MID TERM EXAM

NAME : MUHAMMAD TALHA

ID : 7965

SUBJECT : DIFFERENTIAL EQN'S

SUBMITTED TO : MAM SHUMAILA

SECTION : "B"

DEPARTMENT : BE (CIVIL)

SEMESTER : (4)

DATE : 14-4-2020

Day: M T W T F S S

Scanned with CamScanner

Date:

Day: M T W T F S S

QUESTION : 01

(1) The order of matrix A is $m \times p$ and order of B is $p \times n$. Then order of matrix AB is ?

The order of Matrix AB $= m \times n$

(2) The no of non-Zero rows in an Echelon form ?

No of Non-zero Rows = Rank of Matrix

(3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is Singular Matrix then $a = ?$

A matrix B, such that $|B| = 0$ is Called Singular Matrix.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$$(1 \times a) - (4 \times 2) = 0$$

$$a - 8 = 0$$

$$a = 8$$

Scanned with CamScanner

Date:
Day: M T W T F S S

(4) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ the $|A| = ?$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= (2i \times (-i)) - (i \times i)$

$= -2i^2 - i^2 \qquad \therefore i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$|A| = 3$

(5) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a Scalar matrix.

(6) Solution of $dy/dx + 2xy = y$

$\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$

Dividing y on both sides

$\frac{1}{y} dy = (1-2x) dx$

Scanned with CamScanner

Date:

Day: S M T W T F S

Integrating

$$\int \frac{1}{y}\, dy = \int (1-2x)\, dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x - x^2 + c}$$

$$y = e^{x - x^2 + c}$$

(7) The order and Degree of Differential Equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Order of Equation $= 1$
Degree of Equation $= 6$

(8) The Degree and order of $\frac{d^2y}{dx^2} - 4xy$

$= \sin \frac{d^2y}{dx^2}$ is ?

Order of Equation $= 2$
Degree of Equation $=$ undefine

Scanned with CamScanner

(4)

(4) The differential Equation $2\frac{dy}{dx} + x^2y = 2x+3$

, $y(0) = 5$.

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$2\frac{dy}{dx} = 2x+3-x^2y$$

$$2dy = (2x+3-x^2y)\,dx$$

Taking Integration of both sides

$$\int 2dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{\cancel{2}x^2}{\cancel{2}} + 3x - \frac{x^3y}{3} + c$$

$$2y + \frac{x^3y}{3} = x^2+3x+c$$

$$y\left(2+\frac{x^3}{3}\right) = x^2+3x+c$$

$$y\left(6+\frac{x^3}{3}\right) = x^2+3x+c$$

$$y = (x^2+3x+c) \div \left(\frac{6+x^3}{3}\right)$$

$$y = (x^2+3x+c) \times \frac{3}{6+x^3} \quad - (1)$$

put $x=0$ and $y=5$

$$5 = \left((0)^2+3(0)+c\right) \times \frac{\cancel{3}\,1}{\cancel{6}\,2+(0)^3}$$

Scanned with CamScanner

$$5 = C \times \frac{1}{2}$$

$C = 10$ (Hence Equation is Homogeneous)

Now put the value of C in eq (1)

$$\Rightarrow y = (x^2 + 3x + 10) \times \frac{3}{6 + x^3}$$

$$y = \frac{3x^2 + 9x + 30}{6 + x^2}$$

Hence $y = \dfrac{3x^2 + 9x + 30}{6 + x^2}$

Scanned with CamScanner

(10) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Solution :-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1-1 & a-b & a^2-b^2 \\ 1-1 & a-c & a^2-c^2 \end{vmatrix} \begin{matrix} \\ R_1-R_2 \\ R_1-R_3 \end{matrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

Expand By C_1

$$= 1 \begin{vmatrix} a-b & a^2-b^2 \\ a-c & a^2-c^2 \end{vmatrix} - 0 + 0$$

$$= \begin{vmatrix} a-b & (a-b)(a+b) \\ a-c & (a-c)(a+c) \end{vmatrix}$$

Taking $(a-b)$ and $(a-c)$ Common from R_1 and R_2, therefore

$$= (a-b)(a-c) \begin{vmatrix} 1 & (a+b) \\ 1 & (a+c) \end{vmatrix}$$

Scanned with CamScanner

$$= (a-b)(a-c)\{(a+c)-(a+b)\}$$

$$= (a-b)(a-c)\{\cancel{a}+c-\cancel{a}\mp b\}$$

$$= (a-b)(a-c)(c-b)$$

Therefore, the result would be,

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(a-c)(c-b)$$

Scanned with CamScanner

QUESTION : 02

(A) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ as the product of

factors which are linear in a,b,c.

Solution :-

$$A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$|A| = a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$|A| = a(b^2c^3 - c^2b^3) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$|A| = ab^2c^3 - ac^2b^3 - a^2bc^3 + a^3bc^2 + a^2cb^3 + a^3cb^2$$

Taking a, b and c Common

$$= abc\,(bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b)$$

$$= abc\,\{cb(c-b) - ac(c-a) + ab(b-a)\}$$

Hence the determinant is simplified.

