MID TERM EXAM

NAME ⇒ MUHAMMAD TALHA

ID NO ⇒ 7965

SUBJECT ⇒ MOS(II)

SUBMITTED TO ⇒ SIR SAQIB

SECTION ⇒ "B"

DEPART ⇒ BE (CIVIL)

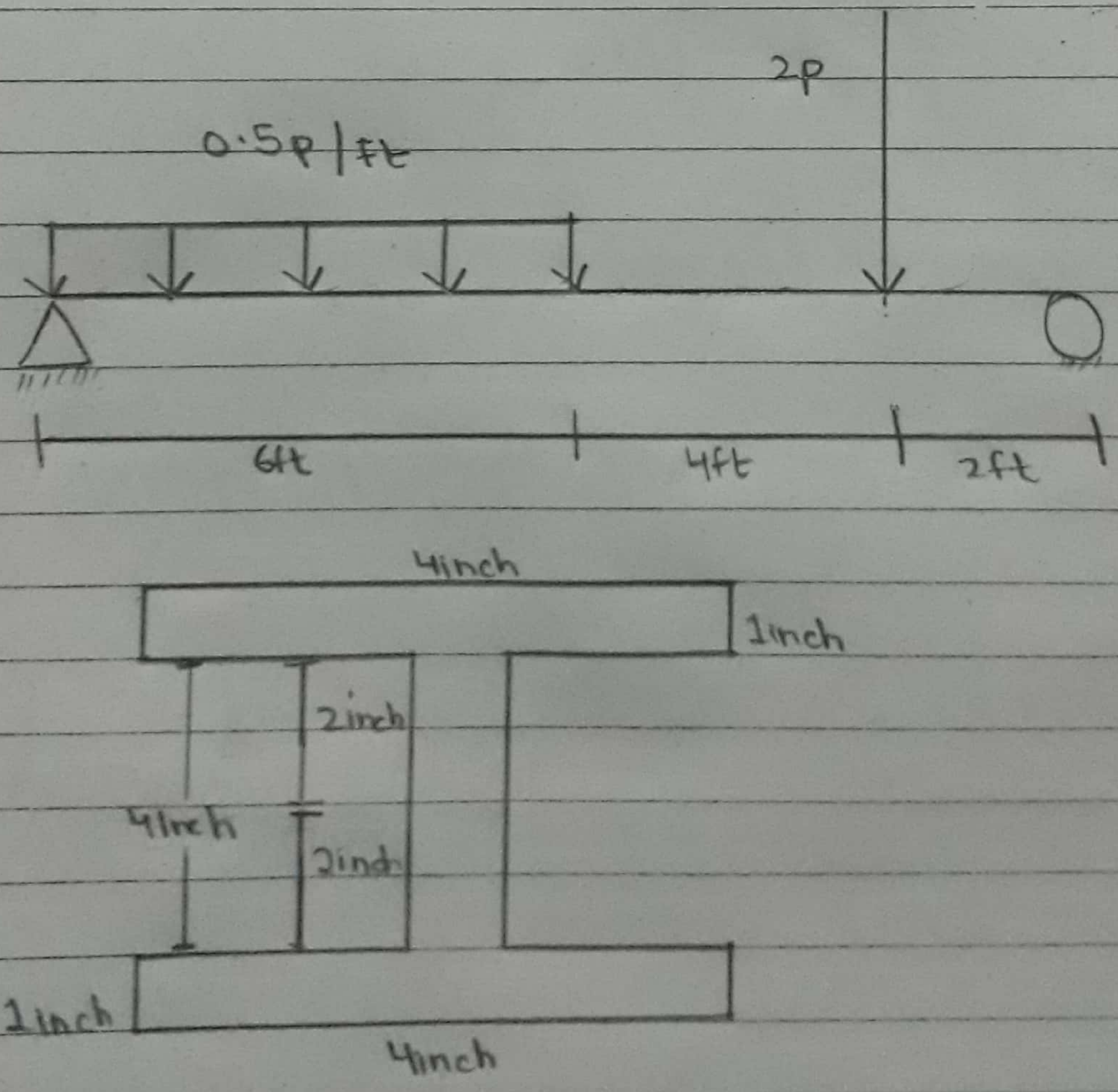
SEMESTER ⇒ "4"

DATE ⇒ 19-04-2020

QUESTION : 02

Construct the Mohr's Circle Diagram and Find the principal Stress and maximum in plane Shear Stress For the stress State of a point C Located at the Centre of UDL and 1 inches . . . .

. . . . equations ?



REACTIONS:

$\Sigma F_y = 0$ ↑+ upward is positive

$$R_A + R_B - 32.5 \times 6 - 130 = 0$$
$$R_A + R_B = 325 \quad - (A)$$

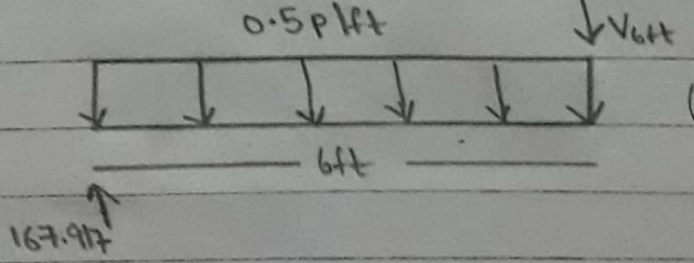
$\Sigma M_A = 0$ ↺ + Anticlock wise is +tive

$$R_B(12) - 130(10) - 32.5(6) \times 3 = 0$$
$$R_B(12) - 1885 = 0$$
$$R_B = 1885/12$$
$$R_B = 157.083 \text{ lb}$$

Now,

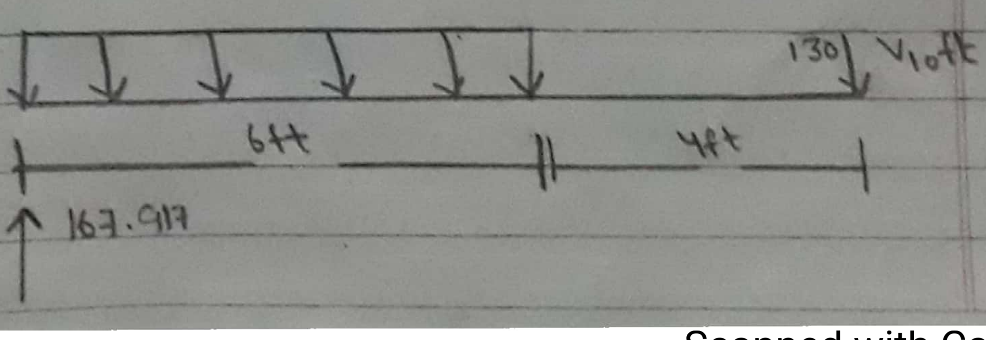
$$R_A = 325 - 157.083$$
$$R_A = 167.917 \text{ lb}$$

