Date

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Subject # Calculus.

Section # A.

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Q1:- Find PQ were P is the point in three dimensional space with co-ordinate (4,1,3) and the point Q with co-ordintes (1,2,4) Find the distance b/w P and Q. Further, find the position vector of the point dividing PQ in the ratio 1:3

Sol:-

Given that:,

co-ordintes of P = (4, 1,3)

$OP = 4i + j + 3K$

OR

$OQ = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 4K) - 4i + j + 3.4)$

$= -3i + j + 4 \longrightarrow (*)$

Now distance b/w

P and Q = |PQ|

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$\sqrt{11} \longrightarrow (**)$

let M be the point which divided PQ in ratio 1:3

Then by ratio theoram:

Position vector of $M = \vec{OM}$

$$= \frac{3(4i+j+3k)+(1)(i+2j+4k)}{1+3}$$

$$= \frac{12i+3j+9k+i+2j+4k}{4}$$

$$= \frac{13i+5j+13k}{4} \quad (***)$$

Ans:.

Hence, eq (*), eq (**) and eq (***) are the required solution:.

Q No - 3

a) $\int_0 x^2 e^x dx$

Now first find integration.

$= \int x^2 e^x dx$

$= x^2 \int e^x dx - \int \left(e^x dx \cdot \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \int x \int e^x dx - \int \left(e^x dx \frac{d}{dx} x \right) dx$

$= x^2 e^x - 2 \left\{ x e^x - \int e^x dx \right\}$

$= x^2 e^x - 2x e^x + 2e^x$

Now Put limits.

$= (x^2 e^x - 2xe^x + 2e^x)$

$= (2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0))$

$= (\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2)$

$= 2e^2 - 2$ Ans:

(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

First find Integration:

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \longrightarrow ①$

let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\boxed{2dy = \frac{1}{\sqrt{x}}\, dx}$ Puting ①

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$$\int \sin(y)\,(2dy) = 2\int \sin(y)\,dy$$

$$= 2\,(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put limits.

$$= -2\left|\cos\sqrt{x}\right|^{2} = -2(\cos\sqrt{2} - \cos 1$$

$$= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans:-}$$

X ———————— ———————— X

Q4 :-

Verify that

$$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three - dimentional laplaces equations :-

Sol :-

The laplace equation is 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A).$$

So $u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$= u(x, y, z) = (x^2+y^2+z^2)^{-1/2}$$

$$= \frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= \frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

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$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$= \frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$= \frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$= \frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (2)$$

$$= \frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z).$$

$$= \frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}.$$

$$= \frac{\partial u^2}{\partial z^2} = 3z(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (3)$$

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Putting ①, ②, ③, in eq Ⓐ

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left(\cancel{3x^2}-\cancel{x^2}-\cancel{x^2}-y^2-\cancel{z^2}+\cancel{3y^2}-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}+\cancel{3z^2}-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}\right)$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution of laplace equation.

X ———— X

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Q 2 :-

Evaluate :-

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\, dx$$

Sol :-

$$\begin{array}{r|l} & \quad 2x - 1 \\ \hline 2x + x & \cancel{4x^3} + 10x + 4 \\ & \underline{\pm \cancel{4x^3}} \\ & -2x^2 + 10x + 4 \\ & \underline{\mp 2x^2 \quad \mp x} \\ & 11x + 4 \end{array}$$

So

$$2x - 1 + \frac{11x + 4}{2x^2 + x} = \frac{4x^3 + 10x + 4}{2x^2 + x}.$$

$$\therefore \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \rightarrow$$

$$= 2 \int x \, dx - \int 1\, dx + \int \frac{11x + 4}{2x^2 + x}\, dx$$

$$= \frac{2x^2}{2} x + \int \frac{11x+4}{x(2x-1)} dx \longrightarrow 2$$

Now find

$$\int \frac{11x+4}{x(2x+1} \cdot dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow A$$

$$\frac{11x+4}{x(2x+1} = \frac{A(2x+1) + B\cancel{x}x}{x(2x+1)}$$

$$11x+4 = A(2x+1) + Bx \longrightarrow 3$$

Put $x = 0$ in 3

$$\boxed{4 = A}$$

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Now Put $x = -\frac{1}{2}$ in (3)

$$11\left(-\frac{1}{2}\right) + 4 = B\left(-\frac{1}{2}\right)$$

$$= -\frac{11}{2} + 4 = -\frac{B}{2}$$

$$= \frac{-11 + 8}{\cancel{2}} = -\frac{B}{\cancel{2}}$$

$$= -3 = -B \Rightarrow \boxed{B = 3}$$

Putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on (6).

$$\int \frac{11x+4}{x(2x+1)} dx = \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

$$= 4\int \frac{1}{x}\, dx + 3 \int \frac{1}{2x+1}\, dx$$

$$= -4 \ln |x| + \frac{3}{2} \ln |2x+1|$$

Putting there value in (2)

$$= x^2 - x + 4 \ln |x| + \frac{3}{2} \ln (2x+1)$$

Now put there value in eq (1)

$$\boxed{\int \frac{4x^3 + 10x + 4}{2x^2 + x}\, dx = x^2 - x + 4\ln |x| + \frac{3}{2} \ln |2x+1| + c}$$ Ans.