[CALCULAS & ANALYTICAL GEOMETRY]

[Final Exam Paper]

JUNE 27, 2020

SUBMITTED BY SAAD ALI
[ID no: 16880]
[BS-SE]
SUBMITTED TO MUHAMMAD IBRAR KHAN

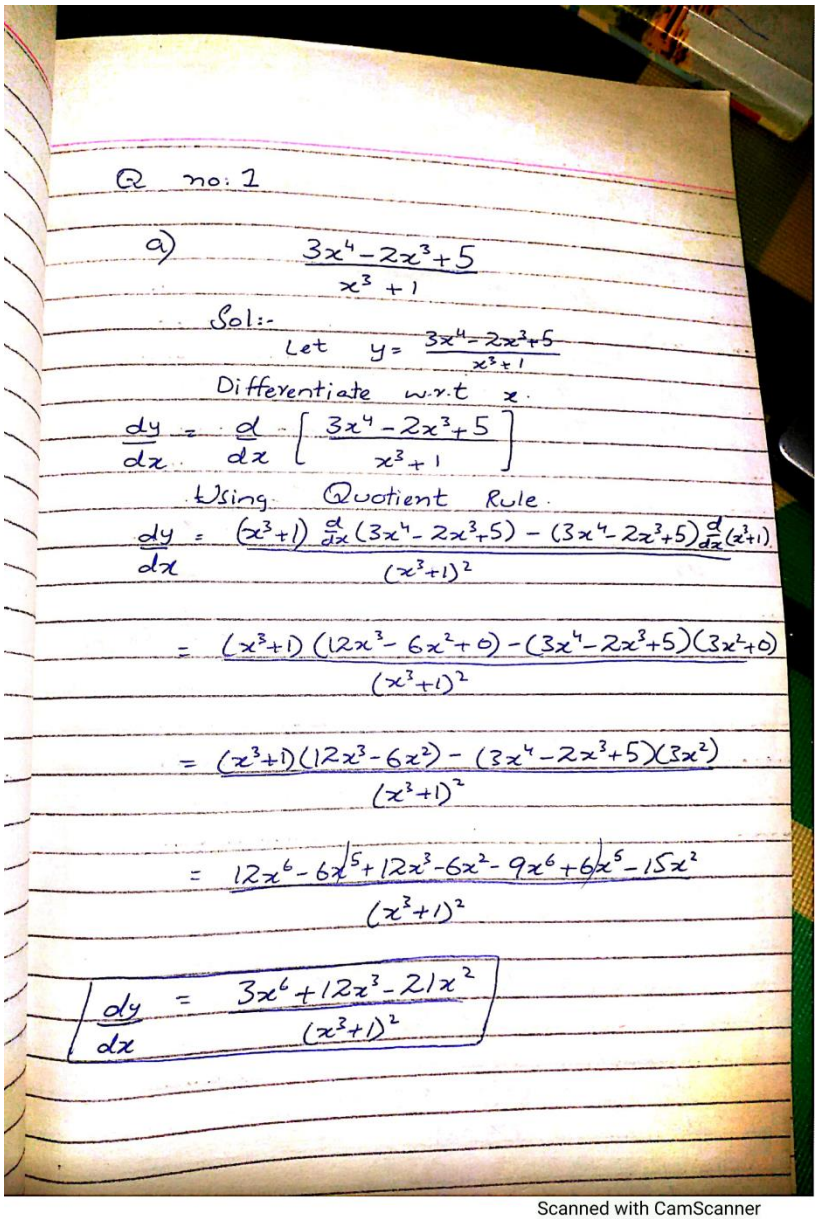
FINAL EXAM ASSIGMENT

Q.1

a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.

b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to x.

a) **Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to x.**



Q no: 1

a) $\frac{3x^4-2x^3+5}{x^3+1}$

Sol:-

Let $y = \frac{3x^4-2x^3+5}{x^3+1}$

Differentiate w.r.t x.

$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{3x^4-2x^3+5}{x^3+1}\right]$

Using Quotient Rule.

$\frac{dy}{dx} = \frac{(x^3+1)\frac{d}{dx}(3x^4-2x^3+5) - (3x^4-2x^3+5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$