SHEAR FORCE:



(At 6 feet)

(At 10 feet)



$\Sigma F_y = 0 \uparrow +$ (At 6ft from Left Support)

$$-V_{6ft} + 167.917 - 32.5 \times 6 = 0$$
$$V_{6ft} = -27.083 \text{ lb}$$

$\Sigma F_y = 0$ (At 10 ft from Left Support)

$$167.917 - 32.5 \times 6 - 130 - V_{10ft} = 0$$
$$-157.083 - V_{10ft} = 0$$
$$-V_{10ft} = 157.083$$
$$V_{10ft} = -157.083 \text{ lb}$$

BENDING MOMENT :

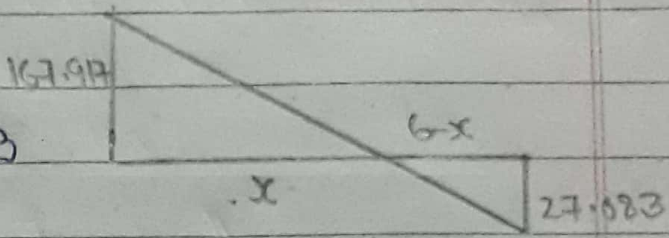
$$\Sigma M_{6ft} = 0$$
$$\Sigma M_{6ft} = -167.917(6) + 32.5(6)(6/3)$$
$$\Sigma M_{6ft} = 617.502 \text{ lb ft}$$

Now we have to find Moment at 3ft

$$\Sigma M_{3ft} = 0$$
$$\Sigma M_{3ft} = -167.917(3) + 32.5(6)(3)$$
$$= 81.249 \text{ lb ft}$$

4 A

Now we have to find the Moment at change point :-



$$\frac{167.917}{x} = \frac{27.083}{(6-x)}$$

$$167.917(6-x) = 27.083(x)$$

$$1007.502 - 167.917x = 27.083x$$

$$1007.502 = 27.083x + 167.917x$$

$$1007.502 = 195x$$

$$x = 5.166 \text{ ft}$$

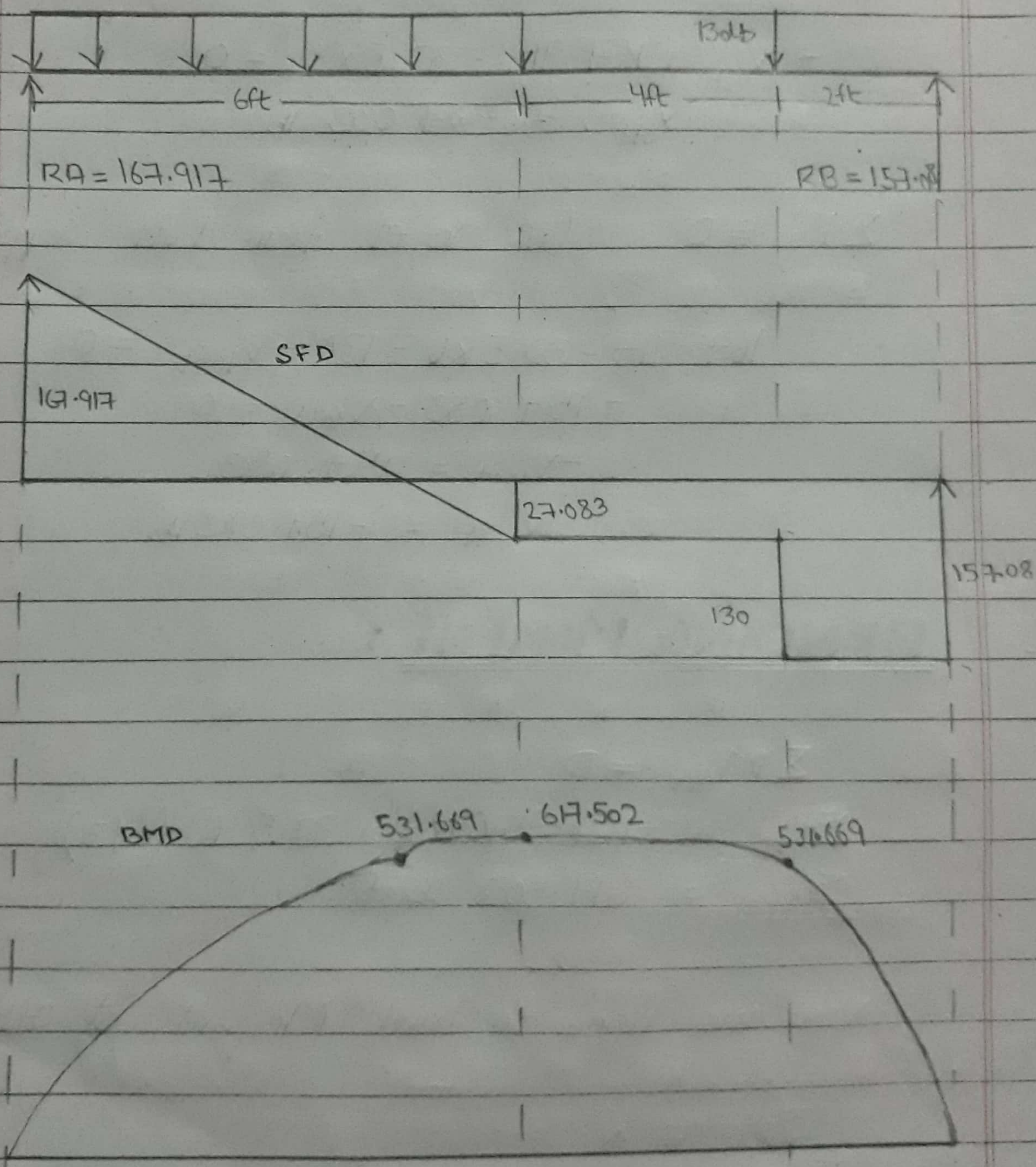
$$\Sigma M_{5.166} = 0 \; ↺+$$

$$\Sigma M_{5.166} = -167.917(5.166) + 32.5(6)(5.166/3)$$

$$\Sigma M_{5.166} = 531.669 \text{ lbft}$$

Scanned with CamScanner

SHEAR FORCE, BENDIGG MOMENT DIAGRAM



MOMENT OF INERTIA :

