Differential Equation
Assignment

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11470
BE(E)
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(1)

Q2: $x^2y'' - 4xy' + 6y = 0$, $y(1) = 0.4$, $y'(1) = 0$

Solution: let's substitute

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

The equation becomes

$x^2 m(m-1)x^m \cdot x^{-2} - 4x\, m x^m x^{-1} + 6x^m = 0$

x^m is a common factor

$x^m [m(m-1) - 4m + 6] = 0$

$\Rightarrow m^2 - 5m + 6 = 0$

So, $y = x^m$ is a solution of given ODE if m is a root of equation.

To find the root of equation

$m^2 - 5m + 6 = 0$

$$m = \frac{5 \pm \sqrt{(-5)^2 + (4)(6)}}{2}$$

$$m = \frac{5 \pm 1}{2}$$

Roots are distinct & real

$m_1 = 3$, $m_2 = 2$

Real different roots gives the real solutions.

$y_1 = x^{m_1} = x^3$

$y_2 = x^{m_2} = x^2$

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^3 + c_2 x^2$

$y' = 3c_1 x^2 + 2c_2 x$

we have to determine c_1 & c_2 from IVP

$$\begin{cases} 0.4 = y(1) = c_1 \cdot 1^3 + c_2 \cdot 1^2 \\ 0 = y(2) \quad 3c_1 \cdot 1^2 + 2c_2 \cdot 1 \end{cases}$$

$$= \begin{cases} 0.4 = c_1 + c_2 & \text{--- (1)} \\ 0 = 3c_1 + 2c_2 & \text{--- (2)} \end{cases}$$

$0.4 - c_2 = c_1$ Put in (2)

$0 = 3(0.4 - c_2) + 2c_2$

$0 = 1.2 - c_2$

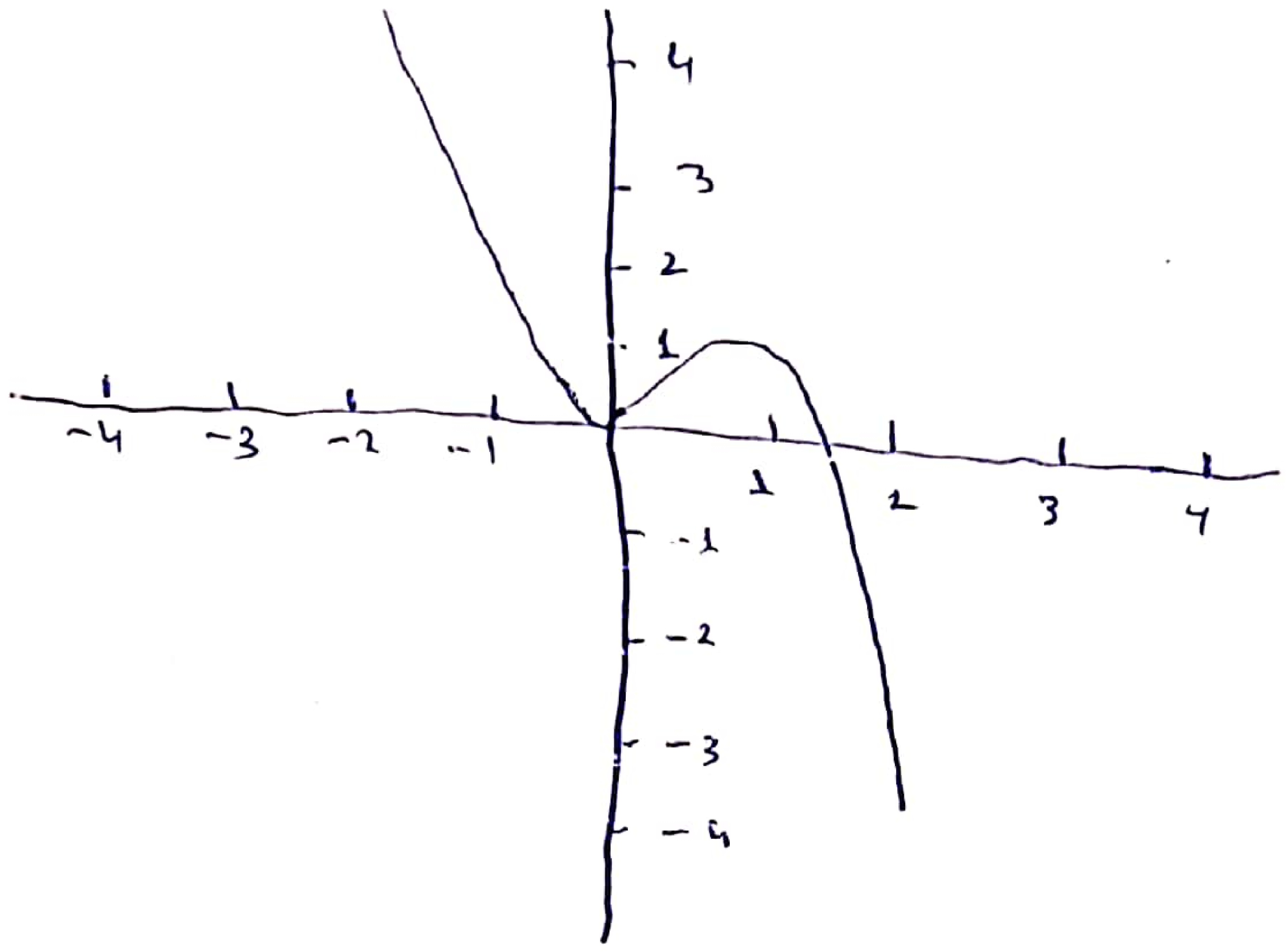
$\boxed{c_2 = 1.2}$

Put in 1

$\boxed{c_1 = -0.8}$

The Particular Solution of IVP is

$y = -0.8x^3 + 1.2x^2$



13:- $x^2 y'' + 3xy' + 0.75y = 0, \quad y(1) = 1, \; y'(1) = -1.5$

Solution:

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

The equation becomes

$x^2 m(m-1)x^{m-2} + 3x\, m x^{m-1} + 0.75x^m = 0$

$\cancel{x^2}\, m(m-1)x^m \cdot \cancel{x^{-2}} + 3\cancel{x}\, m x^m \cdot \cancel{x^{-1}} + 0.75x^m = 0$