$= \frac{(x^3+1)(12x^3-6x^2+0) - (3x^4-2x^3+5)(3x^2+0)}{(x^3+1)^2}$

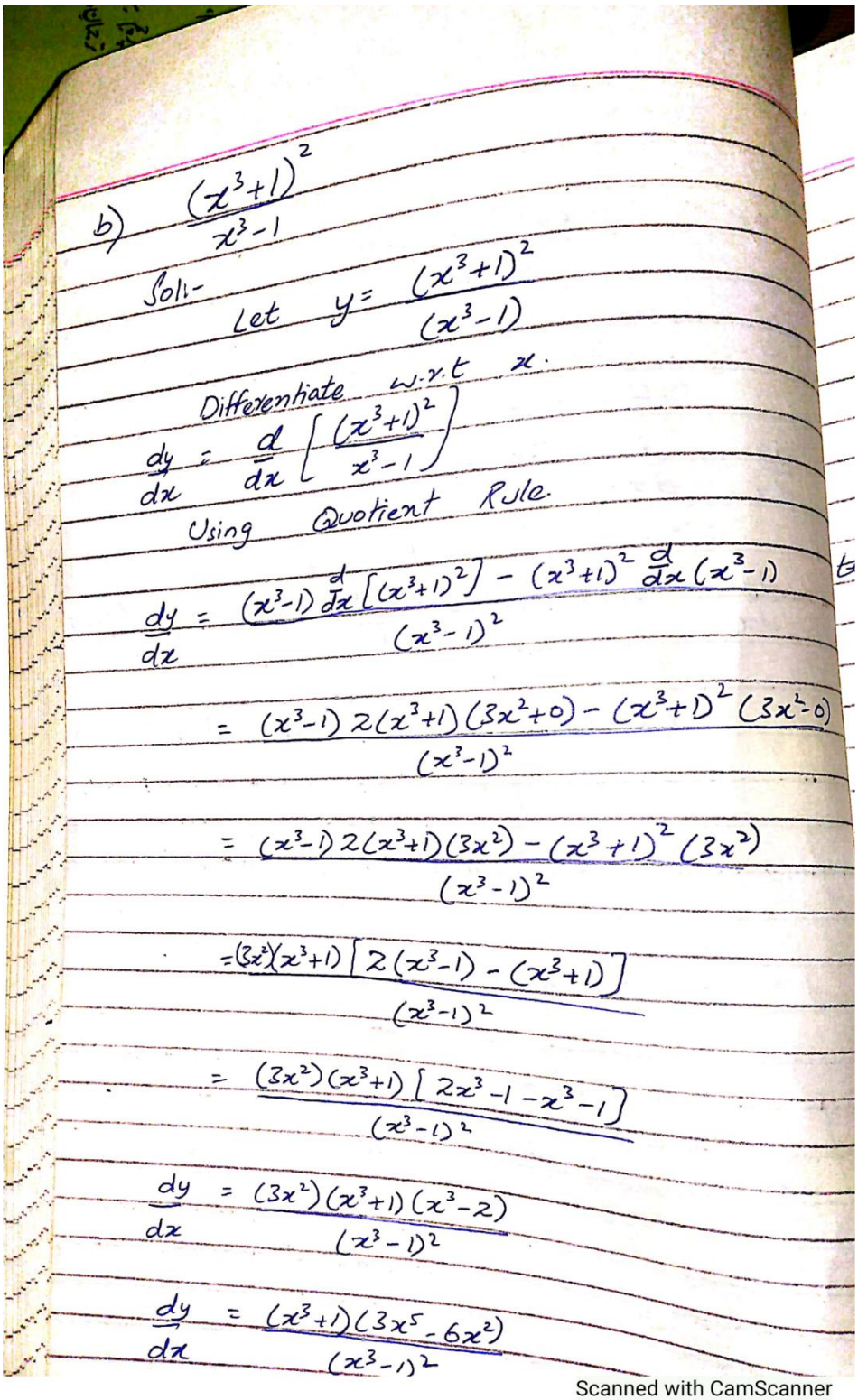
$= \frac{(x^3+1)(12x^3-6x^2) - (3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$

$= \frac{12x^6-6x^5+12x^3-6x^2-9x^6+6x^5-15x^2}{(x^3+1)^2}$

$\frac{dy}{dx} = \frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$

Scanned with CamScanner

b) **Differentiate** $\frac{(x^3+1)^2}{x^3-1}$ **with respect to x.**



b) $\frac{(x^3+1)^2}{x^3-1}$

Soln-

Let $y = \frac{(x^3+1)^2}{(x^3-1)}$

Differentiate w.r.t x.

$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x^3+1)^2}{x^3-1}\right]$

Using Quotient Rule.

$\frac{dy}{dx} = \frac{(x^3-1)\frac{d}{dx}[(x^3+1)^2] - (x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$

$= \frac{(x^3-1)\,2(x^3+1)(3x^2+0) - (x^3+1)^2(3x^2-0)}{(x^3-1)^2}$

$= \frac{(x^3-1)\,2(x^3+1)(3x^2) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$

