



[CALCULAS & ANALYTICAL GEOMETRY]

[Final Exam Paper]



JUNE 27, 2020

SUBMITTED BY SAAD ALI

[ID no: 16880]

[BS-SE]

SUBMITTED TO MUHAMMAD IBRAR KHAN

FINAL EXAM ASSIGNMENT

Q.1

- a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.
b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to x.
a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.

Q no. 1

a) $\frac{3x^4-2x^3+5}{x^3+1}$

Sol:-
Let $y = \frac{3x^4-2x^3+5}{x^3+1}$
Differentiate w.r.t x.

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{3x^4-2x^3+5}{x^3+1} \right]$$

Using Quotient Rule.

$$\frac{dy}{dx} = \frac{(x^3+1) \frac{d}{dx}(3x^4-2x^3+5) - (3x^4-2x^3+5) \frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$
$$= \frac{(x^3+1)(12x^3-6x^2+0) - (3x^4-2x^3+5)(3x^2+0)}{(x^3+1)^2}$$
$$= \frac{(x^3+1)(12x^3-6x^2) - (3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$
$$= \frac{12x^6-6x^5+12x^3-6x^2-9x^6+6x^5-15x^2}{(x^3+1)^2}$$
$$\frac{dy}{dx} = \frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$$

b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to x.

$$b) \frac{(x^3+1)^2}{x^3-1}$$

Soln- Let $y = \frac{(x^3+1)^2}{(x^3-1)}$

Differentiate w.r.t x.

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{(x^3+1)^2}{x^3-1} \right]$$

Using Quotient Rule.

$$\frac{dy}{dx} = \frac{(x^3-1) \frac{d}{dx} [(x^3+1)^2] - (x^3+1)^2 \frac{d}{dx} (x^3-1)}{(x^3-1)^2}$$

$$= \frac{(x^3-1) 2(x^3+1)(3x^2+0) - (x^3+1)^2 (3x^2-0)}{(x^3-1)^2}$$

$$= \frac{(x^3-1) 2(x^3+1)(3x^2) - (x^3+1)^2 (3x^2)}{(x^3-1)^2}$$

$$= \frac{(3x^2)(x^3+1) [2(x^3-1) - (x^3+1)]}{(x^3-1)^2}$$

$$= \frac{(3x^2)(x^3+1) [2x^3-1-x^3-1]}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(3x^2)(x^3+1)(x^3-2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(3x^5-6x^2)}{(x^3-1)^2}$$

Q.2

- a) Find the Integration of $\int \frac{1}{\sqrt{x^5}} dx$.
b) Find the Integration of $\int \frac{1}{(8x+7)^8} dx$.

- a) Find the Integration of $\int \frac{1}{\sqrt{x^5}} dx$.

Q.2
a) Find the Integration of $\int \frac{1}{\sqrt{x^5}} dx$.

Sol

$$\int \frac{1}{\sqrt{x^5}} dx$$
$$\Rightarrow \int \frac{1}{(x^5)^{1/2}} dx$$
$$\Rightarrow \int \frac{1}{x^{5/2}} dx$$
$$\Rightarrow \int x^{-5/2} dx$$
$$\Rightarrow \int x^a dx = \frac{x^{a+1}}{a+1} + C$$
$$\Rightarrow \frac{x^{-5/2+1}}{-5/2+1} + C$$
$$\Rightarrow \frac{x^{-3/2}}{-3/2} + C$$
$$\Rightarrow -\frac{2x^{-3/2}}{3} + C$$

b) Find the Integration of $\int \frac{1}{(8x+7)^8} dx$.

$$b) \int \frac{1}{(8x+7)^8} dx$$

Sol:

$$\int \frac{1}{(8x+7)^8} dx$$

$$\int (8x+7)^{-8} dx$$

$$\frac{1}{8} \int (8x+7)^{-8} (8) dx$$

$$\frac{1}{8} \frac{(8x+7)^{-8+1}}{-8+1} + C$$

$$\frac{1}{8} \frac{(8x+7)^{-7}}{(-7)} + C$$

$$= \frac{(8x+7)^{-7}}{56} + C$$

Q.3

- a) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6} dx$ by Partial fractions.
 b) Find the Integration of $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} dx$ by Partial fractions.

a) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6} dx$ by Partial fractions.