$m(m-1)x^m + 3mx^m + 0.75x^m = 0$

$x^m[(m-1)m + 3m + 0.75] = 0$

$m(m-1) + 3m + 0.75 = 0$

$m^2 + 2m + 0.75 = 0$

$$m = \frac{-2 \pm \sqrt{4 - (4)(0.75)}}{2}$$

$$m = \frac{-2 \pm 1}{2}$$

$m_1 = \frac{-1}{2}$, $m_2 = -3/2$

Real Differential Roots m_1 & m_2 Provide two Real Solutions

$y_1 = x^{m_1} = x^{-1/2} = x^{-0.5}$

$y_2 = x^{m_2} = x^{-3/2} = x^{-1.5}$

So the General Solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^{-0.5} + c_2 x^{-1.5}$

$\Rightarrow y' = 0.5 c_1 x^{-1.5} - 1.5 c_2 x^{-1.5}$

we have to determine c_1 & c_2 from IVP

$y(1) = 1 = c_1 \cdot 1^{-0.5} + c_2 \cdot 1^{-1.5}$

$1 = c_1 + c_2$

$-1.5 = y'(1) = -0.5 c_1 \cdot 1^{-1.5} - 1.5 c_2 \cdot 1^{-1.5}$

$-1.5 = 0.5 c_1 - 1.5 c_2$

$c_1 + c_2 = 1$ — (1)

$3 = c_1 + 3c_2$ — (2)

From (1) $c_1 = 1 - c_2$ Put in (2)

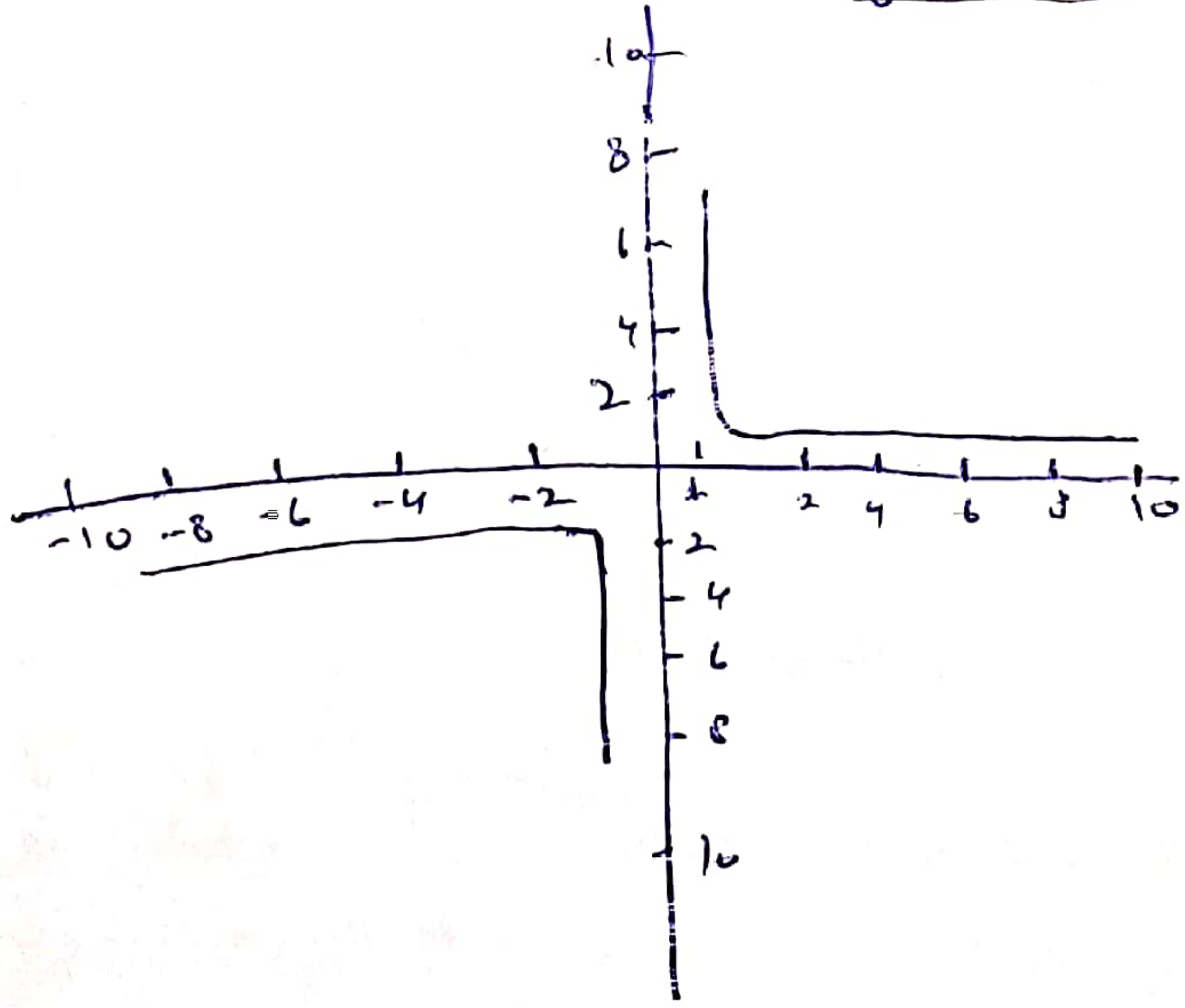
$3 = 1 - c_2 + 3c_2$

$2 = 2c_2$

$\boxed{c_2 = 1}$ Put in (1)

$\boxed{c_1 = 0}$

Particular Solution of IVP is $\boxed{y = x^{-1.5}}$



14:- $x^2y'' + xy' + 9y = 0 \quad y(1) = 0,\ y(1) = 2.5$

Solution :-

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)\,x^{m-2}$

equation Becomes

$x^2\, m(m-1)\,x^{m-2} + mx^{m-1} \cdot x + 9x^m = 0$

$x^2\, m(m-1)\,x^m \cdot x^{-2} + mx^m \cdot x^{-1}\, x + 9x^m = 0$

$x^m[m(m-1) + m + 9] = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$\sqrt{m^2} = \sqrt{-9}$

