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SubJect Differential

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Q1 :- Estimate General solution of

$$4y'' - 20y' + 25y = 0$$

Sol:- The roots of charalteristic equations

$$\Rightarrow 4r^2 - 20r + 25 = 0$$

$$r_1 = r_2 = \frac{5}{2}$$

$$\therefore y = c_1 y_1 + c_2 y_2$$

$$= c_1 e^{\frac{5}{2}t} + c_2 t e^{5/2 t}$$

$$\boxed{= c_1 e^{\frac{5}{2}t} + c_2 t e^{5/2 t}}$$

Ans

Q2:- (a) Calculate the initial value Problem $y'' + 2y' + y = 0$

$y(0) = 4 \quad y'(0) = -6$

Sol:-

$$\frac{d^2 y(x)}{dx^2} + 2\frac{dy(x)}{dx} + y(x) = 0$$

such that $y(0) = 4 \quad y'(0) = -6$

Assume a solution will be Proportional to $e^{\lambda x}$ for some Constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation

$$\frac{d^2}{dx^2}(e^{\lambda x}) + 2\frac{d}{dx}(e^{\lambda x}) + e^{\lambda x} = 0$$

Solve for the unknown constant using the initial conditions.

Compute $\frac{d\,y(x)}{dx}$:

$$\frac{d\,y(x)}{dx} = \frac{d}{dx}\left(c_1 e^{-x} + c_2 e^{-x} x\right)$$

Substitute $y(0) = 4$ into $y(x)$

$$y(x) = e^{-x} c_1 + e^{-x} x\, c_2$$

Substitute $y'(0) = -6$ into

$$\frac{d\,y(x)}{dx} = -e^{-x} c_1 + e^{-x} c_2 - e^{-x} x\, c_2$$

$$-c_1 + c_2 = -6$$ solve the system

$$c_1 = 4 \qquad c_2 = -2$$

Substitute $c_1 = 4$ and $c_2 = -2$ into

$$y(x) = e^{-x} c_1 + e^{-x} x\, c_2$$

$$\boxed{y(x) = -2e^{-x}(x-2)}$$

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Q2 (i) Analyze the general solution of $x^2 y'' + 3xy' + y = 0$

Sol:

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = 0 \quad \text{---}$$

$a_0 = x^2 \quad a_0' = 2x \quad a_0'' = 2$

$a_1 = 3x \quad a_1' = 3$

$a_2 = 1 \quad f(x) = 0$

Condition for exactness is

$a_0'' - a_1' + a_2 = 0$

Consider $a_0'' - a_1' + a_2 = 2 - 3 + 1$

$= 3 - 3 = 0$

To find the first integrat

$a_0 y' + (a_1 - a_0')y = \int f(x)\, dx + c_1$

$x^2 y' + (3x - 2x)y = \int 0 + C_1$

$x^2 y' + xy = c_1$

$y' + \frac{y}{x} = \frac{c_1}{x^2}$

Int. factor $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

Solution is given by

$yx = \int \frac{c_1}{x^2} \cdot x + c_2$

$\boxed{yx = c_1 \log x + c_2}$ Ans

Q3:- Examine the method of undetermined Coefficient method

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$

Sol:-

$y'' + y' - 6 = 0$

Auxillary equation

$r^2 + r - 6\cancel{y} = 0$

$r^2 + 3r - 2r - 6 = 0$

$(r+3) - 2(r+3) = 0$

$r + 3 = 0, \quad r - 2 = 0$

Root are real and distanct

$y = C_1 e^{-3x} + C_2 e^{2x}$

$y_P = K_3 x^3 + K_2 x^2 + K_1 x + K_0$

$y'_P = 3K_3 x^3 + 2Kx^2 + K_1$

$y'_P = 6K_3 x - 2K_2$ ~~Put in~~ (1)

Put in equation

$6K_3x - 2K_2 + 3K_3x^2 + 2K_2x + K_1 - 6K_3x^3$
$-6K_2x^2 - 6K_1x - 6K_0$
$= 6x^3 - 3x^2 + 12x$

Comparing.

$-6K_3 = 6$

$\boxed{K_3 = -1}$

$-6K_2 + 3K = -3 = -6K_2 + 3(-1) = -3$
$= 6K_2 - 3 = -3 \quad = -6K_2 = -3 - 3$

$\boxed{K_2 = 0}$

$6K_3 + 2K_2 + K_1 = 12$
$6(-1) + 2(0) + K_1 = 12$

$\boxed{K_1 = -2}$

$-2K_2 + K_1 + K_0 = 0$
$-2(0) \cancel{+} - 2 + K_0 = 0$

$\boxed{K_0 = 2}$ Hence

$y = y_h + y_c$

$y = c_1 e^{-3x} + c_2 e^{2x} - x^3 + 0x^2 - 2x + 2$

Q4:- Examine method of variation of Parameters

$$y'' - 4y' + 4y = x^2 e^{2x}$$

Sol:- For equation

$$y'' - 4y' + 4y = 0$$

$$r^2 - 4r + 4 = 0$$

$$r^2 - 2r - 2r + 4 = 0$$

$$r(r-2) - 2(r-2) = 0$$

$$(r-2)(r-2) = 0$$

$$r = 2 \;,\; r = 2$$

Root are real and equal

$$y = (c_1 + c_2 x) e^{2x}$$

$$y_1 = c_1 e^{2x} + c_2 x e^{2x}$$

$$y_1 = e^{2x} \;,\; y_2 = x e^{2x}$$

$$y'_1 = 2e^{2x} \;,\; y'_2 = e^{2x} + 2x e^{2x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix}$$

$$= e^{4x} + \cancel{2xe^{4x}} - \cancel{2xe^{4x}}$$

$$W = e^{4x}$$

$$y_p = -y_1 \int \frac{y_2\, r(x)}{W} + y_2 \int \frac{y_1\, r(x)}{W}$$

$$y_p = -e^{2x} \int \frac{xe^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx + xe^{2x} \int \frac{e^{2x} \cdot x^2 e^{2x}}{e^{4x}}$$

$$= -e^{2x} \int \frac{x^3 \cancel{e^{4x}}}{\cancel{e^{4x}}} + xe^{2x} \int \frac{x^2 \cancel{e^{4x}}}{\cancel{e^{4x}}} \cdot dx$$

$$= -e^{2x} \int x^3\, dx + xe^{2x} \int x^2\, dx$$

$$y_p = -e^{2x} \cdot \frac{x^4}{4} + x\, e^{2x} \cdot \frac{x^3}{3}$$

so

$$y = y_h + P_p$$

$$y = C_1 e^{2x} + C_2 x e^{2x} - e^{2x} \frac{x^4}{4} + xe^{2x} \frac{x^3}{3}$$

Q5:: Identify an ODE

$y'' + ay' + by = 0$ for the basis 1

e^{-3x}

Sol::

$y'' + ay' + by = 0$ basis 1, e^{-3x}

$y = C_1 e^{x} + C_2 e^{-3x}$

$\lambda = 0 \qquad \lambda = -3$

$\lambda_1 - 0 = 0 \qquad \lambda + 3 = 0$

$\lambda(\lambda + 3) = 0$

$\lambda^2 + 3\lambda = 0$

$a = 3 \; ; \; b = 0$

$y'' - 3y' + 0y = 0$

$$\boxed{y'' - 3y' = 0}$$

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