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Section : A

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Date: 23 Sep 2020

BE Civil Engineering

Q# Apply both Eulers. method

Given Data $\frac{dy}{dx} = 2x$; $y(0) = 1$

for $0 \leq x \leq 0.5$ using $h = 0.1$ compare

Solution: By Eulers method

$y(0) = 1$, $h = 0.1$, $x_0 = 0$

by formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h[2x_n]$$

1st Iteration"

put $n = 0$ $x_0 = 0$

$y_1 = y_0 + h(2x_0)$

$y_1 = 1 + 0.1 (2(0))$

$y_1 = (1 + 0) +$

$\boxed{y_1 = 1}$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

2nd iteration

$n = 1$

$y_2 = y_1 + h(2x_1)$

$y_2 = 1 + 0.1(2(0.1))$

$y_2 = 1 + 0.1(0.2)$

$y_2 = 1 + 0.02$

$\boxed{y_2 = 1.02}$

$x_{n+1} = x_n + h$

put $n = 1$

$x_{1+1} = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

3rd iteration

put $n = 2$

$y_{n+1} = y_n + h\,(2x_n)$

$y_{2+1} = y_2 + h\,(2x_2)$

$y_3 = 1.02 + 0.1\,[2\,(0.2)]$

$y_3 = 1.02 + 0.1\,(0.4)$

$y_3 = 1.02 + 0.04$

$y_3 = 1.06$

$x_{n+1} = x_n + h$

Put $n = 2$

$x_{2+1} = x_2 + h$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

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b) By Euler's Modified Method

$$\frac{dy}{dx} = 2x$$

Given Data

$y(0) = 1$, $x_0 = 0$, $h = 0.1$

formula

$$y^*_{n+1} = y_n + h[f(x_n)]$$

$$y^*_{n+1} = y_n + h(2x_n) \rightarrow ①$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

$$= y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$$

$$= y_n + \frac{h}{2}\left[4x_n\right]$$

1st iteration

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + \frac{h}{2}(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$\boxed{y_1 = 1}$

2nd iteration

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$y_2 = y_1 + \frac{h}{2}(4x_1)$

$y_2 = 1 + \frac{0.1}{2}(4(0.1))$

$\boxed{y_2 = 1.02}$

3rd iteration

$\boxed{n=2}$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$y = y_2 + \frac{h}{2}(4x_2)$

$= 1.02 + \frac{0.1}{2}(4(0.2))$

$\boxed{y_3 = 1.06}$

Q#2) Use the fourth order Runge-kutta Method to obtain a Solution of

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y=0$ when $x=0$, fo $0 \le x \le 0.6$ with $h = 0.2$ work the throughout to four ~~deci~~ decimal places.

Solution: Given Data = $y(0)$ $y=0$

$x = 0$

$h = 0.2$

$0 \le x \le 0.6$

So

$x_{n+1} = x_n + h$

Put $n=0$

$x_1 = x_0 + h$

$= 0 + 0.2$

$\boxed{x_1 = 0.2}$

Put $n=1$

$x_2 = x_1 + h$

$x_2 = 0.2 + 0.2$

$\boxed{x_2 = 0.4}$

Put $n=2$

$x_3 = x_2 + h$

$\boxed{x_3 = 0.4 + 0.2 = 0.6}$

Put $n = 3$

$x_4 = x_3 + h \Rightarrow 0.6 + 0.2$

$\boxed{x_4 = 0.8}$

$y_{n+1} = y_n + k$

∴ 1st Iteration:

~~Put~~ $n = 0$

$\cancel{y_{0+1} = y_0 + k}$

$\Rightarrow \cancel{y_1 = y_0 + K}$

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$y_{n+1} = y_n + k$, $K = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

put $n = 0$

$y_1 = y_0 + k$

$k_1 = hf(x_n, y_n)$ (x_0, y_0)

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2(0^2 - 0 - 0)$

$\boxed{k_1 = 0}$

$\frac{dy}{dx} = x^2 + x - y$

$f(x,y) = x^2 + x - y$

$f(x_0, y_0) = x_0^2 + x_0 - y_0$

$f(x_1, y_1) = x_1^2 + x_1 - y_1$

$f(x_2, y_2) = x_2^2 + x_2 - y_2$

$f(x_3, y_3) = x_3^2 + x_3 - y_3$

$x_{n+1} = x_n + h$

$y_{n+k} = y_n + k$ —(1)