$m = \pm 3i$

$m_1 = 3i,\ m_2 = -3i$

As $x = e^{\ln x}$

$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i\ln x}$

$x^{m_2} = x^{-3i} = (e^{\ln x})^{-3i} = e^{-3i\ln x}$

As we Know that

$e^z = e^{a+ib}$

$= e^a(\cos b + i\sin b)$ $\to$ Eq

So,

$e^{3i\ln x} = e^0[\cos(3\ln x) + i\sin(3\ln x)]$

$= \cos(3\ln x) + i\sin(3\ln x$

and

$e^{-3i\ln x} = e^0[\cos(3\ln x) - i\sin(3\ln x)$

$= \cos(3\ln x) - i\sin(3\ln x)$

That Gives

$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x)$ — (1)

$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x)$ — (2)

Adding (1) & (2) and $\div$ by 2

$\frac{x^{m_1} + x^{m_2}}{2} = \cos(3\ln x) + i\sin(3\ln x) + \cos(3\ln x) - i\sin 3(\ln x)$

$= \frac{x\cos(3\ln x)}{x} = \cos(3\ln x)$

Now

Subtract (1) & (2) divide by $2i$

$$\frac{x^{m_1} - x^{m_2}}{2i} = \cos 3\ln x + i\sin(3\ln x) - \cos(3\ln x) + i\sin(3\ln x)$$

$$= \frac{2i\sin(3\ln x)}{2i} = \sin(3\ln x)$$

$$y_1 = \cos(3\ln x)$$

$$y_2 = \sin(3\ln x)$$

So, general solution is

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1\cos(3\ln x) + c_2\sin(3\ln x)$$

$$y' = \frac{-3c_1}{x}\sin(3\ln x) + \frac{3c_2}{x}\cos(3\ln x)$$

To determine c_1 & c_2

$$y(1) = 0 = c_1\cos(3\ln 1) + c_2\sin(3\ln 1)$$

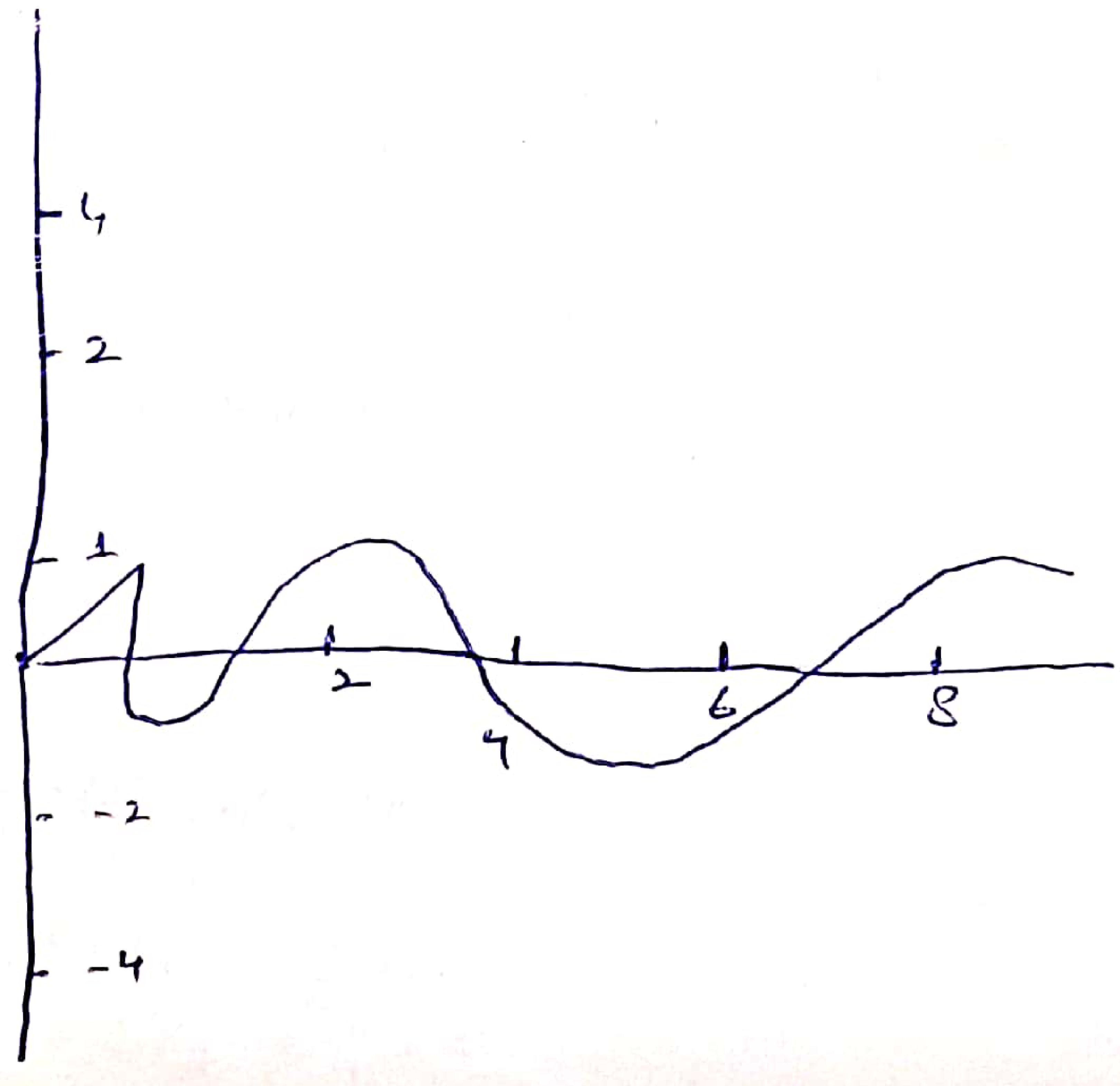
$$\boxed{c_1 = 0}$$

$$2.5 = y'(1) = -3c_1\sin(3\ln 1) + 3c_2\cos(3\ln 1)$$

$$\boxed{c_2 = 5/6}$$

Particular solution of IVP is

$$\boxed{y = 5/6\,\sin(3\ln x)}$$



(15) $x^2y'' + 3xy' + y = 0$, $y(1) = 3.6$, $y'(1) = 0.4$ (6)

Solution:-

let

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

equation becomes

$\cancel{x^2}\, m(m-1)x^m \cdot \cancel{x^2} + 3mx^m + x^m = 0$

$x^m\,[m(m-1) + 3m + x^n] = 0$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$m = 1$

To find second linearly independent solution y_2 by using method of Reduction of Order.