$y_1 = 5.5$

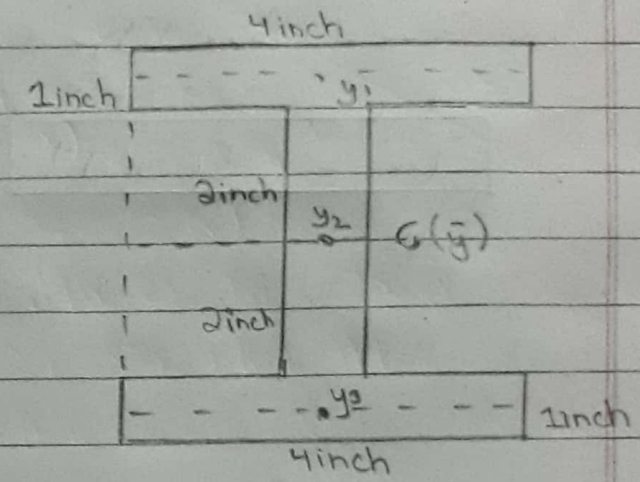
$y_2 = 3$

$y_3 = 0.5$

$A_1 = 4\,inch^2$

$A_2 = 4\,inch^2$

$A_3 = 4\,inch^2$



Now, $\bar{y} = \frac{A_1 \times y_1 + A_2 \times y_2 + A_3 \times y_3}{A_1 + A_2 + A_3}$

$\bar{y} = \frac{4 \times 5.5 + 4 \times 3 + 4 \times 0.5}{4+4+4}$

$\boxed{\bar{y} = 3''}$

$I_1 = \frac{bh^3}{12}$

$I_1 = \frac{4 \times 1^3}{12}$

$I_1 = 0.33\ inch^4$

$I_3 = \frac{b \times h^3}{12}$

$I_3 = \frac{4 \times 1^3}{12}$

$I_3 = 0.33\,inch^4$

$I_2 = \frac{h^3 \times b}{12}$

$I_2 = \frac{1 \times 4^3 \times 1}{12}$

$I_2 = 5.33\ inch^4$

$\boxed{d}$

$d_1 = \bar{y}' - y_1$

$d_1 = 3 - 5.5$

$d_1 = -2.5$

$d_2 = \bar{y}' - y_2$

$d_2 = 3-3$

$d_2 = 0$

$d_3 = \bar{y}' - y_3$

$d_3 = 3-0.5$

$d_3 = 2.5$

$\boxed{A d^2}$

1. $A_1 d_1^2$

$= 4 \times (-2.5)^2$

$= 25 \text{ inch}^4$

2. $A_2 d_2^2$

$= 4 \times 0$

$= 0$

3. $A_3 d_3^2$

$= 4 \times (2.5)^2$

$= 25 \text{ inch}^4$

$I_{1x} = I_1 + A_1 d_1^2$

$I_{1x} = 0.33 + 25$

$I_{1x} = 25.33 \text{ inch}^4$

$I_{2x} = I_2 + A_2 d_2^2$

$I_{2x} = 5.33 + 0$

$I_{2x} = 5.33 \text{ inch}^4$

$I_{3x} = I_3 + A_3 d_3^2$

$I_{3x} = 0.33 + 25$

$I_{3x} = 25.33 \text{ inch}^4$

Now,

$I_{xx} = I_{1x} + I_{2x} + I_{3x}$

$I_{xx} = 25.33 + 5.33 + 25.33$

$\boxed{I_{xx} = 56 \text{ inch}^4}$

SHEAR STRESS :

$\boxed{\text{For } V = 157.083}$

Case NO 1 :

$$\tau_{\text{Top fiber}} = \frac{VQ}{Ib}$$

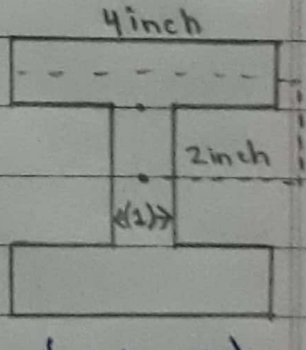
$$= \frac{157.083 \times 0}{56}$$

$$= 0 \text{ psi}$$

Case NO 2A: (For 1 Inch below Top fiber)

$$\tau_{1A} = \frac{157.083 \times 10}{56(4)} \quad (\because b = 4)$$

$$= 7.01 \text{ psi}$$



Case No 2B :

$$\tau_{1B} = \frac{157.083 \times (10)}{56(1)} \quad (\because b = 1)$$

$$\tau_{1B} = 28.05 \text{ psi}$$

$\therefore Q = \bar{y} \times A$

$\bar{y} = 2 + \frac{1}{2}$

$\bar{y} = 2.5$

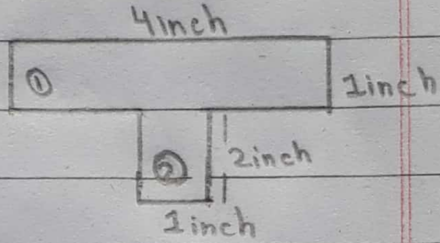
$A = 1 \times 4 = 4$

$Q = 4 \times 2.5$

$Q = 10$

Case No 3 :- (Stress At Centroidal Axis)

$$\tau_{max} = \frac{VQ}{Ib}$$



$Q = Q_1 + Q_2$

$Q = 10 + 1(2)$
$= 12$

So,

$$\tau_{max} = \frac{157.083 \times 12}{56(1)}$$

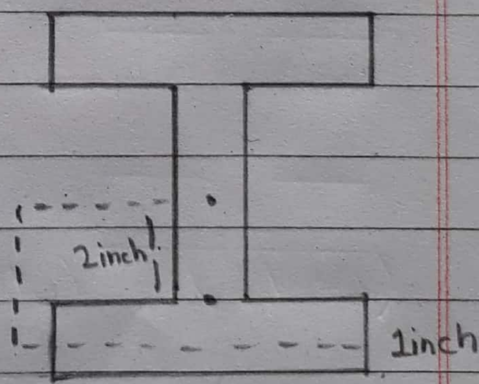
$= 33.66$ psi

Case No 4A :- (1 inch Above Bottom)