Q no: 3

a) Find $\int \frac{-x+9}{2x^2-8x+6} dx$ by Partial fraction

Sol:-

$$\int \frac{-x+9}{2x^2-8x+6} dx$$

Consider

$$\frac{-x+9}{2x^2-8x+6} = \frac{-x+9}{2x^2-6x-2x+6}$$

$$= \frac{-x+9}{2x(x-3)-2(x-3)}$$

$$= \frac{-x+9}{2x(x-3)-2(x-3)}$$

$$= \frac{-x+9}{(x-3)(2x-2)}$$

Let

$$\frac{-x+9}{(x-3)(2x-2)} = \frac{A}{x-3} + \frac{B}{2x-2} \rightarrow (*)$$

Multiplying and dividing both side by $(x-3)(2x-2)$

$$\Rightarrow -x+9 = A(2x-2) + B(x-3) \rightarrow (1)$$

Setting $x-3=0 \Rightarrow x=3$ in (1)

$$-3+9 = A(2(3)-2) + B(3-3)$$

$$6 = A(6-2) + 0$$

$$6 = A(4)$$

$$A = \frac{6}{4}$$

$$A = \frac{3}{2}$$

Setting $2x-2=0 \Rightarrow x=1$ in (1)

$$-1+9 = A(2(1)-2) + B(1-3)$$

$$8 = A(0) + B(-2)$$

$$8 = -2B$$

$$\Rightarrow B = -4$$

Now

$$\Rightarrow \frac{-x+9}{2x^2-8x+6} = \frac{3/2}{x-3} + \frac{-4}{2x-2}$$

Taking integration on both sides

$$\int \frac{-x+9}{2x^2-8x+6} dx = \int \frac{3}{2} \frac{dx}{x-3} - \int \frac{4}{2x-2} dx$$

$$= \frac{3}{2} \int \frac{1}{x-3} dx - 2 \int \frac{1}{x-1} dx$$

$$= \frac{3}{2} \ln|x-3| - 2 \ln|x-1| + C$$

b) Find the Integration of $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} dx$ by Partial fractions.

b) $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} dx$
 Sol:-
 Consider

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{4x^2+8x}{(x^2+1)(x^2+2x+3) \cdot x}$$
 Let

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$$
 Multiplying $(x^2+1)(x^2+2x+3)$ on both sides, we get

$$4x^2+8x = (Ax+B)(x^2+2x+3) + (Cx+D)(x^2+1)$$
 Comparing the co-efficients of x^3, x^2, x, x^0
 $x^3, \quad 0 = A+C \rightarrow (i)$
 $x^2, \quad 4 = 2A+B+D \rightarrow (ii)$
 $x, \quad 8 = 3A+2B+C \rightarrow (iii)$
 $x^0, \quad 0 = 3B+D \rightarrow (iv)$

$D = -3B \rightarrow (v)$
 Putting in (ii)
 $4 = 2A+B-3B$
 $4 = 2A-2B \rightarrow (vi)$
 $(i) \Rightarrow 0 = A+C \rightarrow$
 Adding (i) and (vi)
 $4 = 3A-2B+C$
 $8 = 3A+2B+C$
 $12 = 6A+2C \rightarrow (vii)$
 Subtract (vii) from 2(i)
 $0 = 2A+2C$
 $+12 = 6A+2C$
 $-12 = -4A$
 $A = 3$
 Now
 Putting $A=3$ in (i)
 $0 = 3+C$
 $C = -3$
 Putting $A=3$ in (vi)
 $4 = 6-2B$
 $-2 = -2B$
 $B = 1$

Putting $B=1$ in (v)

$$D = -3$$

$$\frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} = \frac{3x+1}{x^2+1} + \frac{(-3x-3)}{x^2+2x+3}$$