ODE of Standard Form

$y'' + \frac{3}{x}y' + 1/x^2\, y = 0$

$P(x) = 3 \cdot 1/x \rightarrow \int P dx = 3\ln|x|$

$y_2 = uy$.

$u = \int U dx$ & $U = 1/y_1{}^n\, e^{-\int P dx}$

$e^{-\int P dx} = e^{-3\ln x} = (e^{\ln x})^{-3} = x^{-3}$

$U = x^{-3} \cdot 1/x^2 = 1/x$

$y_2 = 1/x \ln x$

So, the general Solution is

$y = c_1 y_1 + c_2 y_2$

$= 1/x\,(c_1 + c_2 \ln x)$

$y' = \frac{1}{x^2}(-c_1 - c_2 \ln x + c_2)$

to determine c_1 & c_2.

$3.6 = y_1 = 1/1\,(c_1 + c_2 \ln 1)$

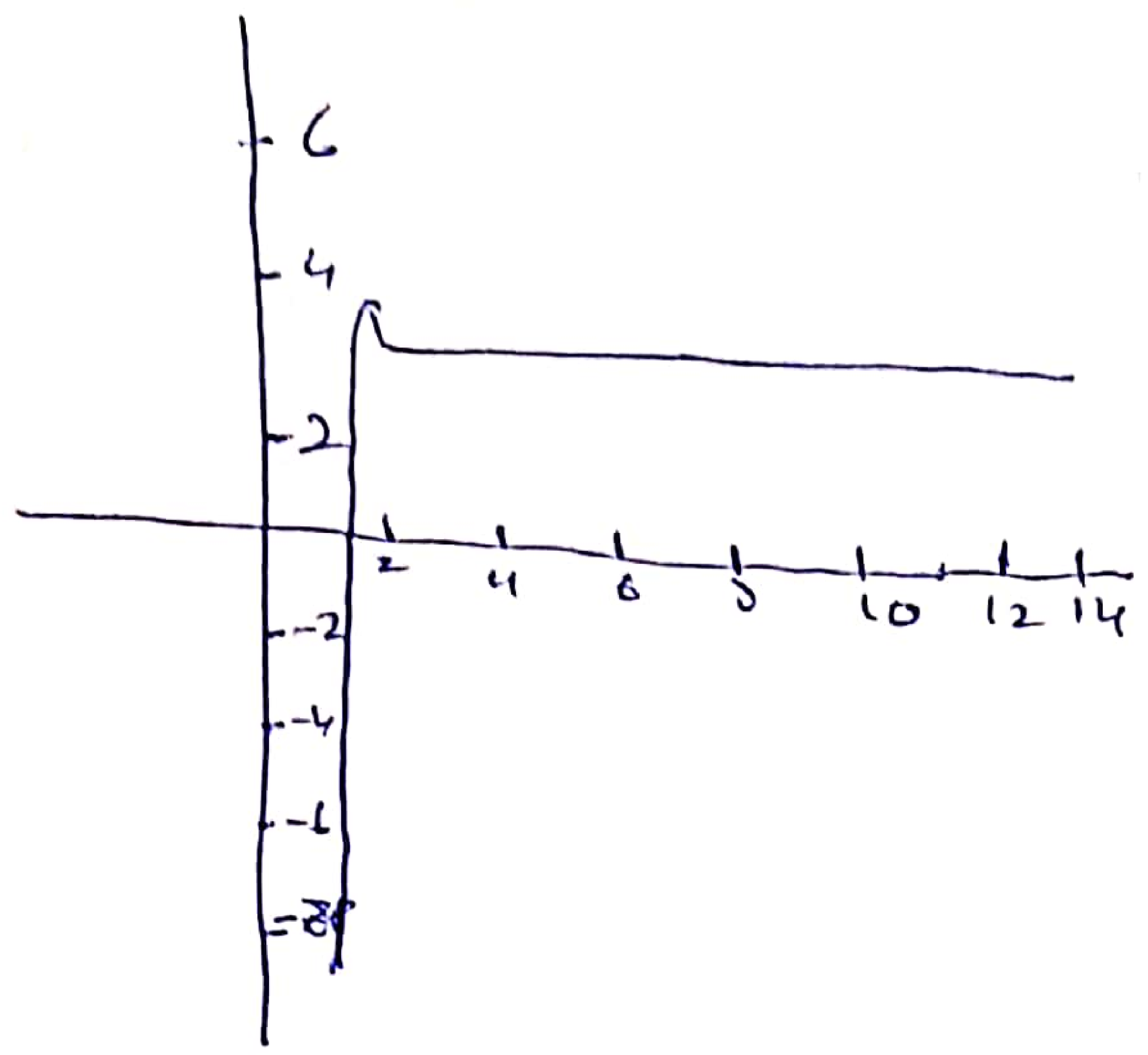
$c_1 = 3.6$

$0.4 = y'(1) = 1/1\,(-c_1 - c_2 \ln 1 + c_2)$

$0.4 = -c_1 + c_2$ $\boxed{C = 4}$

Particular Solution of IVP is

$$y = (3.6 + 4 \ln x)\, 1/x$$



16:-

$$(x^2D^2 - 3xD + 4I)y = 0 \;,\; y(1) = -\pi \;,\; y'(1) = 2\pi$$

$$x^2D^2y - 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y$$
$$= x^2y'' - 3xy' + 4y \quad -(1)$$

let

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

Eq (1) $\Rightarrow$

$$\cancel{x^2}\, m(m-1)x^m \cdot \cancel{x^{-2}} - 3x\, m(x^m)\cancel{x^{-1}} + 4x^m$$

$$x^m(m(m-1) - 3m + 4) = 0$$

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$

It has real double roots

$$m = 2$$

Real Double Root m Provide Real Solution

$$y_1 = x^m = x^2$$

To find y_2 we use method of Reduction of Order

ODE in Standard form

$$y'' - 3/x \cdot y' + 4/x^2 \cdot y = 0$$

$P(x) = 3 \cdot \frac{1}{x} \Rightarrow \int P\,dx = -3\ln|x|$ ⑧

Put $y_2 = u y_1$

$u = \int U\,dx$, $U = \frac{1}{y_1^2} e^{-\int P dx}$

lets find U

$= e^{-\int P dx} = e^{3\ln|x|} = (e^{\ln|x|})^3 = x^3$

$= U = x^3 \cdot 1/(x^2)^2 = 1/x$

By Integrating

$u = \int \frac{dx}{x} \cdot \ln|x|$

$y_2 = u y_1 = y_1 \ln x = x^2 \ln x$.