$$\tau_{2A} = \frac{VQ}{It} \quad \therefore b = 4$$

$$= \frac{157.083 \times 10}{56 \times 4}$$

$= 7.01$ psi



Case No 4B :-

$$\tau_{2B} = \frac{VQ}{It} \quad \therefore b = 1$$

$$= \frac{157.083 \times 10}{56 \times 1}$$

Case No 5 :- (At Bottom fiber)

$$\tau_{Bottom} = \frac{VQ}{It}$$

$$\tau_{Bottom} = \frac{157.083 \times 0}{56(4)}$$

$$= 0 \text{ psi}$$

Case No 6 :-

"Shear force at a distance of 3 ft from left Support of beam along its length"

$$V_{6ft} = -27.083$$
$$Q = 12$$

$$\tau_{max} = \frac{27.083 \times 12}{56(1)}$$
$$= 5.803 \text{ psi}$$

Case No 7 :- (At a distance & 1 inch below)

For $b = 4$ $\quad \tau_A = \frac{27.083 \times 10}{56(4)}$

$$= \boxed{1.2090 \text{ psi}}$$

For $b = 1$

$$\tau_B = \frac{27.083 \times 10}{56(1)}$$

$$= 4.836 \text{ psi}$$

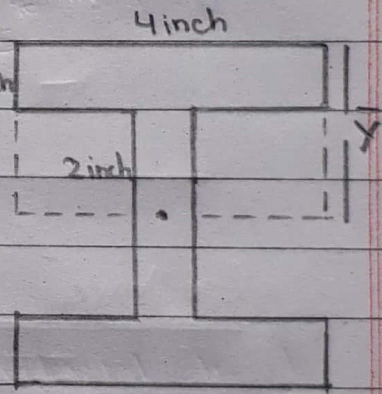
FLEXURAL STRESS :

$$\boxed{\sigma = \frac{My}{I}}$$

Moment = 617.502

Moment of Inertia = I = 56

Case No 1 : (stress at top of fiber)



$$\sigma_{Top} = \frac{My}{I}$$

$$= \cancel{12.21}$$

$$= \frac{617.502 \times 3}{56}$$

$$= 33.080 \text{ psi}$$

Case No 2 : (1 Inch below top fiber)

$$\sigma_1 = \frac{My}{I}$$

$$= \frac{617.502 \times 2}{56}$$

$$\boxed{\sigma_1 = 22.053 \text{ psi}}$$

Case No 3 : (At Geometrical Centroid)

$$\sigma_{Center} = \frac{My}{I} \quad \because \bar{y}(0)$$

$$= 0 \text{ psi}$$

Case No 4 :- (1 inch above the bottom fiber)

$$\sigma = \frac{My}{I}$$

$$= \frac{617.502 \times 2}{56}$$

$$= 22.05 \text{ psi}$$

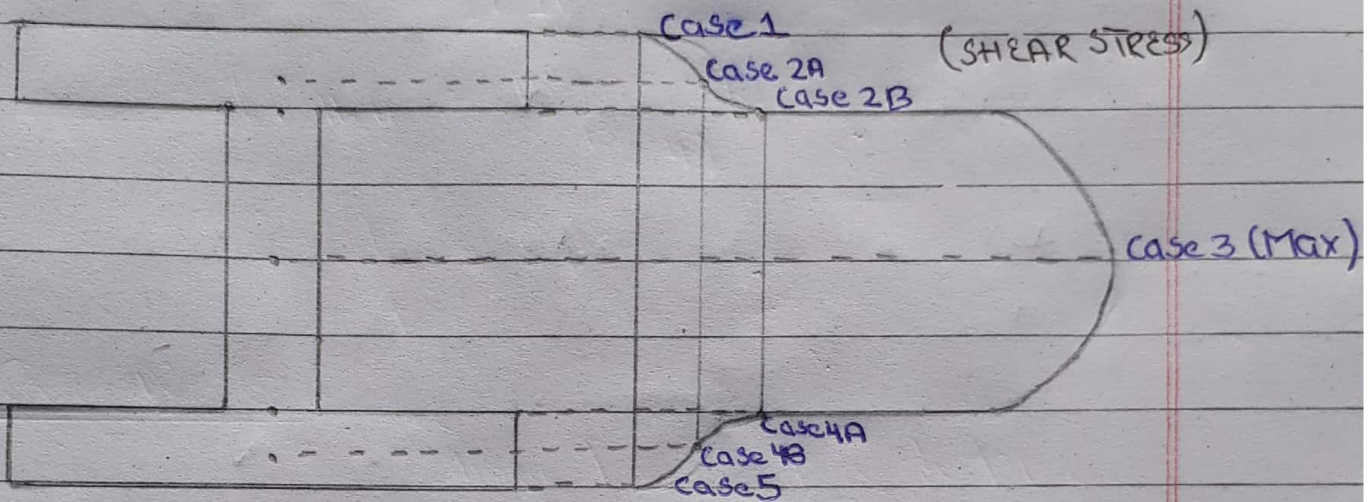
Case No 5 : (At bottom fiber)