$= \frac{(3x^2)(x^3+1)\left[2(x^3-1) - (x^3+1)\right]}{(x^3-1)^2}$

$= \frac{(3x^2)(x^3+1)\left[2x^3-1-x^3-1\right]}{(x^3-1)^2}$

$\frac{dy}{dx} = \frac{(3x^2)(x^3+1)(x^3-2)}{(x^3-1)^2}$

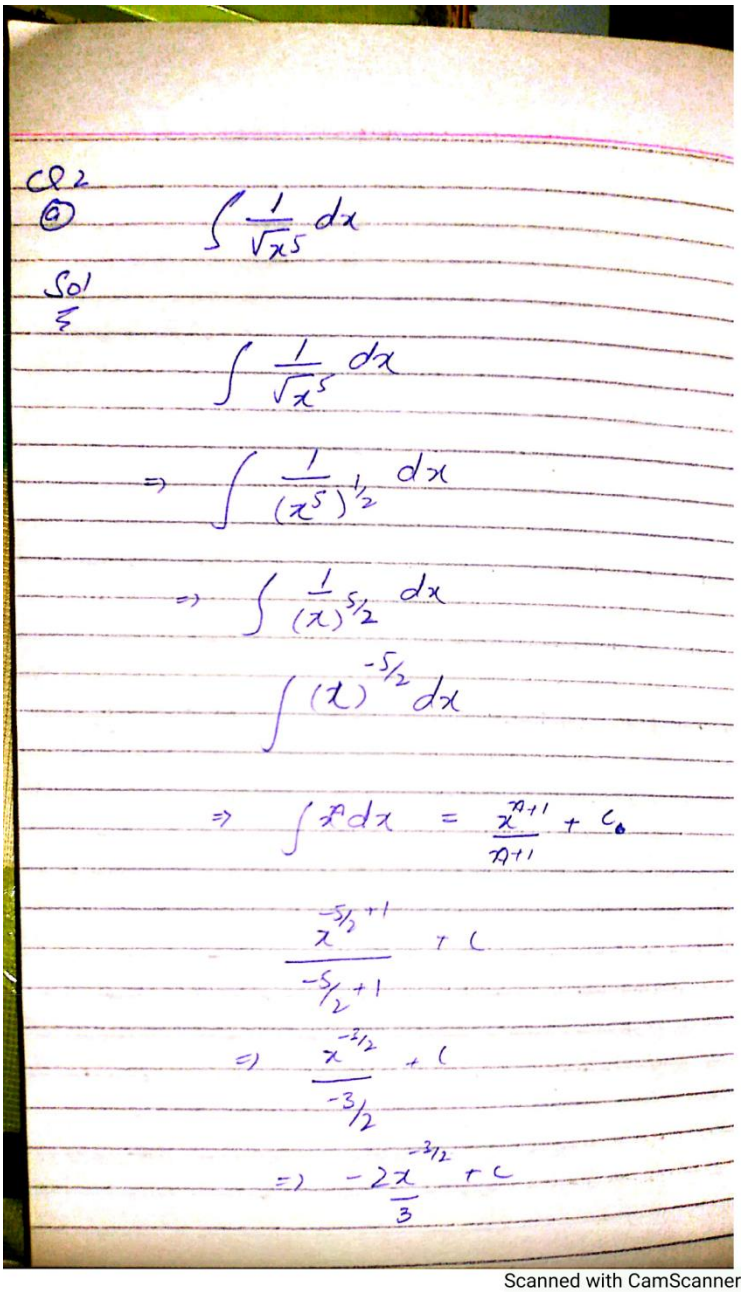
$\frac{dy}{dx} = \frac{(x^3+1)(3x^5-6x^2)}{(x^3-1)^2}$

Q.2

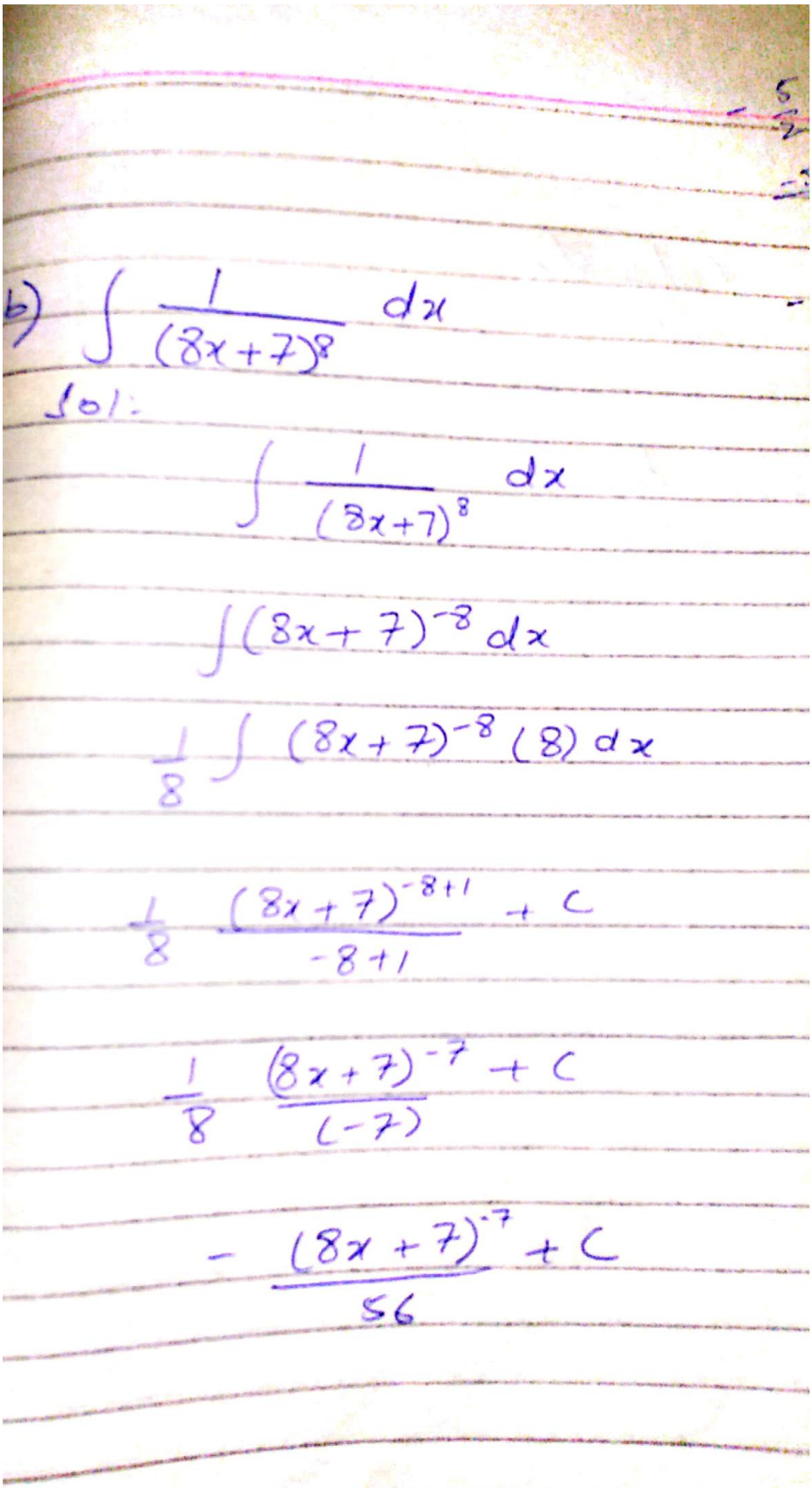
a) Find the Integration of $\int \frac{1}{\sqrt{x^5}}$ dx.