So the general solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^2 + x^2 \ln x$

$= x^2 (c_1 + c_2 \ln x$

$y' = (x^2)'(c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$

$= 2x(c_1 + c_2 \ln x) + c_2 x^2 \cdot 1/x$

$= 2c_1 x + 2c_2 x \ln x + c_2 x$

$= 2c_1 x + c_2 x(2\ln x + 1)$

Now we have to determine c_1 & c_2

$-\pi = y(1) = 1^2(c_1 + c_2 \ln 1)$

$-\pi = c_1$ — ①

$2\pi = y'(1) = 2c_1 + c_2(2\ln 1 + 1)$

$2\pi = 2c_1 + c_2$ — ②

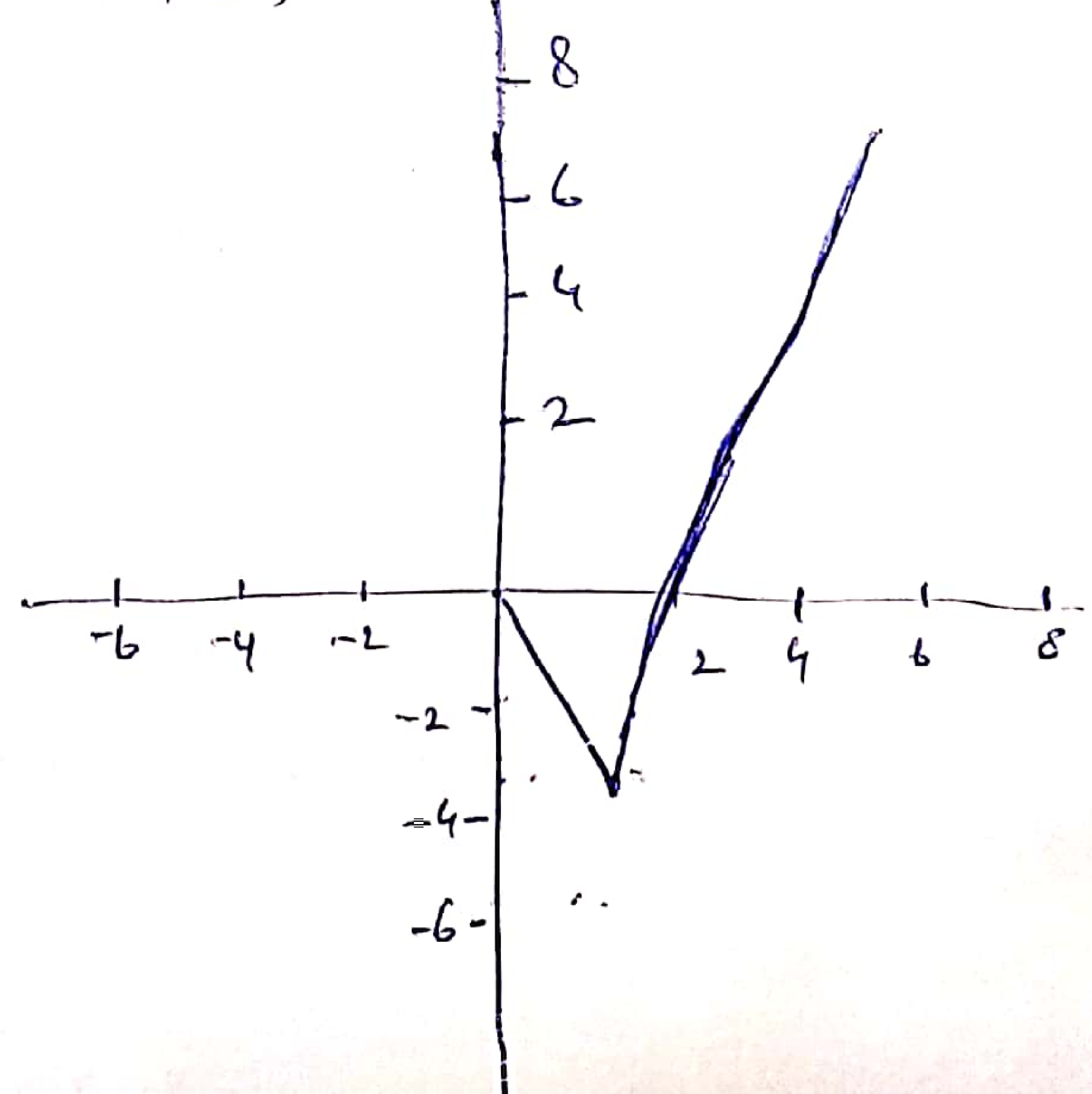
Put ① in ②

$2\pi = 2(-\pi) + c_2$

$c_2 = 4\pi$

Particular solution

$y = x^2(-\pi + 4\pi \ln x)$



(7): $(x^2D^2 + xD + 1)y = 0,\ y(1) = 1,\ y'(1) = 1$

Solution:-

By Applying operator

$x^2 D(Dy) + x\,Dy + y$

$= x^2y'' + xy' + y$ —(1)

let $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

eq (1) Becomes

$x^2 m(m-1)x^{m-2} + x m x^{m-1} + x^m = 0$

$x^m(m(m-1) + m + 1) = 0$

$m(m-1) + m + 1 = 0 \Rightarrow m^2 - m + m + 1 = 0$

$m^2 + 1 = 0 \Rightarrow \sqrt{m^2} = \sqrt{-1} \quad m = \pm i$

$m_1 = i,\ m_2 = -i$

As $x = e^{\ln x}$

$x^{m_1} = x^i = (e^{\ln x})^{-i} = e^{i\ln x}$

$x^{m_2} = x^{-i} = (e^{\ln x})^{-i} = e^{-i\ln x}$

$e^z = e^{a+ib} = e^a(\cos b + i\sin b)\ z \in \phi$

So using

$e^{i\ln x} = e^0[\cos(\ln x) + i\sin(\ln x)]$

$= \cos[\ln x] + i\sin(\ln x)$ —(i)

and $e^{-i\ln x} = \cos(\ln x) - i\sin(\ln x)$ —(ii)

By Adding (1) & (2) ÷ by 2

$x^{m_1} + x^{m_2} = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x) - i\sin(\ln x)$