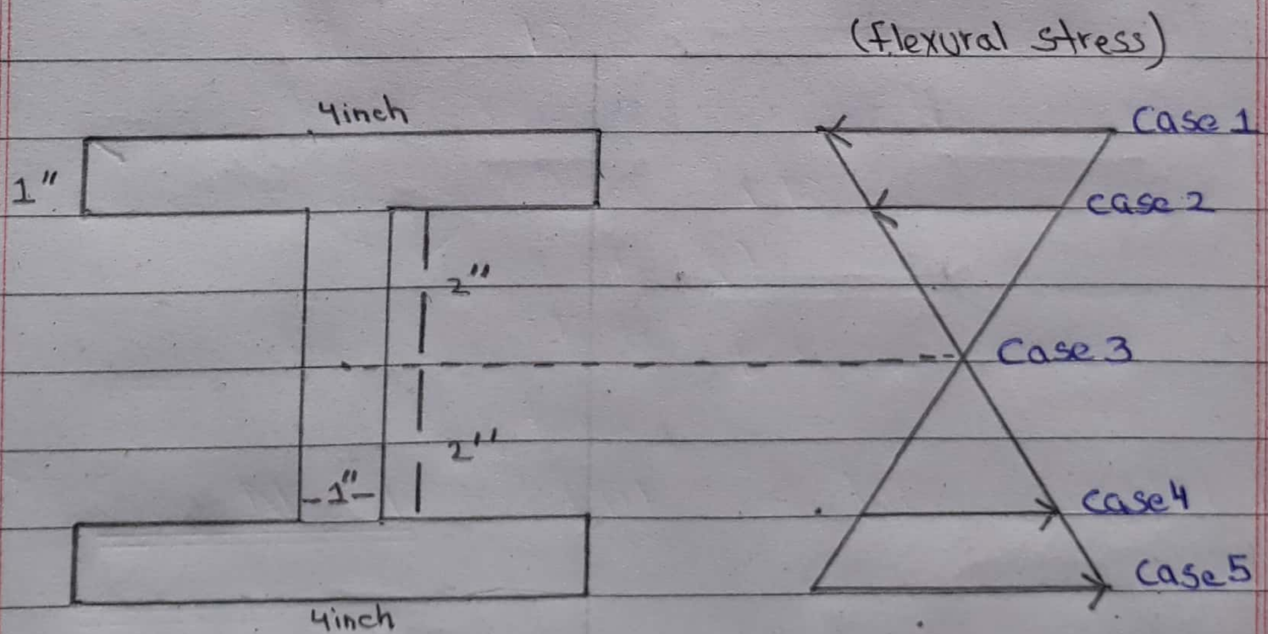
$$\sigma_{Bottom} = \frac{My}{I}$$

$$\sigma_{Bottom} = \frac{617.502 \times 3}{56}$$

$$= 33.08 \text{ psi}$$

SHEAR FORCE AND BENDING STRESS ~~MOMENT~~ DIAGRAM VARIATION





★ Stress State of a point Element :-

We find all stresses, which is acting on the I-Section Beam.
Given stress state Condition is at point C which is the Centre of udl at 3ft.
Thus, Flexural stress at point "C",

$\sigma_x = 22.053$ psi ∴ from Case 2, point C lies in this Case; At 1 inch below top fiber.

Now, Shear stress at point "C",

$\tau_{xy} = 1.2090$ psi ∴ from case No 7.

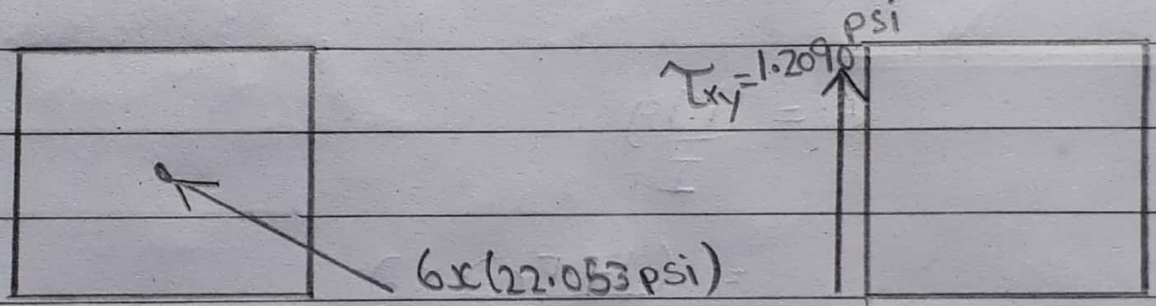
Consider this point "C" is a planar Element.

Day: M T W T F S

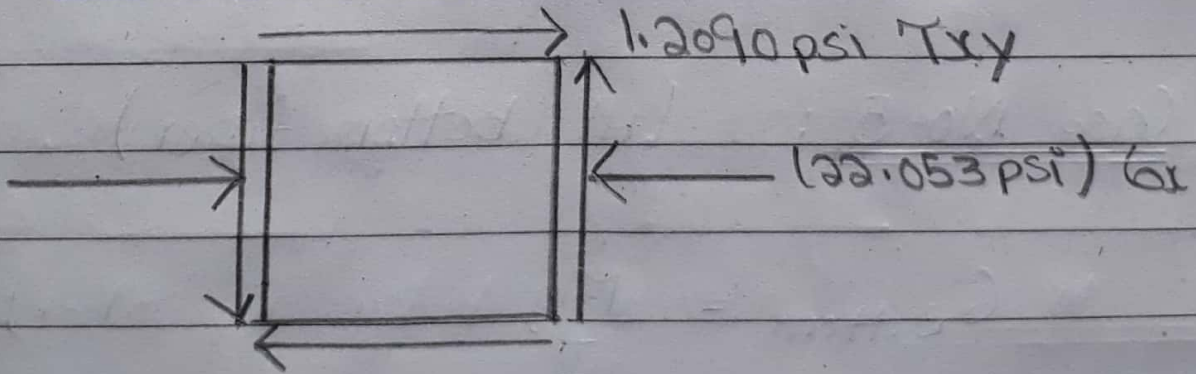
$\sigma_x = -22.053$ (compressive)

-22.053 is compressive Because it (point c) lies in the Centre, and in the Compression Zone of beam Cross Section.

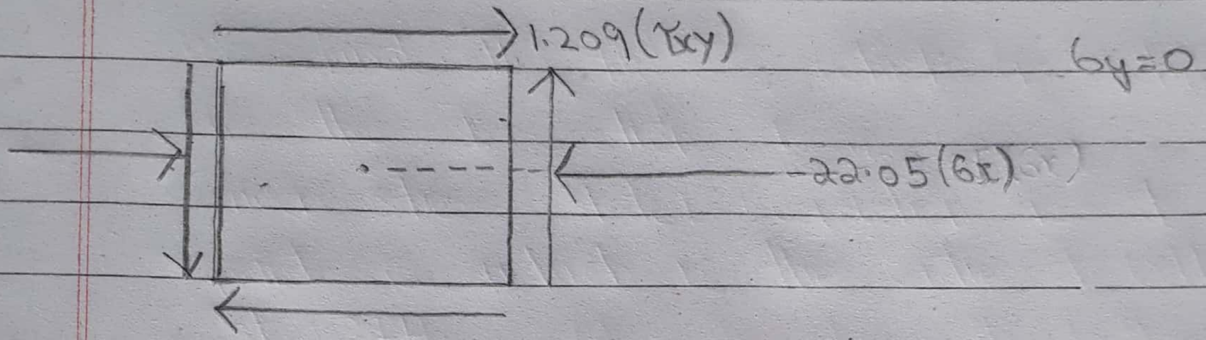
If point c lies below the Centroid then stress would be tensile.