Taking integration on both sides

$$\int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} dx = \int \frac{3x+1}{x^2+1} dx - \int \frac{3x+3}{x^2+2x+3} dx$$

$$= 3 \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$- 3 \int \frac{x+1}{x^2+2x+3} dx$$

$$= \frac{3}{2} \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$- \frac{3}{2} \int \frac{2x+2}{x^2+2x+3} dx$$

$$= \frac{3}{2} \ln|x^2+1| + \tan^{-1}(x)$$

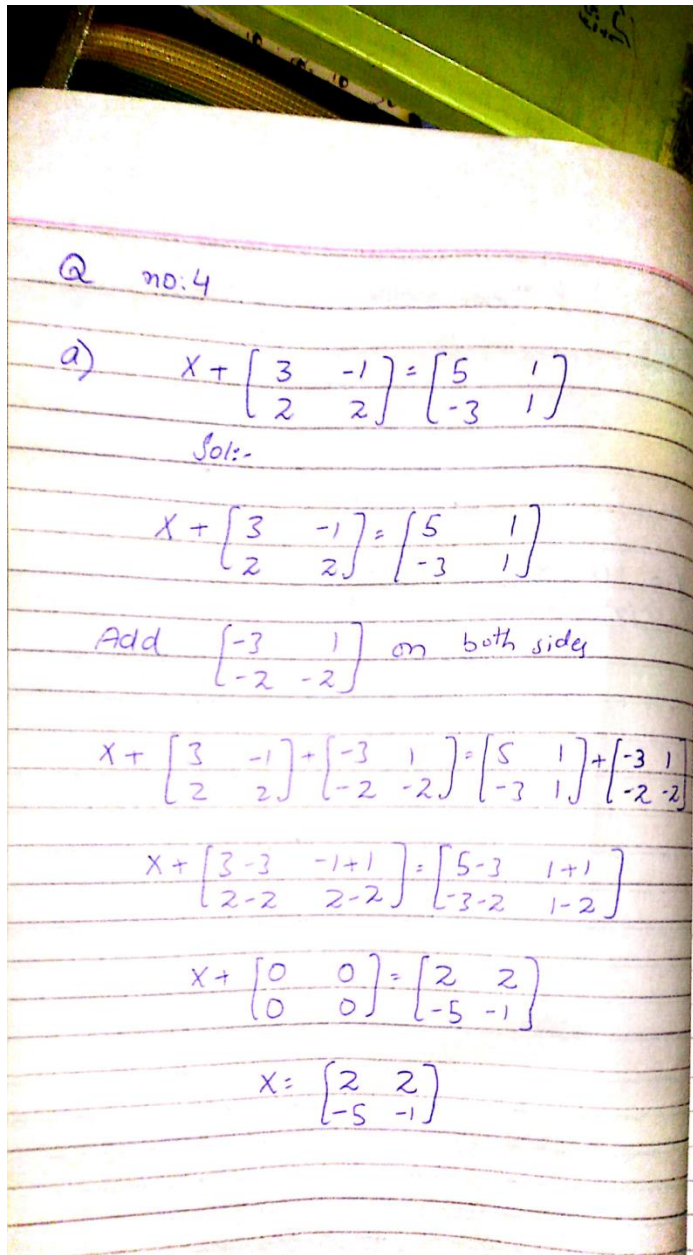
$$- \frac{3}{2} \ln|x^2+2x+3| + C$$

Q.4

Solve each of the following matrix equations:

- a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$
b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$
c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$



$$b) X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$b) X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

Sol:-

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5+0 \end{bmatrix}$$

Add $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ on both sides

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -8 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} -1+0 & 0+0 \\ 0+0 & 2+(-2) \end{bmatrix} = \begin{bmatrix} -2+1 & -2+0 \\ -1+0 & 5-2 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$c) X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$0) X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Sol.

$$X + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Add $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ on both sides

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2-2 & 0+0 \\ 0+0 & 2-2 \end{bmatrix} = \begin{bmatrix} 3-2 & -1+0 \\ 1+0 & 2-2 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

Q.5

a) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ Find $A^2 + BC$

Q no: 5

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Find $A^2 + BC$

Sol:-

$$\begin{aligned} A^2 &= A \cdot A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 4 \times 2 & 1 \times 4 + 4 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 4 + 1 \times 1 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 7 & 8 \\ 4 & 9 \end{bmatrix} \end{aligned}$$

Now

$$\begin{aligned} BC &= \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 \times 1 + 2 \times 0 & -3 \times 0 + 2 \times 2 \\ 4 \times 1 + 0 \times 0 & 4 \times 0 + 0 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} \end{aligned}$$

$$A^2 + BC = \begin{bmatrix} 7 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 4 & 12 \\ 8 & 9 \end{bmatrix}$$