(B) Find the Eigen value,

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution :-

$$C = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$|C - \lambda I| = 0$$

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scanned with CamScanner

$$= \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand By Row 1

$$\Rightarrow 2-\lambda \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1) \begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$+(-1) \begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + (0) = 0 \quad \text{— } D$$

Now,
$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand By Row 1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$+(-1) \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix} = 0$$

Scanned with CamScanner

$$\Rightarrow 3-\lambda\{(3-\lambda)(2-\lambda)-1(-1)(-1)\}+1\{(-1)(2-\lambda)\ldots$$
$$-(-1)(-1)\}-1\{(-1)(-1)-(3-\lambda)(-1)\}=0$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$
$$-1(1+3-\lambda)=0$$

$$\Rightarrow (3-\lambda)(5-5\lambda+\lambda^2)+(-3+\lambda)-(4-\lambda)=0$$

$$\Rightarrow 15-15\lambda+3\lambda^2-5\lambda+5\lambda^2-\lambda^3-3+\lambda-4+\lambda=0$$

$$\Rightarrow -\lambda^3+8\lambda^2-18\lambda+8=0 \quad - (1)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand By Row 1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ 0 & 2-\lambda \end{vmatrix} +(-1)\begin{vmatrix} -1 & 3-\lambda \\ 0 & -1 \end{vmatrix}=0$$

$$\Rightarrow -1\{(3-\lambda)(2-\lambda)-(-1)(-1)\}+1\{(-1)(2-\lambda)-(0)\}$$
$$-1\{(-1)(-1)-(0)\}=0$$

$$\Rightarrow -1\{6-3\lambda-2\lambda+\lambda^2-1\}+1\{-2+\lambda\}-1\{1\}=0$$

$$\Rightarrow -6+3\lambda+2\lambda-\lambda^2+1-2+\lambda-1=0$$

Scanned with CamScanner

$$\Rightarrow \quad -\lambda^2+6\lambda-8=0 \quad — \quad 2$$

Now, $-1\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$

Expand By Column C_1

$$\Rightarrow -1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1(-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} +0\right]=0$$

$$\Rightarrow -1\left[-1\{(-1)(2-\lambda)-(-1)(-1)\}+1\{(3-\lambda)(2-\lambda)-(-1)(-1)\}\right]=0$$

$$\Rightarrow -1\left[-1(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]=0$$

$$\Rightarrow -1(3-\lambda+5-5\lambda+\lambda^2)=0$$

$$\Rightarrow \quad -8+6\lambda-\lambda^2=0$$

$$\Rightarrow \quad -\lambda^2+6\lambda-8=0 \quad — \quad 3$$

putting equation 1, 2, 3 in D

$$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 18\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 8 - 8 = 0$$

$$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

By Synthetic Division :-

$$\begin{array}{c|cccc} & 1 & -10 & 32 & -32 \\ 2 & & 2 & -16 & 32 \\ \hline & 1 & -8 & +16 & 0 \end{array}$$

So, $(\lambda - 2)(\lambda^3 - 8\lambda^2 + 16\lambda)$

$$\Rightarrow \lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$$

$\lambda = 0$	$\lambda - 2 = 0$ $\lambda = 2$	$\lambda^2 - 8\lambda + 16 = 0$ Factorization $\lambda^2 - 4\lambda - 4\lambda + 16 = 0$ $\lambda(\lambda - 4) - 4(\lambda - 4) = 0$ $(\lambda - 4) = 0$ | $(\lambda - 4) = 0$ $\lambda = 4$ | $\lambda = 4$

Conclusion :-

$\lambda = 0$

$\lambda = 2$

$\lambda = 4$

$\lambda = 4$

Scanned with CamScanner

QUESTION : 03

The rate of change in the form of differential equation is given by $(x^2+3y^2)\,dx - 2xy \cdot dy = 0$.
Find the general solution at $x=2$ and $y=6$.

Solution :-

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right) \quad \text{— (C)}$$

Comparing equation C with $\frac{dy}{dx} = g(y/x)$

Equation C is homogeneous equation of degree 1.

put $\frac{y}{x} = v$ or $y = vx$

Differentiating w.r.t x, then

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

put in equation C

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Integrating

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

Scanned with CamScanner

$\ln|1+v^2| = \ln x + \ln c$

$\ln|1+v^2| = \ln xc$ $\quad \because \ln m + \ln n = \ln mn$

$$\boxed{1+v^2 = xc} \quad -① $$

Again,

putting $v = \frac{y}{x}$

$1 + (y/x)^2 = xc$

$1 + y^2/x^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$$\boxed{x^2+y^2 = x^3 c} \quad -②$$

For Initial values, $x=2, y=6$

$(2)^2 + (6)^2 = (2)^3 c$

$4 + 36 = 8c$

$c = 5$

putting value of c in eq ②

$x^2 + y^2 = x^3 \times 5$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Scanned with CamScanner

Taking Under Root

$$\sqrt{y^2} = \sqrt{x^2(5x-1)}$$

$$y = \pm x\sqrt{5x-1}$$

So we have,

$$\boxed{\begin{aligned} y &= +x\sqrt{5x-1} \\ y &= -x\sqrt{5x-1} \end{aligned}}$$

Scanned with CamScanner