$= \frac{x\cos(\ln x)}{x} = \cos\ln x$

Now Subtract (1) & (2) & ÷ By $2i$

$x^{m_1} - x^{m_2} = \cos(\ln x) + i\sin(\ln x) - \cos(\ln x) + i\sin(\ln x)$

$= \frac{2i}{2i}\sin(\ln x) = \sin\ln(x)$

$y_1 = \cos(\ln x)$ & $y_2 = \sin(\ln x)$

The general solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 \cos(\ln x) + c_2/x \cos(\ln x)$

we have to determine $c_1 > c_2$

$1 = y(1) = c_1 \cos((\ln 1)) + c_2 \sin(\ln 1)$

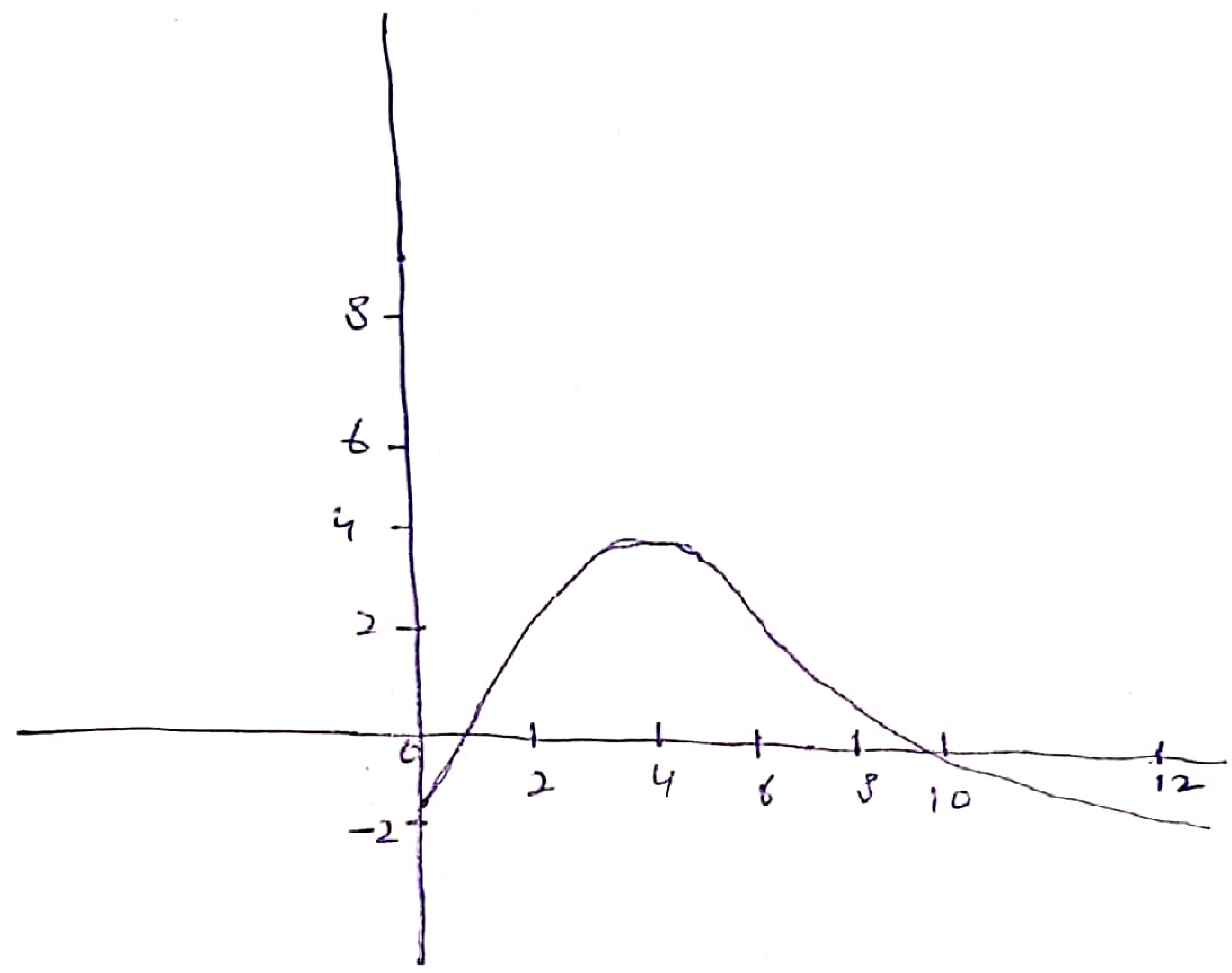
$\boxed{c_1 = 0}$

$1 = y'(1) = -c_1 \sin(\ln 1) + 3c_2 \cos(\ln 1)$

$\boxed{c_2 = 1}$

Particular Solution is

$y = \sin(\ln x) + c_2(\ln x)$



13: $(9x^2 D^2 + 3xD + I)\, y = 0, \; y(1) = 1, \; y'(1) = 0$

Solution:- By Applying operations

$= 9x^2 y'' + 3xy' + y$

let $y = x^m \Rightarrow y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2} + x^m = 0$

$x^m (9m(n-1) + 3m + 1) = 0$

$9m^2 - 6m + 1 = 0$

$n = \frac{6 \pm \sqrt{6^2 - (4)(9)}}{18}$

$n_1 = \frac{6}{18} \Rightarrow m_1 = 1/3$

it has real double Root

$n = 1/3$

To find y_2 we use Method of Reduction of order.