b) Find the Integration of $\int \frac{1}{(8x+7)^8}$dx.

a) Find the Integration of $\int \frac{1}{\sqrt{x^5}}$ dx.



b) Find the Integration of $\int \frac{1}{(8x+7)^8} dx$.



b) $\int \frac{1}{(8x+7)^8} dx$

Sol:

$$\int \frac{1}{(8x+7)^8} dx$$

$$\int (8x+7)^{-8} dx$$

$$\frac{1}{8} \int (8x+7)^{-8} (8) dx$$

$$\frac{1}{8} \frac{(8x+7)^{-8+1}}{-8+1} + C$$

$$\frac{1}{8} \frac{(8x+7)^{-7}}{(-7)} + C$$

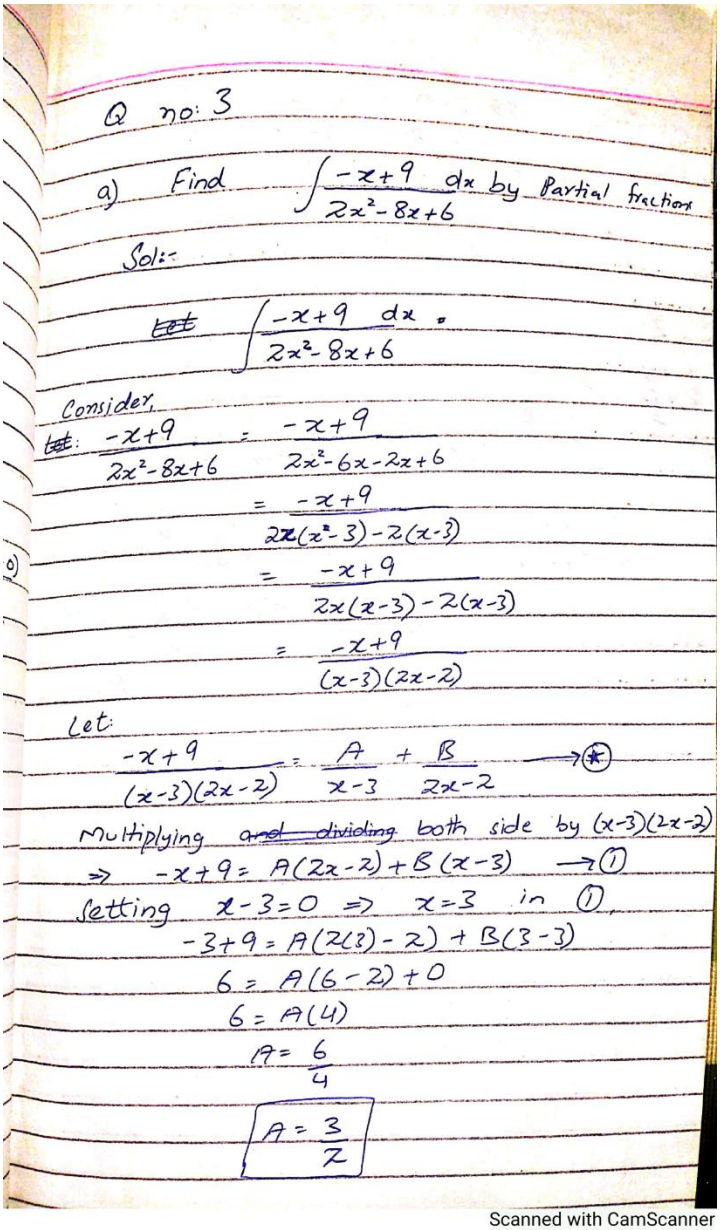
$$-\frac{(8x+7)^{-7}}{56} + C$$

Q.3

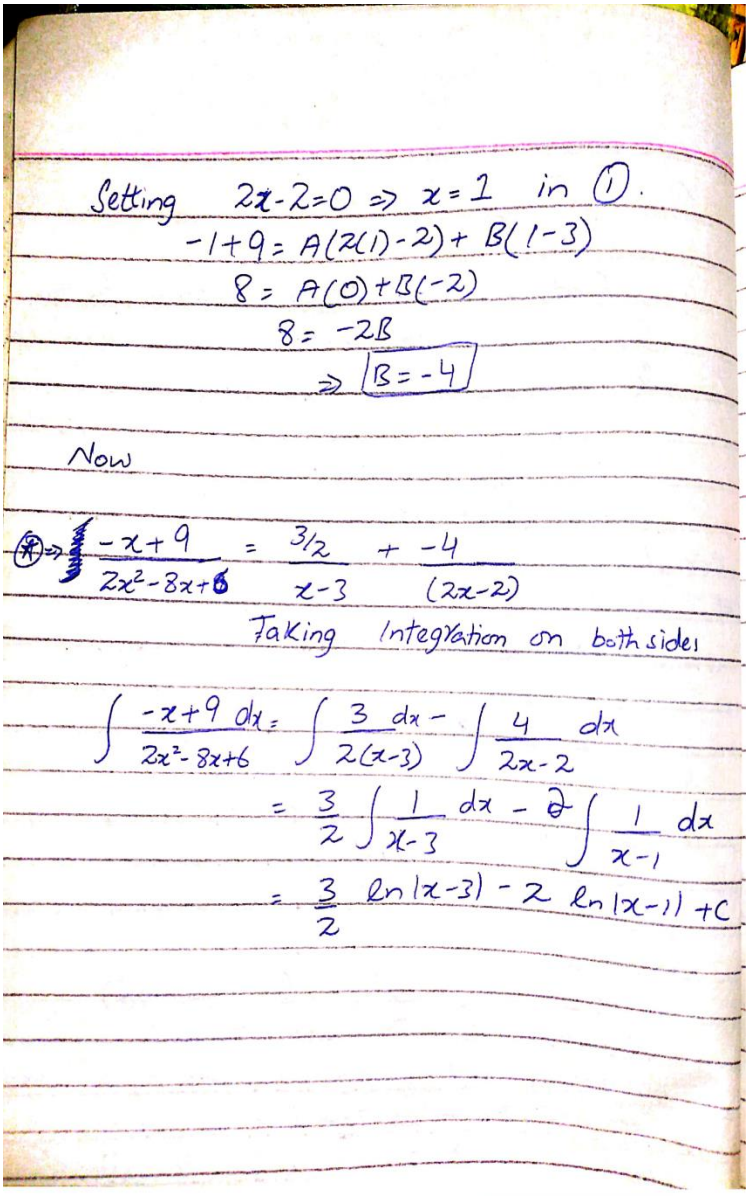
a) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6}dx$ by Partial fractions.

b) Find the Integration of $\int \frac{4x^2+8x}{(x^{2+1})(x^2+2x+3)}dx$ by Partial fractions.

a) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6}dx$ by Partial fractions.

