# Assignment

Q1 comput adjoint of ;

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

(ii)

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

part (i)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1 = 5$$

$$A_{12} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = (-1)^3 |4-3| = -1(1) = -1$$

$$A_{13} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = (-1)^{1+3}(2-9) = 1(-7) = -7$$

$$A_{21} \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = (-1)^{2+1}(4-2) = -1(2) = -2$$

$$A_{22} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (-1)^{2+2}(2-6) = 1(-4) = -4$$

$$A_{23} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (-1)^{2+3}(1-6) = -1(-5) = 5$$

$$A_{31} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = (-1)^{3+1}(2-6) = 1(-4) = -4$$

$$A_{32} \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = (-1)^{3+2}(2-2) = -1(0) = 0$$

$$A_{33} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (-1)^{3+3}(3-4) = (1)(-1) = -1$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -2 & -4 & 5 \\ -4 & 0 & -1 \end{bmatrix}$$

(iii)

$B =$

$$A^T \begin{bmatrix} 5 & -2 & -4 \\ -1 & -4 & 0 \\ -7 & 5 & -1 \end{bmatrix} \rightarrow \text{Adj } A = \begin{bmatrix} 5 & -2 & -4 \\ -1 & -4 & 0 \\ -7 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Part (ii)

$$\text{(ii) } B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

$B_{11} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (-8-(-16) = 8$

$B_{12} \begin{vmatrix} -2 & 5 \\ -4 & -1 \end{vmatrix} = 2-(-20) = 22$

$B_{13} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4-(-5) = -4+5 = 1$

$B_{21} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (32-(-10)) = 42$

$B_{22} \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = 24-25 = -1$

$B_{23} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6-20 = -26$

$A_{31} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 32-10 = +22$

$A_{32} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = 24-10 = 14$

$A_{33} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = -11$

$$B = \begin{bmatrix} 8 & -22 & 1 \\ -42 & +1 & +26 \\ 22 & -14 & -11 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 8 & 42 & 22 \\ -22 & 1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$\text{Adj}B = \begin{bmatrix} 8 & 42 & 22 \\ -22 & 1 & 14 \\ 1 & 26 & -11 \end{bmatrix}$$

Q2 confactors $A_{21}, A_{31}, A_{33}$

Find $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

Solution

Confactor $= (-1)^{R+C} \left| \quad \right|$

we find $A_{21}, A_{31}, A_{33}$

$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

Part 1

$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)^3 [-4-(-9)]$

$(-1) \cdot (-4+9) = (-1)(+5) = -5$

$\boxed{A_{21} = -5}$

Part 2

$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (-1)^4 \cdot (-2-9) = 1(-11)$

$\boxed{A_{31} = -11}$

Part 3: $A_{33} (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (-1)^6 (3-(+4))$

$= +1(3-4)$

$= (1)(-1) = -1$

$\boxed{A_{33} = -1}$

Q3

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad \& \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

Step#1 :- $(\underset{\text{given matrix}}{A} - \underset{\text{constant}}{\lambda} \underset{\text{Identity Matrix}}{I}) X = 0$ - Null matrix

Step#2: $|A - \lambda I| = 0$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 4$$

$$\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 3$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$$

Step#3:-

$$\lambda^3 - \begin{Bmatrix} \text{sum of} \\ \text{diagonal} \\ \text{Element} \end{Bmatrix} \lambda^2 + \begin{Bmatrix} \text{sum of} \\ \text{diagonal} \\ \text{minors} \end{Bmatrix} \lambda - \{ \quad \}$$

$\therefore \lambda^3 - 6\lambda^2 + 12\lambda - 10 = 0$

$\boxed{\lambda = 1, 2, 3}$ - Eigen value

Part 2 Eigen vectors:- put $\lambda = 1$

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 1 & 3-1 & 0 \\ 0 & 0 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 1: Crammer Rules

$1x_1 + 1x_2 + x_3 = 0$ —— (i)

$1x_1 + 2x_2 + 0 = 0$ —— (ii)

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{(0-2)} = \frac{x_2}{(0-1)} \quad \frac{x_3}{(2-1)}$$

$$\frac{x_1}{-2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

Put $\lambda = 2, 3$

$$\begin{bmatrix} 2-2 & 1 & 1 \\ 1 & 3-2 & 0 \\ 0 & 0 & 2-2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 1

$$0x_1 + 1x_2 + 1x_3 = 0$$

$$1x_1 + 1x_2 + 0x_3 = 0$$

Crammer rules

$$\frac{x_1}{0} = \frac{x_2}{\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}} \qquad X_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ |
|---|---|---|
| 0 | -0 | 0 |

Put $\lambda = 3$,

$$\begin{bmatrix} 2-3 & 1 & 1 \\ 1 & 3-3 & 0 \\ 0 & 0 & 2-3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{x_1}$ $\cancel{x_2}$ $\cancel{x_3}$

Cramaer rule

$$1x_1 + 1x_2 + x_3 = 0 \quad — \text{(I)}$$

$$x_1 + 0x_2 + 0x_3 = 0 \quad — \text{(II)}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{x_2}{-1} = \frac{x_3}{-1}$$

$$x_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$