$y'' + \frac{1}{3x} y' + \frac{1}{9x^2} y = 0$

$P(x) = 1/3 \cdot 1/x \Rightarrow \int P dx = 1/3 \ln(x)$

$u = \int U dx$ & $U = \frac{1}{y_1^2} e^{-\int P dx}$

$e^{-\int P dx} = e^{-1/3 \ln(x)} = x^{-1/3}$

$U = x^{-1/3} \cdot 1/(x^{1/3})^2 = 1/x$

$u = \ln(x)$

$y_2 = u y_1 = x^{1/3} \ln x$

General Solution is

$y = c_1 y_1 + c_2 y_2$

$= x^{1/3} (c_1 + c_2 \ln x)$

$y' = 1/3 \, x^{-2/3} (c_1 + c_2 \ln x) + x^{1/3} c_2$

we have to determine c_1 & c_2

$1 = y(1) = 1(1/3) \cdot (c_1 + c_2 \ln 1)$

$1 = c_1$

$4 = y'(1) = 1/3 \cdot 1^{-2/3} (c_1 + c_2 \ln 1) + 1^{1/3} c_2$

$c_2 = -1/3$

Particular Solution is

$\boxed{y = x^{1/3} (1 - 1/3) \ln x}$

13:- $(X^2 D^2 - X D - 15 I) y = 0$, $y(1) = 0.1$, $y'(1) = -4.5$

Solution:-

By applying operator

$X^2 D(Dy) - x Dy - 15 Iy = 0$

$= x^2 y'' - x y' - 15 y$

let $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

Eq becomes

$x^2 m(m-1) x^m x^{-2} - x m x^m x^{-1} - 15 x^m = 0$

$x^m [m(m-1) - m - 15] = 0$

INU

$x^m[m(m-1) - m - 15] = 0$

$m^2 - 2m - 15 = 0$

$m = \frac{2 \pm 8}{2}$

$m_1 = 5, \; m_2 = -3$

Real Different root m_1 & m_2 provide two real solutions

$y_1 = x^{m_1} = x^5$

$y_2 = x^{m_2} = x^{-3}$

So, the General Solution is

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^5 + c_2 x^{-3}$

$y' = 5c_1 x^4 - 3c_2 x^{-4}$

we have to find c_1 & c_2

$0.1 = y(1)$

$= c_1 \cdot 1^5 + c_2 \cdot 1^{-3}$

$= 0.1 = c_1 + c_2$ —— (1)

$-4.5 = y'(1)$

$= 5c_1 1^4 - 3c_2 \cdot 1^{-4}$

$-4.5 = 5c_1 - 3c_2$ —— (2)

By solving (1) and (2) Simultaneously.

$0.1 - c_2 = c_1$ Put in (2)

$\boxed{c_2 = 0.625}$

$\boxed{c_1 = -0.525}$

Particular Solution is

$y = -0.525 x^5 + 0.625 x^{-3}$.

Q2:- Use the Method of seperation of variables to find the General Solution of the following D.E

(i) $x' = \sqrt{x}$

$dx/dy = \sqrt{x}$

$\frac{1}{\sqrt{x}} \cdot dx = dy$

$$\int \frac{1}{\sqrt{x}} \cdot dx = \int (1)\, dy$$

$$\int (x)^{-1/2} \cdot dx = \int (1)\, dy$$

$$\frac{x^{-1/2+1}}{-1/2+1} + C = y$$

$$y = \frac{x^{-1+2/2}}{\frac{-1+2}{2}} + C$$

$$y = \frac{x^{1/2}}{1/2} + C$$

$$y = \frac{1}{2} x^{1/2} + C$$

(b) $x' = e^{-2x}$

Solution:

[illegible]

$$\frac{dx}{dy} = e^{-2x}$$

$$\frac{1}{e^{-2x}}\, dx = (1) \cdot dy$$

$$\int \frac{1}{e^{-2c}}\, dx = \int 1 \cdot dy$$

$$\int e^{2x} \cdot dx = \int 1 \cdot dy$$

$$e^{2x} \cdot \int 2x\, dx = y$$

$$2e^{2x} \cdot \frac{x^{1+1}}{1+1} = y$$

$$y = 2e^{2x} \cdot \frac{x^2}{x} + C$$

$$y = e^{2x} \cdot x^2 + C$$

INU

(c) $y' = 1 + y^2$

Solution:-

$$dy/dx = 1 + y^2$$

$$\frac{1}{1+y^2} \cdot dy = (1)\, dx$$

$$\int \frac{1}{(1+y^2)} \cdot dy = \int (1)\, dy$$

$$\int \frac{1}{(1+y^2)} \cdot dy = \int (1) \cdot dx$$

$$\tan^{-1} y + c = x$$

$$x + c = \tan^{-1} y$$

$$\tan x + c = y$$

$$y = \tan x + c$$

(d) $u' = \frac{1}{5-2u}$

Sol:-

$$\frac{du}{dx} = \frac{1}{5-2u}$$

$$(5-2u) \cdot du = (1) \cdot dx$$

$$\int (5-2u)\, du = \int (1)\, dx$$

$$5 \int (1)\, du - 2 \int u \cdot du = \int 1 \cdot dx$$

$$5u - 2 \frac{u^{1+1}}{1+1} = x$$

$$5u - \frac{2u^2}{2} = x + c$$

$$5u - u^2 = x + c$$

(e) $x' = au + b$, $a, b > 0$

Solution:-

$$\frac{dx}{du} = au + b$$

$$(1)\, dx = (au + b)\, du$$

$$\int 1 \cdot dx = a \int u \cdot du + b \int (1) \cdot du$$

$$c + x = a \cdot \frac{u^{1+1}}{2} + bu \Rightarrow c + x = a \cdot u^2/2 + bu$$

$$\Rightarrow \tfrac{1}{2} a u^2 + bu = x + c$$

INU

(f) $Q' = \frac{Q}{4+Q^2}$

Sol:-

$$\frac{d\varphi}{dn} = \frac{\varphi}{4+\varphi^2}$$

$$\frac{4+\varphi^2}{\varphi} \cdot d\varphi = (1) \cdot dn$$

$$\left(\frac{4}{\varphi} + \frac{\varphi^{2}}{\varphi} \right) \cdot d\varphi = (1) \cdot dn$$

$$\left(\frac{4}{\varphi} + \varphi \right) \cdot d\varphi = (1) \cdot dn$$

$$4 \int \frac{1}{\varphi} \, d\varphi + \int \varphi \, d\varphi = \int 1 \cdot dn$$

$$4 \ln |\varphi| + \frac{\varphi^2}{2} = n + c$$

x ——— x ———

(g) $n' = e^{n^2}$

Sol:-

$$dn/dy = e^{n^2}$$

$$\frac{1}{e^{n^2}} \cdot dn = (1)\, dy$$

$$\int e^{-n^2} dn = \int (1) \cdot dy$$

$$e^{-n^2} \cdot \int (-n^2)\, dn = y$$

$$y = -e^{-n^2} \cdot \frac{n^{2+1}}{2+1} + c$$

$$y = -e^{-n^2} \cdot \frac{n^3}{3} + c$$

$$y = \frac{-1}{3} e^{-n} \cdot n^3 + c$$

——— x ——— x ———

(h) $y' = r(a-y)$

Sol:- $dy/dn = ra - ry$

$$dy/dn = r(a-y)$$

$$1/_{a-y}\, dy = r \cdot dr$$

$$\int \frac{1}{a-y} \cdot dy = \int r \cdot dr$$

INU

$\int \frac{1}{a-y} \cdot dy = \int r \cdot dr$

$= \frac{r^2}{2} + c$

$-\int \frac{-1}{a-y} \cdot dy$

$-\ln|a-y| = \frac{r^2}{2} + c$

(2) - Solve $y' = r(a-y)$, where r and a are constants.