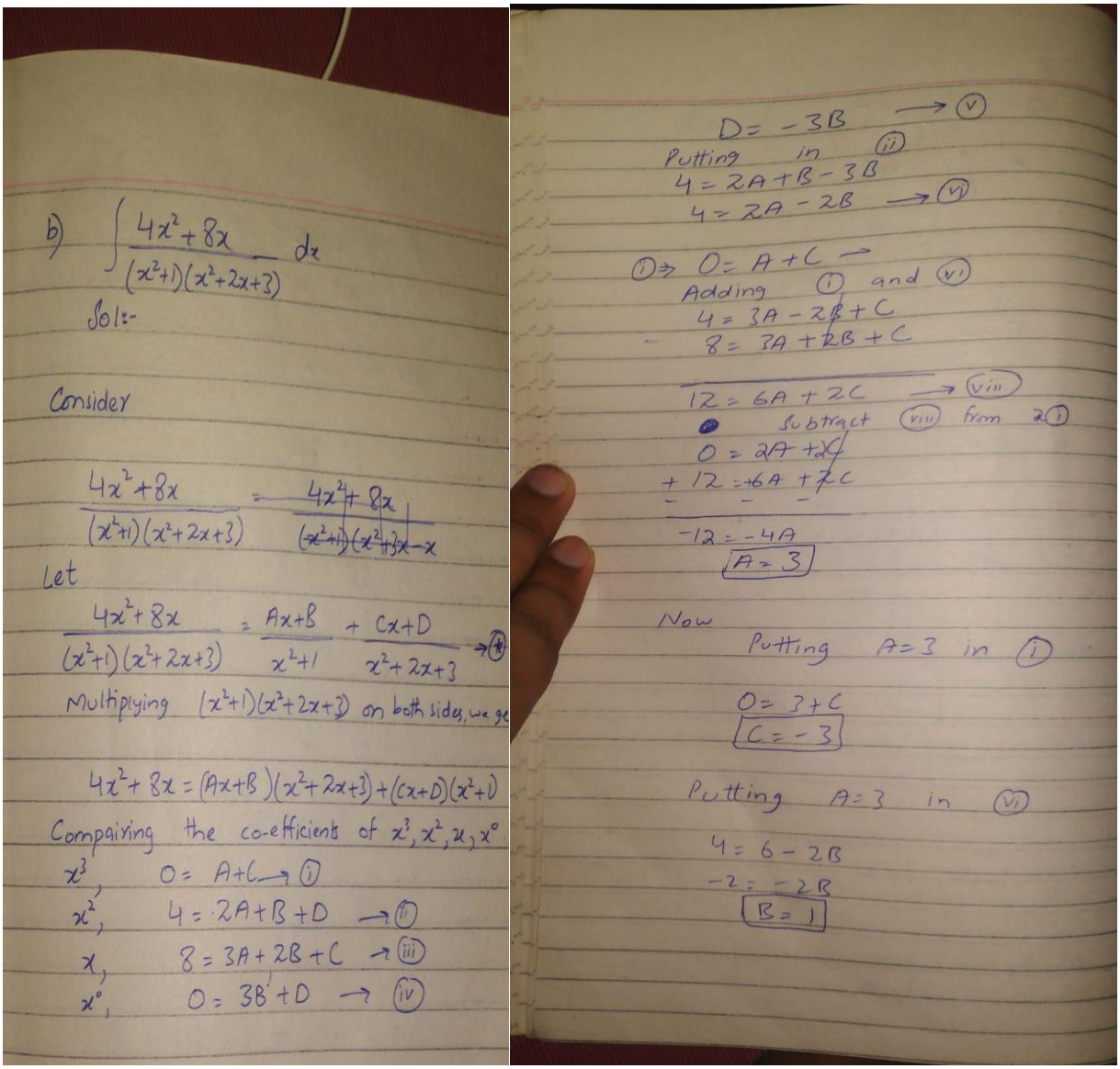


Scanned with CamScanner



Scanned with CamScanner

b) Find the Integration of $\int \frac{4x^2+8x}{(x^{2+1})(x^2+2x+3)}dx$ by Partial fractions.



b) $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} dx$

Sol:-

Consider

$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)-x}$

Let

$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$ → (*)

Multiplying $(x^2+1)(x^2+2x+3)$ on both sides, we ge

$4x^2+8x = (Ax+B)(x^2+2x+3)+(Cx+D)(x^2+1)$

Compairing the co-efficients of x^3, x^2, x, x^0

x^3, $0 = A+C$ → (i)

x^2, $4 = 2A+B+D$ → (ii)

x, $8 = 3A+2B+C$ → (iii)

x^0, $0 = 3B+D$ → (iv)

$D = -3B$ → (v)

Putting in (ii)

$4 = 2A+B-3B$

$4 = 2A-2B$ → (vi)

(i) ⇒ $0 = A+C$ →

Adding (i) and (vi)

$4 = 3A-2B+C$

$8 = 3A+2B+C$

$12 = 6A+2C$ → (viii)

Subtract (viii) from 2(i)

$0 = 2A+2C$

$+12 = +6A+2C$

$-12 = -4A$

$A = 3$

Now

Putting $A=3$ in (i)

$0 = 3+C$

$C = -3$

Putting $A=3$ in (vi)

$4 = 6-2B$

$-2 = -2B$

$B = 1$

Putting $B=1$ in (v)

$\boxed{D=-3}$

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{3x+1}{x^2+1} + \frac{(-3x-3)}{x^2+2x+3}$$

Taking Integration on both sides

$$\int \frac{4x^2+8x\,dx}{(x^2+1)(x^2+2x+3)} = \int \frac{3x+1}{x^2+1}\,dx - \int \frac{3x+3}{x^2+2x+3}$$

$$= 3\int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$$- 3\int \frac{x+1}{x^2+2x+3}\,dx$$

$$= \frac{3}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$$- \frac{3}{2}\int \frac{2x+2}{x^2+2x+3}\,dx$$