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$

$= 0.2 f\left(\left(x_0 + \frac{h}{2}\right), y_0 + \frac{h}{2}\right)$

$k_2 = 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$

$k_2 = 0.2 f(0.1, 0.1)$ (x_0, y_0)

$k_2 = 0.2((0.1)^2 + \cancel{0.1} - \cancel{0.1})$

$k_2 = 0.2(0.1)^2$

$$K_2 = 0.2(0.1)^2$$

$$K_2 = 0.2(0.01)$$

$$\boxed{K_2 = 0.002}$$

$$K_3 = hf\left(x_n + \frac{h}{2}\right), \ y_n + \frac{K_2}{2}\Big)$$

$$K_3 = 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$

$$K_3 = 0.2 f\big(\underset{x_1}{0.1}, \underset{y_1}{0.001}\big)$$

$$K_3 = 0.2\left((0.1)^2 + 0.1 - 0.001\right)$$

$$K_3 = 0.2\left(0.01 + 0.099\right)$$

$$\boxed{K_3 = 0.0218}$$

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Now

$K_4 = hf(x_n + h, y_n + K_3)$

$K_4 = 0.2 f(0 + 0.2, 0 + 0.0218)$

$K_4 = 0.2 f(0.2, 0.0218)$

$K_4 = 0.2 \left((0.2)^2 + 0.2 - 0.0218\right)$

$\boxed{K_4 = 0.04364}$

Now

$K = \frac{1}{6}(K_1 + 2K_2 + 3K_3 + K_4)$

$K_1 = 0$

$K_2 = 0.002$

$K_3 = 0.0218$

$K_4 = 0.04364$

$$K = \frac{1}{6}\left[0.2 + 2(0.002) + 3(0.0218) + 0.04364\right]$$

$$K = \frac{1}{6}\left[0.2 + 0.004 + 0.0654 + 0.04364\right]$$

$$K = \frac{1}{6}\left[0.31304\right]$$

$$K = 0.1666666(0.31304)$$

$$\boxed{K = 0.052173}$$

Answer

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Q#3

Given Data

$a=0$

$b=10$

$G=n=10$

$h=\frac{b-a}{n} \Rightarrow \frac{10-0}{10}=1$

$\boxed{h=1}$

Solution

x	0	1	2	3	4	5	6	7	8	9	10
fx	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.12	62.1

Using formula

$$f(x)dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3) - f(x_9)\right) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2\left(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2 + 62.1\right)\right]$$

$$= f(x)dx = 412.8 \quad \text{Ans}$$

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A Q#4 Estimate the ~~following~~ value of the following integral using Simpson's Rule.

$$\int_2^3 \ln(x^3+1)\,dx$$

Use 10 strips

$\Delta x = \frac{b-a}{2n} = \Delta x \Rightarrow \frac{3-2}{2(5)} = \frac{3-2}{10}$

$\Delta x = \frac{1}{10} = \boxed{0.1}$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
fx	2.19	2.32	2.45	2.57	2.69	2.81	2.9	3.02	3.13	3.23	3.33

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$$A = \frac{\Delta x}{3}\Big[f(x_0) + 2\big[f(x_2) + f(x_4) + f(x_6)\ f(x_8)\big] + 4\big[f(x_1) + f(x_3)$$

$$+ f(x_5) + f(x_7) + f(x_9)\big] + f(x_{10})\Big]$$

$$A = \frac{0.1}{3}\Big[2.19 + 2\big[2.45 + 2.69 + 2.92 + 3.13\big] + 4\big[2.32 + 2.57 +$$

$$2.81 + 3.02 + 3.23\big] + 3.33\Big]$$

$$A = 0.3\big[2.19 + 22.38 + 58.8 + 3.33\big]$$

$$A \approx 2.511$$

$$\boxed{\int_2^3 \ln(x^3+1)\,dx = 2.511}$$

Ans