Solution:-

$dy/dx = ra - ry$

$\int \frac{1}{ra-ry} \cdot dy = \int (1) \cdot dx$

$\frac{1}{r} \int \frac{1}{a-y} \cdot dy = x$

X'ing by (−)

$\frac{1}{r} \int \frac{-1}{a-y} \cdot dy = -x$

$\frac{1}{r} \ln|a-y| = -x + c$

$\frac{1}{r} \ln \cdot |a-y| = -x + c$

(3) In Exercise 1 (a) - (b) find the solution to the Resulting IVP when $x(0) = 1$

Sol:- (a) $x = \sqrt{x}$

$dx/dy = \sqrt{x}$

$\frac{1}{\sqrt{x}} \cdot dx = dy$

$\int \frac{1}{\sqrt{x}} \cdot dx = \int (1) dy$

$\frac{(x)^{-1/2+1}}{-\frac{1}{2}+1} + c = y$

$y = \frac{1}{2} x^{1/2} + c$

$x(0) = 1$

$y = 1/2 \, (0)^{1/2} + c$

$\boxed{y = c}$

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(b) $x' = e^{-2x}$

$\frac{dx}{dy} = e^{-2x}$

$\frac{1}{e^{-2x}} dx = (1) \cdot dy$

$\int \frac{1}{e^{-2x}} \cdot dx = \int 1 \cdot dy$

$\int e^{2x} dx = \int 1 \cdot dy$

$y = e^{2x} \cdot x^2 + c$

$x(0) = 1$

$y = e^{2(0)} \cdot (0)^2 + c$

$y = 1 \times 0 + c$

$y = c$

$y = e^{2(0)} \cdot (0)^2 + c$

$\boxed{y = 0}$

(7):– Find the general solution.

(a) $x' = \frac{2x}{t+1}$

Sol:– $\frac{dx}{dt} = \frac{2x}{2t+1}$

$\int \frac{1}{2x} \cdot dx = \int \frac{1}{1+t} \cdot dt$

$\frac{1}{2} \int \left(\frac{1}{2}\right) \cdot dx = \int \frac{1}{1+t} dt$

$\frac{1}{2} \ln|x| + c = |\ln| 1+t$

$\left[\frac{1}{2} \ln|x|\right] + c = 1+t$

$\ln|1+t| = \frac{1}{2} [\ln|x|] + c$

(b) $Q' = t\sqrt{t^2+1} \sec Q$

Sol:– $\frac{dQ}{dt} = \sqrt{t^2+1} \cdot \sec Q$

$\frac{1}{\sec Q} \cdot dQ = \pm \sqrt{t^2+1} \cdot dt$

$\int \frac{1}{\sec\varphi} d\varphi = \int \sqrt{1+t^2} \cdot dt$

$\int \frac{1}{\sec\varphi} \cdot d\varphi = \int \sqrt{1+t^2} \cdot dt$

$\int \cos\varphi \cdot d\varphi = \int \sqrt{1+t^2} \cdot dt$

$\sin\varphi + c = \frac{(1+t^2)^{1/2+1}}{\frac{1}{2}+1} \cdot \int (1+t)^2 \cdot dt$

$\sin\varphi + c = \frac{(1+t^2)^{3/2}}{3/2} \cdot \int 1 \cdot dt \int t^2 \cdot dt$

$\sin\varphi + c = \frac{2}{3} (1+t^2)^{3/2} \cdot t \cdot t^3/3$

$\sin\varphi + c = 2/9 (1+t^2)^{3/2} \cdot t^4$

x — x — x

c) $(2u+1)\, u' - (t+1) = 0$

Sol:- $(2u+1)u' = (t+1)$

$u' = \frac{(t+1)}{2u+1}$

$\frac{du}{dt} = \frac{t+1}{2u+1}$

$\int (2u+1) \cdot du = \int (t+1) dt$

$2\int u \cdot du + \int 1 \cdot du = \int t \cdot dt + \int 1 \cdot dt$

$\frac{2u^2}{2} + u = t^2 + t$

$u^2 + u + \frac{t^2}{2} = t$

$t = u^2 + u \cdot \frac{t^2}{2} + c$

x — x —

(d) $R' = (t+1)(R^2+1)$

Sol:- $\frac{dr}{dt} = (t+1)(R^2+1)$

$1/R^2+1 \cdot dr = (t+1) dt$

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$\int \frac{1}{R^2+1} \cdot dR = \int t \cdot dt + \int 1 \cdot dt$

$= \tan^{-1} R = \frac{t^2}{2} + t + c$

$R = \tan \left[\frac{t^2}{2} + t\right] + c$

——— x ——— x ——— x

e) $y' + y + \frac{1}{y} = 0$

Sol:- $y' = -y - \frac{1}{y}$

$dy/dx = -y - \frac{1}{y}$

$(y + \frac{1}{y}) dy = -(1) \cdot dx$

$\int y \cdot dy + \int \frac{1}{y} dy = \int 1 \cdot dx$

$y^2/2 + \ln |y| = x$

$\frac{1}{2} y^2 + \ln |y| + c = x$

——— x ——— x ——— x

(f) $(t+1) x' + x^2 = 0$

Sol:- $\cancel{(t+1)} \frac{x'}{\cancel{(t+1)}} + x^2 = 0$

$\cancel{(t+1)} \frac{x'}{\cancel{(t+1)}} = \frac{-x^2}{(t+1)}$

$\frac{dx}{dt} = \frac{-x^2}{(t+1)}$

$\int \frac{1}{x^2} dx = \int \frac{1}{1+t} dt$

$\int \frac{1}{x^2} dx = \int \frac{1}{1+t} dt$

$\int x^{-2} dx = \int \frac{1}{1+t} \cdot dt$

$-x^{-1} = \ln |t| + c$

$-x^{-1} = + \ln |t| + c$