$$= \frac{3}{2}\ln|x^2+1| + \tan^{-1}(x)$$

$$- \frac{3}{2}\ln|x^2+2x+3| + C$$

Q.4

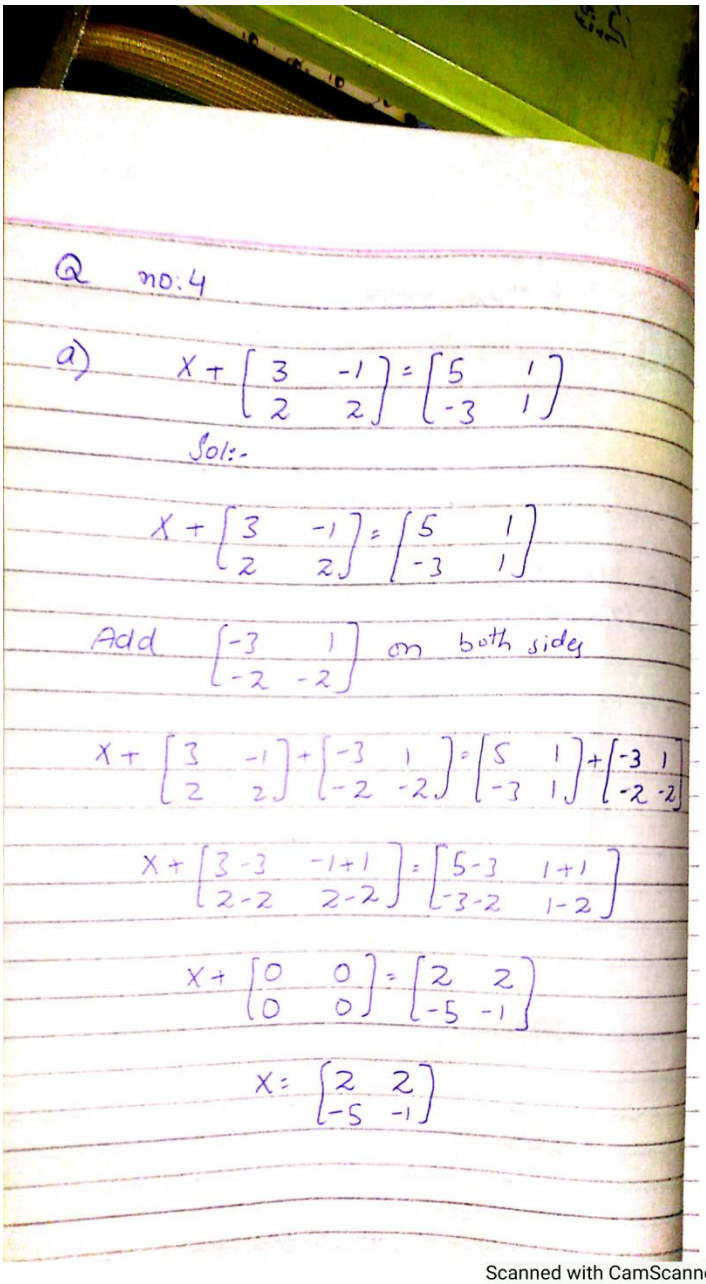
Solve each of the following matrix equations:

a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

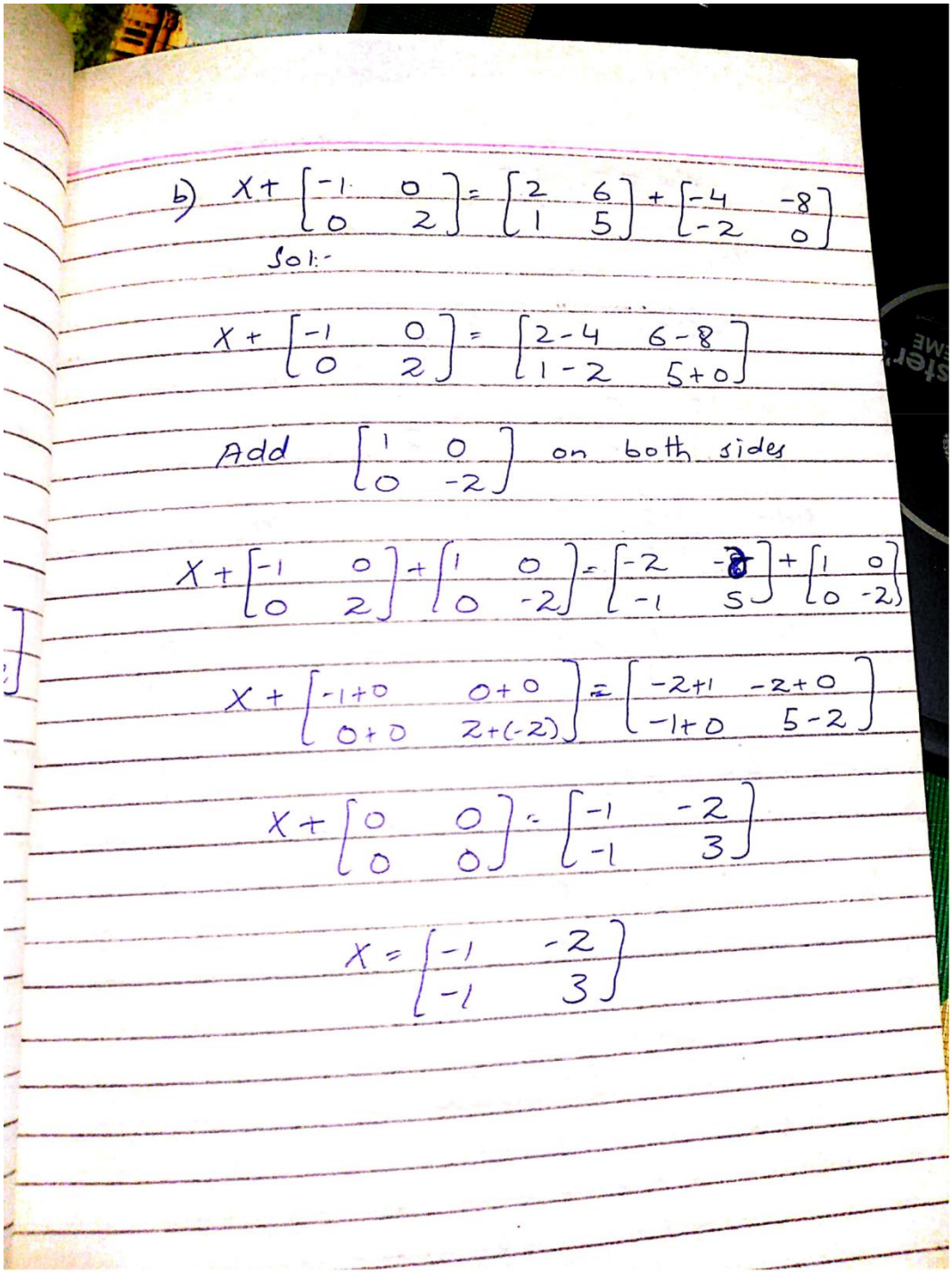
b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

