COURSE :- Signal and System

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Part :- B QUESTION NO = 01

If $x(n) = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h(n) = 3\delta[n] + \delta[n-1] + 2\delta(n-2)$

Solution :

$Y(Z) = H(Z)X(Z)$

find $Y[n]$

$X(Z) = 2 - 4Z^{-2} + 2Z^{-3}$

$H(Z) = 3 + 1Z^{-1} + 2Z^{-2}$

Now

$Y(Z) = H(Z) * X(Z)$

$= 2(-4Z^{-2} + 2Z^{-3})(3 + Z^{-1} + 2Z^{-2})$

$6 + 2Z^{-1} + 4Z^{-2} - 12Z^{-2} - 4Z^{-3} - 8Z^{-4} + 6Z^{-3} + 2Z^{-4} + 4Z^{-5}$

$6 + 2Z^{-1} - 8Z^{-2} - 2Z^{-3} + 6Z^{-4} + 4Z^{-5}$

Part (A):-

Fourier Transform of differention Integration of Continous - Time

Let $x(t)$ be a continuous - time signal with a Fourier transform of $X(jw)$, i.e

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jwt} \, dw$$

Differentiating both side with respect to (t).

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \frac{d}{dt} \left\{ e^{jwt} \right\} dw$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\omega}^{\omega} X(j\omega)\left\{e^{j\omega t} \cdot j\omega\right\} \cdot d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\omega}^{-\omega} \left\{j\omega X(j\omega)\right\} e^{j\omega t}\, d\omega$$

$$\Rightarrow \mathcal{F}\left\{\frac{d}{dt} x(t)\right\} = j\omega X(j\omega).$$

Question # 2

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx\,dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} F(x) \cos nx\,dx + \int_{0}^{\pi} F(x) \cos nx\,dx$$

$$= \frac{1}{\pi} \int^{0} -\pi/2 \cos nx\,dx + \int_{0}^{\pi} \pi/2 \cos nx\,dx$$

$$= \frac{1}{\pi} \left[\pi/2 \frac{\sin nx}{n} \Big|_{-\pi}^{0} + \pi/2 \frac{\sin nx}{n} \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi} \left[-\pi/2 \sin n(0) - \sin n(-x) \right]$$

$$+ \pi/2 \left[\sin n(\pi) - \sin n(0) \right]$$

$$= \frac{1}{n\pi} \left[-\pi/2 (0) + \pi/2 (0) \right]$$

$$= \frac{1}{n\pi} (0)$$

$$a_n = 0)$$

Now

$$bn = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} F(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi/2 \sin nx\,dx + \int_{0}^{\pi} \pi/2 \sin nx\,dx\right.$$

$$F(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

As

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} F(x)dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\pi/2\,dx + \int_{0}^{\pi} \pi/2\,dx$$

$$= \frac{1}{2\pi}\left[-\pi/2 \int_{-\pi}^{0} 1dx + \pi/2 \int_{0}^{\pi} 1dx\right.$$

$$= \frac{1}{2\pi}\left[-\pi/2 \Big|_{-\pi}^{0} + \pi/2\, x \Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi/2\left[0-(-\pi)\right]+\pi/2\left[\pi-0\right]\right]$$

$$= \frac{1}{2\pi}\left[-\pi/2\,(\pi)+\pi/2\,(x\right].$$

$$= \frac{1}{2\pi}\left[\frac{-\pi^2}{2}+\pi^2/2\right]$$

$$= \frac{1}{2\pi}\left[0/2\right] \cancel{=} (a_0 \Rightarrow 0).$$

$$= \frac{1}{\pi}\left[-\pi/2\int_{-\pi}^{0}\sin nx\,dx+\pi/2\int_{0}^{\pi}\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\pi/2\;\frac{-\cos nx}{n}\Big|_{-\pi}^{0}+\pi/2\;\frac{-\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{\pi}\left[-\pi/2\;\frac{-\cos nx}{n}\Big|_{-\pi}^{0}+\pi/2\;\frac{-\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\pi/2\left[-\cos n(0)+\cos n(-\pi)\right]+\pi/2\left[-\cos n(\pi)+\cos n(0)\right]\right.$$

$$= \frac{1}{n\pi}\left[-\pi/2\left[-1+\cos n\cdot(-\pi)\right)+\pi/2\left[-\cos n\pi+\cos n(0)\right)\right]$$

$$= \frac{1}{n\pi}\left[-1\left[+1+\cos n(-\pi)\right]+1\left[-\cos n x+1\right)\right)$$

$$= \frac{1}{2n}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now :-

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 4/2n & \text{if } n \text{ is odd} \end{cases}$$

$$\left\{ b_n = 4/2n \right\}$$

$$F(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

$$F(x) = (0) + (0)\cos x + 0\cos(2x) + 0\cos 3x + \cdots$$
$$= 4/2 \sin x + (0)\sin^2 x + \frac{4}{3(2)} \sin 3x + \cdots$$

$$\left\{ 4/2 \sin x + 4/6 \sin 3x + \cdots \right\}$$

Question # 3 .

$A + B = 2$, $3A - B = 0$;

$B = -B/3$ $A = B/3$.

$B + B/3 = 2$

$4B = 6$

$B = 3/2 = 1.5$

$A = 3/2 \times 1/3 = 1/2 = 0.5$

$$\frac{2z^2+2z}{z^3+2z-3} = \frac{0.5}{(z-1)} + \frac{1.5z}{(z+3)}$$

$$X(z) = 0.5\frac{z}{z-1} + 1.5\frac{z}{z+3}.$$

$$x[n] = 0.5\,u[n] + 1.5(-3)^k.$$

$$x[z] = \frac{2z^2+2z}{z^2+2z-3}$$

$$\frac{2z^2+2z}{z^2+2z-3} = \frac{}{z(z+2)-3}$$

$z^2 + 3z - z - 3$

$z(z+3) - 1(z+3)$

$(z-1)(z+3)$

$$\frac{2z+2z}{(z+1)(z+3)} = \frac{A}{z-1} + \frac{B}{z+3}$$

$$2z^2 + 2z = A(z+3) + B(z-1)$$

$$2z^2 + 2z = Az + 3A + Bz - B$$

$$2z^2 + 2z = (A+B)z + 0(3A-B)$$

Question NO #4.

Express the transfer function using the given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [$$

Solution :-

We know that

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1}B + D$$

Putting the value

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix}\left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\text{Adj} = (s+2)s+1 = s^2+s+2s+2$$

$$\Rightarrow s^2+3s+2$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2+3s+2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+3s+2} \begin{bmatrix} s & 0 \\ 1 & \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix}}{s^2+3s+2} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

Question No # 5
Answer

Solution :- The Fourier transform of the given $x(t)$ is given by,

$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-jwt}\, dt$$

Note :-

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{For } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$\therefore$

$$X(jw) = \int_{-\infty}^{0} e^{at}\, e^{-jwt}\, dt + \int_{0}^{\infty} e^{-at} e^{-jwt}\, dt$$

$$w) = \int_{-w}^{0} e^{(a-jw)t}\, dt + \int_{0}^{\infty} e^{-(a+jw)t}\, dt$$

$$= \left.\frac{e^{(a-jw)t}}{a-jw}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+jw)t}}{-(a+jw)}\right|_{0}^{\infty}$$

$$= \frac{1}{(a-jw)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+jw)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-j\omega)}\boxed{1-0} - \frac{1}{(a+j\omega)}\boxed{0-1}$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+j\omega+a-j\omega}{a^2-(j\omega)^2}$$

$$\boxed{X(j\omega) = \frac{2a}{a^2+\omega^2}}$$