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Subject: Linear Algebra.

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Qi $x_1 - 3\gamma dID\ x_2 + x_3 = 0$

$2x_2 - 8x_3 = 8$

$5x_1 - 5x_3 = 10$

Sollection:

$x_1 - 8x_2 + x_3 = 0$

$2x_2 - 8x_3 = 8$

$5x_1 - 5x_3 = 10$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$= \left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & +40 & -10 & 10 \end{array}\right] \quad R3 - 5R1$$

$$= \left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right] \quad \begin{array}{l} R2/4 \\ R3/10 \end{array}$$

$$= \left[\begin{array}{cccc} 1 & -8 & 1 & 10 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & -15 \end{array}\right] \quad R3 - 4R_2$$

Consistent because this triangle

$-15x_3 = -15$

$x_3 = 1$

$x_2 - 4x_3 = 4$

$x_2 = 4 + 4x_3$

$x_2 = 8$

$\Rightarrow x_1 - 8x_2 + x_3 = 0$

$x_1 = 8x_2 - x_3$

$x_1 = 64 - 1$

$x_1 = 65$

Afshar

(4) Find the Inverse $\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & \text{4lh-1D} \\ 5 & -2 & 7 \end{bmatrix}$ by adjoint method.

"Sollucation"

$$A \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 3 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A = 3(-7+6) - 4(14-15)\ 5(-4+5)$$
$$= 3(-1) - 4(-1) \cdot (1)$$
$$= -3+4+5$$
$$= 6$$

of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 3 \\ 5 & -2 & 7 \end{bmatrix}$

$a_{11} = (-1)^{1+1}\ (-7+6) = (-1)^2 = -1$

$a_{12} = (-1)^{1+1}\ (14-15) = +1$

$a_{13} = (-1)^{1+2}\ (-4+5) = 1$

$a_{21} = (-1)^{2+1}\ (28+10) = 38$

$a_{22} = (-1)^{2+2}\ (21-25) = -4$

$a_{23} = (-1)^{2+3}\ (-6-20) = -26$

$a_{31} = (-1)^{3+1}\ (12+5) = 17$

$a_{32} = (-1)^{3+2}\ (9-10) = -1$

$a_{33} = (-1)^{3+3}\ (-3-8) = -12$

$$\begin{bmatrix} -1 & +1 & 1 \\ 38 & -4 & -26 \\ 17 & -1 & -12 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 38 & 17 \\ 1 & -4 & -1 \\ 1 & -26 & -12 \end{bmatrix}$$ Answer

Q3 Solve the following system of linear equation by Gauss-jordan method.

$$2x+2y+4z=18$$
$$x+3y+2z=13$$
$$3x+2y-3z=14$$

Solution:

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_1=\frac{1}{2}R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\begin{matrix} 2=R_2-R_1 \\ 3=R_3-3R_1 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \xrightarrow{R_2=\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -2 & -3 \end{bmatrix}$$

$$R_3+2R_2 \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow[R_3=-\frac{1}{3}R_3]{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1-2R_3 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \begin{matrix} x=1 \\ y=2 \\ z=3 \end{matrix}$$

So $z=3$ Answer

"Question 4"

Show that is Diagoisable.

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

"Solution"

materix A is diagoisable if

$A = CDC^{-1}$

$\det(A - \lambda I_3) = 0$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$\Rightarrow 4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$\Rightarrow 4-\lambda((3-\lambda)(1-\lambda)-8) - 2(-5(1-\lambda)+4) - 2(-20+2(3-\lambda)) = 0$

$\Rightarrow 4-\lambda[3-3\lambda-\lambda+\lambda^2-8] - 2[-5+5\lambda+4] - 2[-20+6-2\lambda] = 0$

$\Rightarrow 4-\lambda[\lambda^2-4\lambda-5] - 2[5\lambda-1] - 2[-14-2\lambda] = 0$

$\Rightarrow 4\lambda^2+16\lambda-20-\lambda^3+4\lambda^2+5\lambda-10\lambda+2+28+4\lambda = 0$

$\Rightarrow -\lambda^3+8\lambda^2+15\lambda+10 = 0$

$\lambda = 9.65$

$\lambda = -0.82$

$\lambda = -0.229$

for $\lambda = 9.65$

P.t.o

4 $A - \lambda I_3 \begin{bmatrix} -5.65 & 2 & -2 \\ -5 & -6.65 & 2 \\ -2 & 4 & -8.65 \end{bmatrix}$

for $\lambda = -0.82$

$A - \lambda I_3 = \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$

In end or by solving only 2 eigenspace or 2 bais vector in total

So, matrix A is not diagonalizable.

Q5 $3x_1 + 5x_2 - 4x_3 = 0$
$-3x_1 - 25x_2 + 4x_3 = 0$
$6x_1 + x_2 - 8x_3 = 0$

System has non-trivial sollution if and only if its determinded is non-zero.

$Ax = 0$

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix}$$

det A =

$$3\begin{vmatrix} -25 & 4 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -3 & 4 \\ 6 & -8 \end{vmatrix} - 4\begin{vmatrix} -3 & -25 \\ 6 & 1 \end{vmatrix}$$

$= 3(200-4) - 5(24-24) - 4(-3+150)$
$= 588 - 5(6) - 588$
$= 0$

As determinent comes out zero then this system has either no non-trivial sollutions or an infinite number of soluations.

Q6

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sollution:

Maximum Possible Rank for materix A is 3 if $|A| = 0$

$$|A| = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Rank = No of non-zero rows.

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_3 - R_1$
$1-1=0$
$3-3=0$
$4-4=0$
$0-3=-3$

$$= \begin{bmatrix} 1 & 3 & 11 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 11 & 3 \\ 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_2/3$$

$R_2 - R_1$
$1-1=0$
$3-3=0$
$4-4=0$
$3-3=0$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

Rank = 2

Ans