NAME: DANISH HAYAT

STUDENT ID: 14566

SUBJECT: ELECTRO MAGNETIC FIELD THEORY

INSTRUCTOR: Dr RAFEEQ MANSOOR

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Q.1 The value of E at $P(\rho = 2, \phi = 40^\circ, Z = 3)$ is given as $E = 100\,a_\rho - 200\,a_\phi + 300\,a_z$ V/m. Determine the incremental work required to move a $20\,\mu C$ charged a distance of $6\,\mu m$.

Ans in the direction of a_ρ: The incremental work is given by $dW = -qE \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6 \times 10^{-6}\, a_\rho$. Thus

$$dW = -(20 \times 10^{-6} C)(100\, V/m)(6 \times 10^{-6} M) = -12 \times 10^{-9} J = -12 nJ$$

* in the direction of a_ϕ: in this case $dL = 2\,d\phi\, a_\phi = 6 \times 10^{-6}\, a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6}) = 2.4 \times 10^{-8} J = \underline{24 nJ}$$

* in the direction of a_z: Here, $dL = dz\, a_z = 6 \times 10^{-6}\, a_z$,

$$dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6}) = -3.6 \times 10^{-8} J = \underline{-36 nJ}$$

* in the direction of E: Here, $dL = 6 \times 10^{-6}\, a_E$, where

$$a_E = \frac{100\,a_\rho - 200\,a_\phi + 300\,a_z}{[100^2 + 200^2 + 300^2]^{1/2}} = 0.267\,a_\rho - 0.535\,a_\phi + 0.802\,a_z$$

Thus

$$dW = -(20 \times 10^{-6})[100\,a_\rho - 200\,a_\phi + 300\,a_z] \cdot [0.267\,a_\rho - 0.535\,a_\phi + 0.802\,a_z](6 \times 10^{-6}) = \underline{-44.9 nJ}$$

⇒ in The direction of $G = 2a_x - 3a_y + 4a_z$: in This case, $dl = 6\times10^{-6}\, a_G$, where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z$$

So now

$dw = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.371a_x - 0.557a_y + 0.743a_z](6\times10^{-6})$

$= -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9](6\times10^{-6})$

Where, at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos(40^\circ) = 0.766$, $(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$, and $(a_\phi\cdot a_x) = -\sin(40^\circ) = -0.643$. Subsituting These result in

$dw = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = \underline{-41.8\,nJ}$

Q2 Let $E = 400a_x - 300a_y + 500a_z$ in The neighborhood of point $P(6, 2, -3)$. Find The incremental work done in moving a 4-C charge a distance of 1 mm in The direction Specified by:

$\Rightarrow$ $a_x + a_y + a_z$: we write

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot \frac{(a_x + a_y + a_z)}{\sqrt{3}} (10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{3}} (400 - 300 + 500 = -1.39 \text{ J}$$

$\Rightarrow$ $-2a_x + 3a_y - a_z$: The computation is similar to that of part a, but we change the direction.

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot$$

$$\frac{(-2a_x + 3a_y - a_z)}{\sqrt{14}} (10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = \underline{2.35 \text{ J}}$$

Q3: if $E = 120\,a_\rho$ V/m, Find The incremental amount of work done in moving a 50 μm charge a distance of 2mm From.

Ans: $P(1,2,3)$ Toward $Q(2,1,4)$: The vector along This direction will be $Q-P=(1,-1,1)$ from which $a_{PQ} = [a_x - a_y + a_z]\,/\sqrt{3}$. we now write

$$dW = -qE\cdot dL = -(50\times10^{-6})\left[120\,a_\rho\cdot\left(\frac{a_x-a_y+a_z}{\sqrt{3}}\right)\right](2\times10^{-3})$$

$$= -(50\times10^{-6})(120)\left[(a_\rho\cdot a_x) - a_\rho\cdot a_y)\right]\frac{1}{\sqrt{3}}(2\times10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho\cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho\cdot a_y) = \sin(63.4) = 0.894$. Subsituting These, we obtain $dw = \underline{3.1\,\mu J}$.

→ $Q(2,1,4)$ Toward $P(1,2,3)$: A Little Thought is in order here: Note That The field has only a radial component and does not depend on ϕ or z. Note also That P and Q are at The Same radius $(\sqrt{5})$ from The z axis, but have different ϕ and z coordinates. We could Just as well Position The Two Points at The Same z location and The Problem would not change.

if This were So, Then moving along a straigth.

line Between P & Q would involve moving along a chord of a circle whose radius $\sqrt{5}$. Halfway along this line point of symmetry in field (make a sekch to see this). This means that when starting from either Point the intial force will be same. Thus the answer $dW = 31\,\mu J$ as Part a. This is also found by going through the same procedure as part a, but with the direction (role of P & Q) reversed.

Q_4 Compute The value of G - - - - - - - - - using The Path.

Ans⇒ Striaght line of Segments $A(1,-1,2)$ To $B(1,1,2)$ To $P(2,1,2)$ In general we have

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The Change of x occurs when moving b/w B and P during which $y = 1$ Thus

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \boxed{2}$$

(B) Straigh line Segment $A(1,-1,2)$ $C(2,-1,2)$ To $P(2,1,2)$ In case The change in x occur when moving From A To C, during which $y = -1$ Thus

$$\int_A^P G \cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \boxed{-2}$$

Q5 For $G = 3xy^3 a_x + 2z a_y$. Now Things - - -
- - - - - in That path does matter.

Ans Straight line $y = x-1, Z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2z dy = \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1) dy = \boxed{90}$$

B. Parabola $6y = x^2 + 2, Z = 1$ We obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2Z dy$$

$$\Rightarrow \int_2^4 \frac{1}{12} x (x^2+2)^2 dx + \int_1^3 2(1) dy = \boxed{82}$$