Combine Stress in 2D Element :

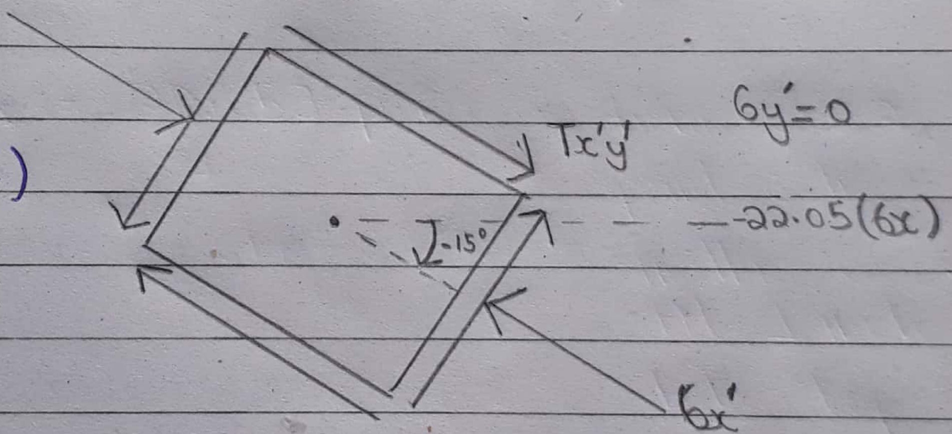


STRESS TRANSFORMATION :



Let assume teta (θ) = -15° For New Orientation

$\theta = -15$
(clockwise -)



Now,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{x'} = \frac{-22.05 + 0}{2} + \frac{-22.05 - 0}{2} \cos 2(-15) + 1.209\{\sin 2(-15)\}$$

$$= -11.025 - 11.025(0.866) + 1.209(-0.5)$$

$$= -21.177 \text{ psi}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \left(\frac{-22.05+0}{2}\right) - \left(\frac{-22.05-0}{2}\right)\cos 2(-15) - 1.209\{\sin 2(-15)\}$$

$$= -11.025 + 11.025(0.866) - 1.209(-0.5)$$

$$= -0.8 \text{ psi}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-22.05-0}{2}\right)\sin 2(-15) + 1.209\{\cos 2(-15)\}$$

$$= 11.025(-0.5) + 1.209(0.866)$$

$$\tau_{x'y'} = -4.46 \text{ psi}$$

PRINCIPLE STRESS :

$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}}$$

$\sigma_x = -22.05$, $\sigma_y = 0$, $\tau_{xy} = 1.209$

$$\tan 2\theta_p = \frac{1.209}{(-22.05-0)/2}$$

$$= \frac{1.209}{-11.025}$$

$$= -0.109$$

$$\tan \theta_p = -\frac{0.109}{2}$$

$$= -0.05$$

$$\theta_p = \tan^{-1}(-0.05)$$

$$\theta_p = -2.86^\circ$$

Now,

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_x' = \left(\frac{-22.05+0}{2}\right) + \left(\frac{-22.05-0}{2}\right) \cos 2(-2.86)$$

$$+ 1.209 \{\sin 2(-2.86)\}$$

$= -11.025 - 11.025\,(0.99) + 1.209\,(-0.09)$

$= -22.04$ psi

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \left(\frac{-22.05 + 0}{2}\right) - \left(\frac{-22.05 - 0}{2}\right)\cos 2(-2.86) - (1.209)\sin 2(-2.86)$$

$= -11.025 + 11.025\,(0.99) - 1.209\,(-0.09)$

$= -0.001$ psi

SHEAR STRESS :

$$\boxed{\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}}$$

$$\tan 2\theta_s = -\left(\frac{(-22.09-0)/2}{\tau_{xy}}\right)$$

$$= -\frac{(-22.09-0)/2}{1.209}$$

$$\tan 2\theta_s = -(-9.11)$$

$$\tan \theta_s = 9.11/2$$

$$\tan \theta_s = 4.55$$

$$\theta_s = \tan^{-1}(4.55)$$
$$= 77.60^\circ$$

Now, $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$

$$\tau_{x'y'} = -\left(\frac{-22.05-0}{2}\right) \sin 2(77.60) + 1.209 \{\cos 2(77.60)\}$$

$$= 11.025(0.419) + 1.209(-0.907)$$

$$\tau_{x'y'} = \cancel{3}\,5.22 \text{ psi}$$

ate:
Day: S M T W T F S S

MOHR'S CIRCLE :

Mohr's Circle Center Coordinates :-

$$(h,k) = \left[\frac{\sigma_x+\sigma_y}{2}, 0\right]$$

$$= \left[\frac{-22.05+0}{2}, 0\right]$$

$$= \left[-11.025, 0\right]$$

$$\text{Radius} = r = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-22.05-0}{2}\right)^2 + (1.209)^2}$$

