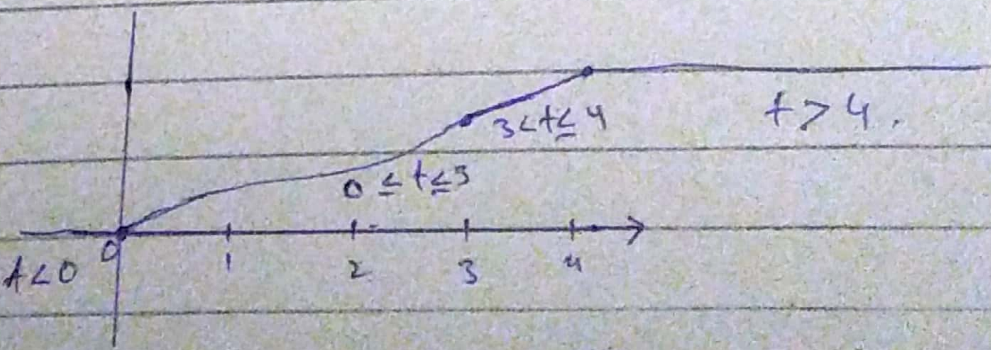
(a) State any point of discontinuity.

(b) Find, if they exist.

(i) $\lim_{t \to 3} g$.

Sol. (a):

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4. \\ 12 & t > 4. \end{cases}$$



From the graph, we conclude that the function $g(x)$ is continuous at every point. There is no point of discontinuity.

Part ① b

Sol:

$$g(x) = \begin{cases} t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t < 4 \end{cases}$$

R.H.S:

$$\lim_{t \to 3} g(x) = \lim_{t \to 3} (2t+3)$$

$$= 2(3)+3$$

$$= 9$$

L.H.S

$$\lim_{t \to 3} g(x) = \lim_{t \to 3} (t^2)$$

$$(3)^2 =$$

$$9.$$

So L.H.S = R.H.S.

So limits exist.

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Q.2 Find the Maclaurin's Series for $y(x) = x^2 + \sin x$.

Sol: $f(x) = x^2 + \sin x$:

Maclaurin's series

$f(x) = x^2 + \sin x$ $\qquad f(0) = 0$

$f'(x) = 2x + \cos x$ $\qquad f'(0) = 1$.

$f''(x) = 2 - \sin x$ $\qquad f''(0) = 2$.

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$.

$f^{iv}(x) = \sin x$ $\qquad f^{iv}(0) = 0$

$f^{v}(x) = \cos x$ $\qquad f^{v}(0) = 1$

$f^{vi}(x) = -\sin x$ $\qquad f^{vi}(0) = 0$.

$f^{vii}(x) = -\cos x$. $\qquad f^{vii}(0) = -1$

$f^{viii}(x) = \sin x$. $\qquad f^{viii}(0) = 0$.

Maclaurine Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0) + \frac{x^6}{6!} f^{vi}(0) + \frac{x^7}{7!} f^{vii}(0) + \cdots$$

$$= x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \cdots$$

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Q3 Part 1:

Find y'' given

$1 + xy = x^2 + y^2$.

Sol: $1 + xy = x^2 + y^2$.

Using implicit Differentiation w.r.t x.

$$0 + xy' + y.1 = 2x + 2yy'$$

$$xy' - 2yy' = 2x - y \quad \text{dividing } x-2y \text{ b.s.}$$

$$\frac{(\cancel{x-2y})\,y'}{\cancel{x-2y}} = \frac{2x-y}{x-2y}$$

$$y' = \frac{2x-y}{x-2y} \longrightarrow (1)$$

Differentiate again using quotient rule

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)\left(2 - \frac{2x-y}{x-2y}\right) - (2x-y)\left(1 - 2\,\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{(\cancel{x-2y})\left(\frac{2x-4y-2x+y}{\cancel{x-2y}}\right) - \frac{(2x-y)(x-2y-4x+2y)}{x-2y}}{(x-2y)^2}$$

$$= \frac{-3y - \frac{(2x-y)(-3x)}{x-2y}}{(x-2y)^2}$$

$$= \frac{-3y(x-2y) - \{-6x^2 + 3xy\}}{(x-2y)\,(x-2y)^2}$$

$$= \frac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)^2}$$

$$= \frac{-6xy + 6x^2 + 6y^2}{(x-2y)^2}$$

$$y'' = \frac{6(xy - x^2 - y^2)}{(x-2y)^2}$$

Ans

Q3 part 2:

Find y by using logarithmic differentiat

$y = x^3 (1+x)^9 e^{6x}$.

Sol:

$y = x^3 (1+x)^9 e^{6x}$.

natural log on both sides.

$\ln y = \ln [x^3 (1+x)^9 e^{6x}]$.

$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$

Differentiating on both sides.

$\frac{1}{y}(y') = \frac{3}{x} + \frac{9}{x+1} + 6$

$y' = y \left[\frac{3}{x} + \frac{9}{x+1} + 6\right]$.

$y = x^3 (1+x)^9 + e^{6x}$

$y' = \left[x^3 (1+x)^9 e^{6x}\right] \left[\frac{3}{x} + \frac{9}{x+1} + 6\right]$.

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