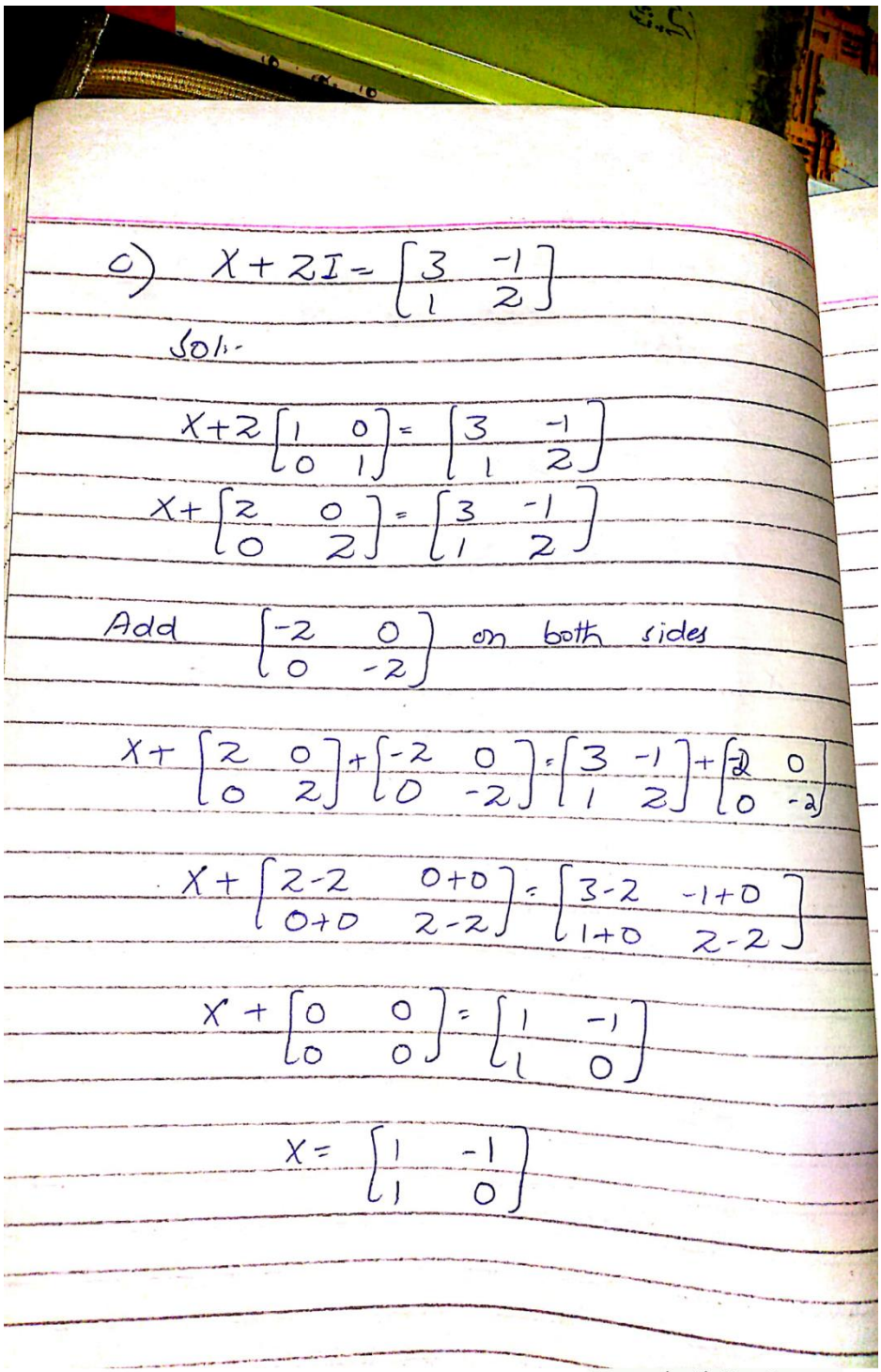
a) $\mathbf{X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}}$



b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$



c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$



c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Sol:-

$$X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Add $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ on both sides

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2-2 & 0+0 \\ 0+0 & 2-2 \end{bmatrix} = \begin{bmatrix} 3-2 & -1+0 \\ 1+0 & 2-2 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

Q.5

a) If A = $\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ Find A^2+BC

Q no: 5

$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Find $A^2 + BC$

Sol:-

$A^2 = A \cdot A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times1+4\times2 & 1\times4+4\times1 \\ 2\times1+1\times2 & 2\times4+1\times1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 8 \\ 4 & 9 \end{bmatrix}$

Now

$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3\times1+2\times0 & -3\times0+2\times2 \\ 4\times1+0\times0 & 4\times0+0\times2 \end{bmatrix}$

$= \begin{bmatrix} -3 & +4 \\ 4 & 0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 7 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 4 & 12 \\ 8 & 9 \end{bmatrix}$