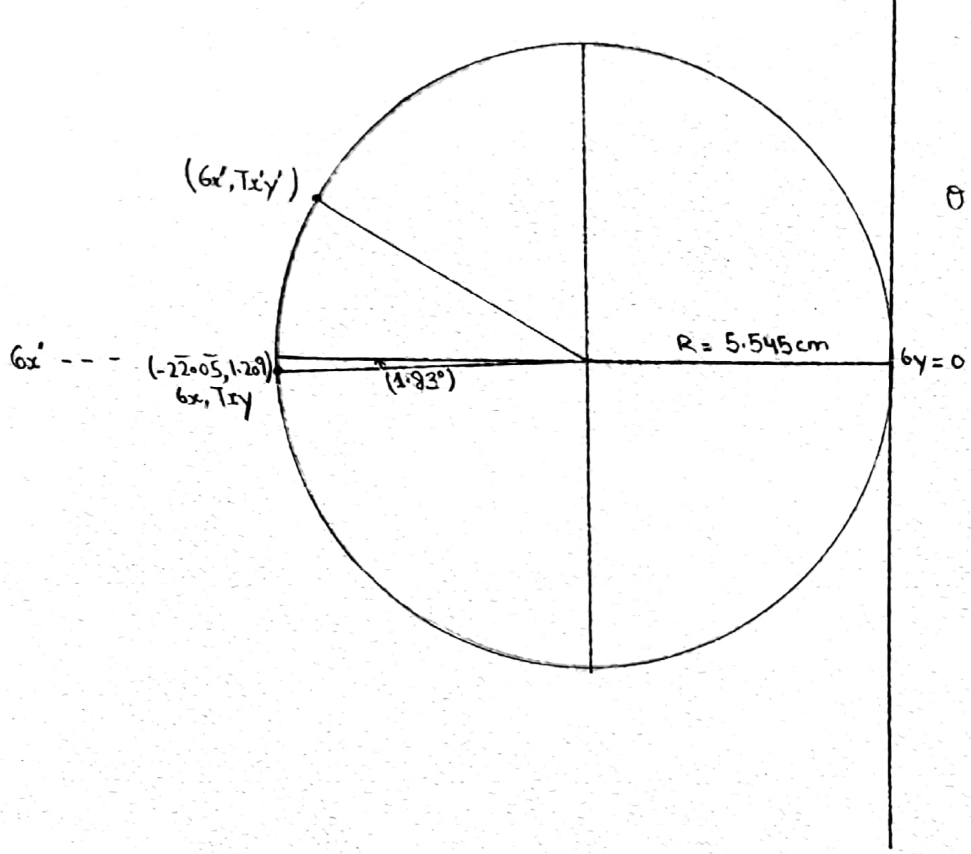
$$= \sqrt{(-11.025)^2 + (1.209)^2}$$

Radius = 11.09 psi

MOHR'S CIRCLE

$\sigma_x = -22.05$ psi , $\sigma_y = 0$, $\tau_{xy} = 1.209$ psi

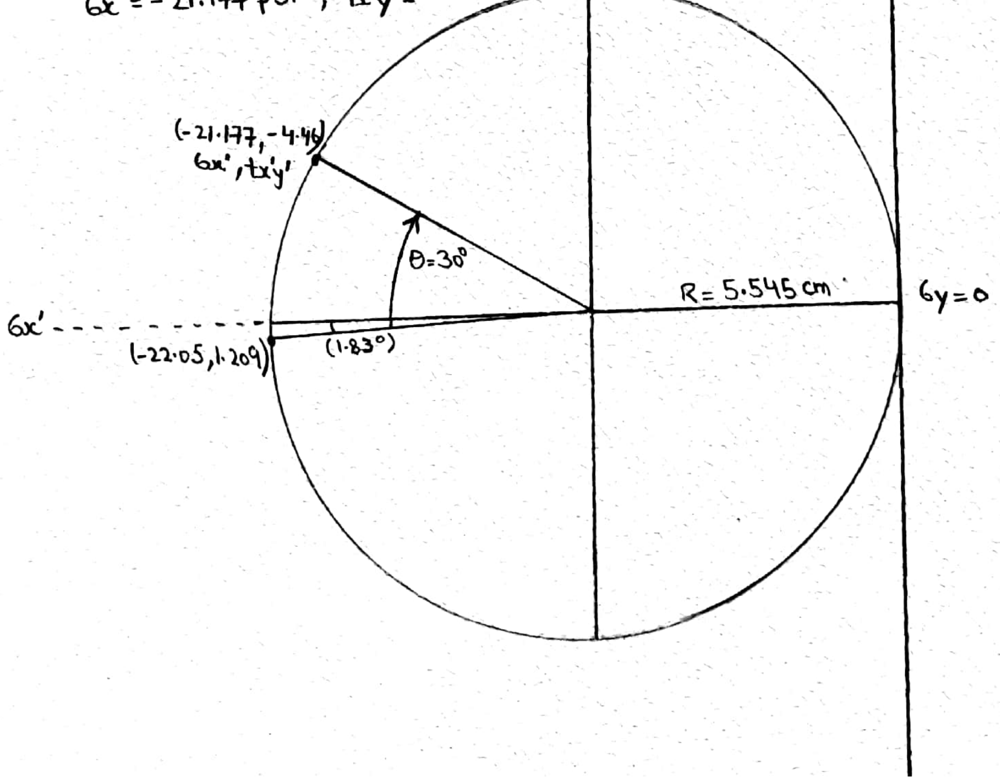
Scale ⇒ 1psi = 2cm
So, Radius = $11.09/2$
= 5.545 cm



$$\theta = \tan^{-1} \frac{1.209}{22.05+15}$$
$$= \tan^{-1}(0.032)$$
$$= 1.83°$$

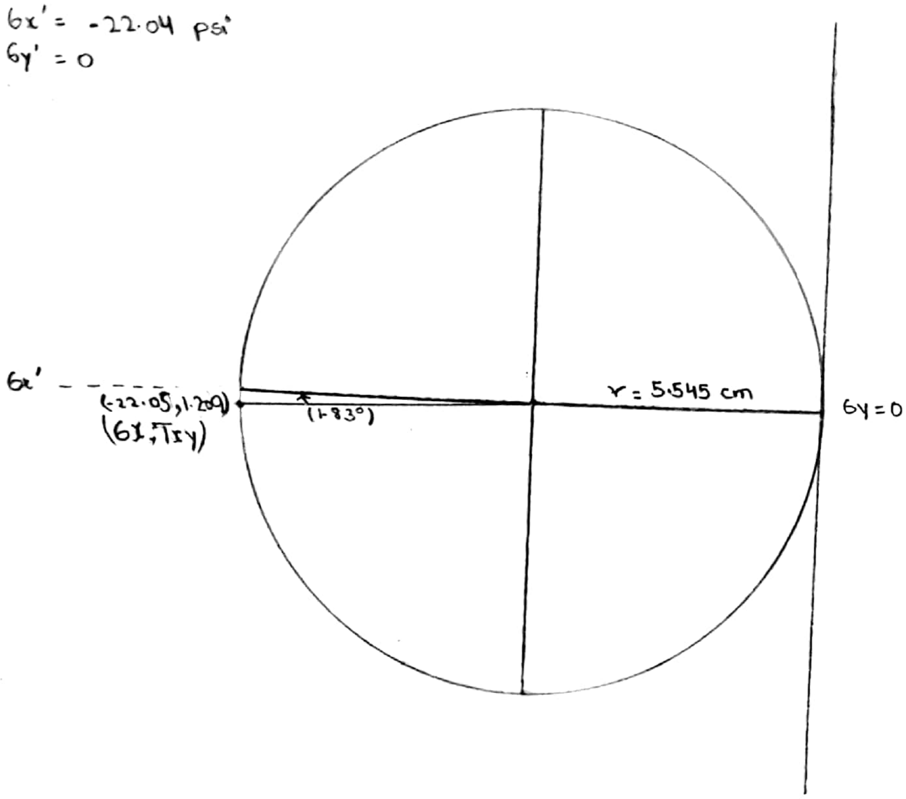
For New Orientation (-15° teta) clockwise :-

$\sigma_{x'} = -21.177$ psi , $\tau_{x'y'} = -4.46$ psi



For principle stresses :-

$\sigma_x' = -22.04$ psi
$\sigma_y' = 0$



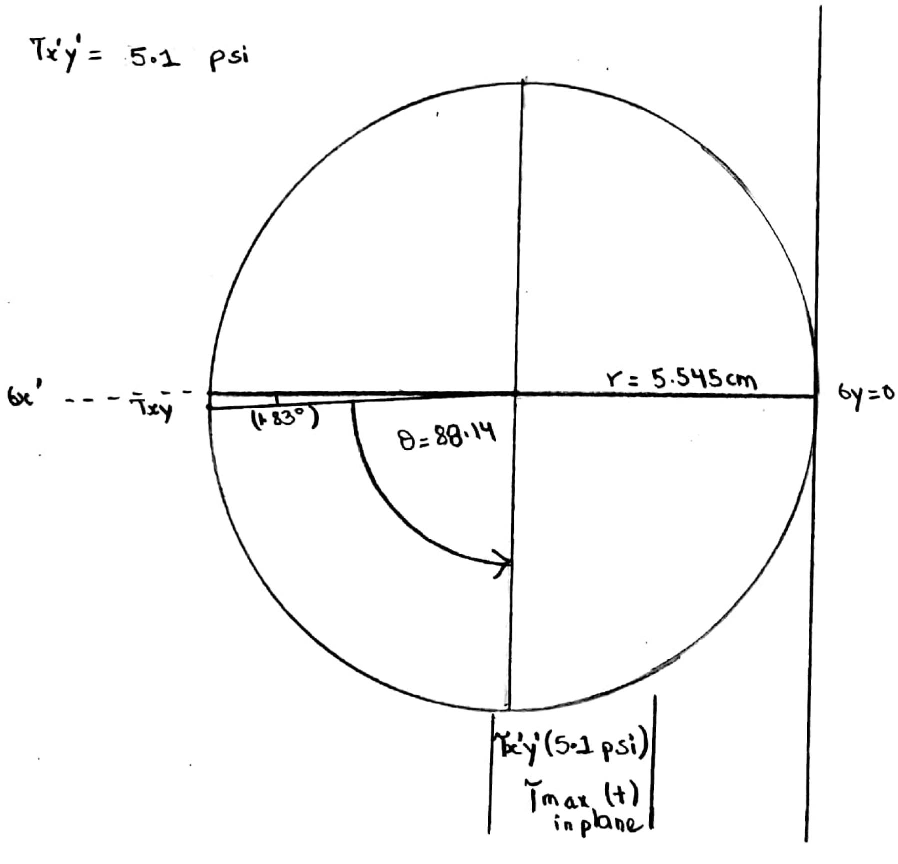
Scale ⇒ 1 psi = 2 cm
So, Radius = $11.09/2$
= 5.545 cm

$\theta_p = \frac{1}{2} \cdot 1.83^\circ$

$= 0.915^\circ$

For shear stresses :-

$\tau_{x'y'} = 5.1$ psi

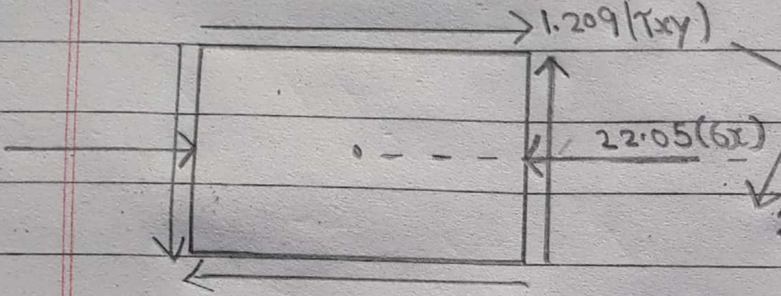


$\theta_s = \frac{1}{2}(88.14)$

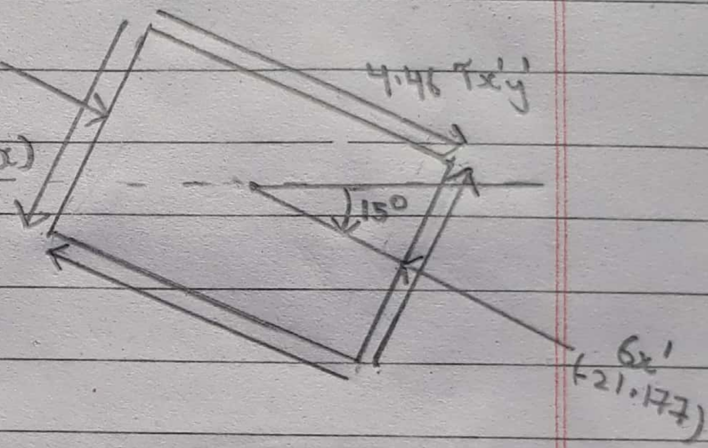
$= 44.085^\circ$

RESULTS COMPARISON :-

$\theta = 0^\circ$

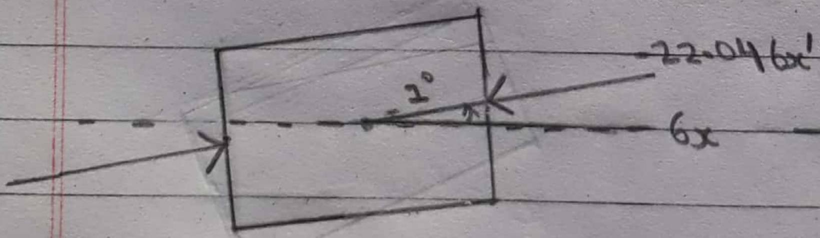


$\theta = -15^\circ$ (clockwise -)



PRINCIPLE AND PLAN SHEAR STRESS :-

$\theta = 1^\circ$ (Anticlock wise +)



$\theta = 44.08^\circ$