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SUBJECT DIFFRINTIAL EQUATIO

SECTION A.

Assingmnt 3

(1) Application of partial differential equation.

Many engineering problems are governed by different types of partial differential equation, and some of the more important types are given below.

Tricomi Equation:-

$$y \frac{\partial^4 u}{\partial x^2} + \frac{\partial^x u}{\partial y^2} = 0$$

$$\begin{cases} y > 0 : \text{elliptic} \\ y < 0 \ \text{hyperbolic} \end{cases}$$

Laplace Equation:- (or varients) $= \partial_2 \phi \, \partial x \partial + \partial_2 \phi \, \partial y_2 = \nabla_2 \phi = 0$

Poisson's Equation: $\partial_2 \phi \, \partial x_2^2 + \partial_2 \phi \, \partial y^2 = \cancel{\nabla_2 \phi} \neq \cancel{0} = f(x, y)$

Helmholt Equation:- $\partial_2 \phi \, \partial x_2 + \partial_2 \phi \, \partial y^2 = \cancel{\phi(x, y)} + c_2 \phi = 0$

Plate bending:- $\nabla_2 \nabla_2 w = \nabla_4 w = q^0$

2)

WAVE EQUATION:-

$$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

FOURIER EQUATION:-

$$\partial T \ \partial t = q(\partial_2 T \ \partial_{x_2})$$

Separable differential equation:- For equation which can be expressed in separable from as as shown below, the solution can be obtained easily as:

$$\frac{dy}{dx} = F(x,y) \ \frac{dy}{(y)} = f(x) \ dx \int \frac{dy}{n(y)}$$

$$\int f(x) \ dx + c$$

$M(x,y)\ dx + N(x,y)\ dy = 0 \ M(x)\ dx = N(y)\ dy$

then

$$\int M(x)\ dx = -\int N(y)\ dy + c$$

(3) Separable differential equation:- For equation which can be expressed in separable from as shown below, the solution can be obtained easily as.

③

$\frac{dy}{dx} = f(x, y)$ $\frac{dy}{n(y)} = f(x)\,dx$ $\int \frac{dy}{n(y)} = \int F(x)\,dx + c$

$M(x, y)\,dx + N(x, y)\,dy = 0$ $M(x)\,dx = -N(y)\,dy$

Then $\int M(x)\,dx = -\int N(y)\,dy + c$

Example :-

$\frac{dy}{dx} = x^3 + (y^2+1) \Rightarrow \frac{dy}{y^2+1} = x^3\,dx$

$\int \frac{dy}{y^2+1} = \int x^3\,dx + c \Rightarrow \tan^{-1} y = \frac{1}{4}x^4 + c$

$\Rightarrow y = \tan\left(\frac{1}{4}x^4 + c\right)$

Example :-

$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$ subject to $y(0) = -1$

(4)

Since is a seperable function the problem can be solved as.

$2(y-1)\,dy = (3x^2+4x+2)\,dx$

$y^2 - 2y = x^3 + 2x^2 + 2x + c$

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$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

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Based on the boundary condition c=

hence $y^2 - 2y = x^3 + 2x^2 + 2x + 3$

This quadratic equation in y^2 can be solved with two solution by the quadratic equation as.

$y = 1 \sqrt{x^3+2x^2+2x+4}$ and $y = 1 + \sqrt{x^3+2x^2+2x}$

Since the second solution doesnot satisfy the boundary equation, it will not be accepted, hence the solution to this differential equatio is obtained.

VARIATION OF PERAMETER :-

For the following equation form, it is possible to solve it by variation of perameter

for
$$\frac{dy}{dx} = P(x)\,y + Q(x)$$

put $y = C(x)\, e^{\int P(x)\,dx}$ by differentiating it given.

$$\frac{dy}{dx} = \frac{dc}{dx} e^{\int P(x)\,dx} + \underbrace{C(x)\,P(x)\,e^{\int P(x)\,dx}}_{P(x)\,y}$$

Substitute it to the original ODE

$$\frac{dc(x)}{dx} = Q(x)\,e^{-\int P(x)\,dx}$$ comparing the terms, it given
$$C(x) = \int Q(x)\,e^{-\int P(x)\,dx}\,dx + c$$

Example:
$$(x+1)\frac{dy}{dx} - ny = e^x (x+1)^{n+1}$$

This equation is now expressed as.

$$\frac{dy}{dx} = P(x)\,y + q(x)$$

$$\frac{dy}{dx} = \frac{n}{x+1}\,y + e^x \frac{(x+1)^n}{(x)}$$

for $x \neq -1$
solving the homogenous part of the ODE

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$$\frac{dy}{dx} = \frac{n}{x+1} y \quad \text{then} \quad \frac{dy}{y} = \frac{n}{x+1} dx$$

$$\ln(y) = n \ln |x+1| + c_1$$

$$y = c(x+1)^n$$

Look for solution $y = c(x)(x+1)^n$

where $c(x)$ is the variation of perameters substitute it to the ODE.

$$\frac{dc(x)}{dx}(x+1)^n + nc(x)(x+1)^{n-1} = nc(x)$$

$$(x+1)^{n-1} = nc(x)(x+1)^{n-1} + e^x (x+1)^n$$

Comparison givens $\frac{dc(x)}{dx} = e^x$

Integrations of this equation gives.

$$c(x) = e^x + \vec{c}$$

General solution is hence given by

$$y = (x+1)^n (e^x + \vec{c})$$

The bernulli equation is an important equation type which can be solved in a similar way by variation of perameters consider the following from the equation.

$$\frac{dy}{dx} = P(x)\, y + Q(x)\, y^n$$

step 1: put $z = y^{1-n}$

step 2: Then $\frac{dz}{dx} = (1-n)\, y^{-n} \frac{dx}{dy}$

$$\frac{dz}{dx} = (1-n)\, P(x)\, z + (1-n)\, Q(x)$$

The non linear ODE now becomes linear ODE. It can be solved by formula.

Step 3: $n = -1$, z, y^2. Inverting to get y

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$$

$$\frac{dz}{dx} = \frac{1}{x} z + x^2$$

$$z = c\int \frac{1}{x}\,dx \left(\int x^2 c - \int \frac{1}{x}^{dx}\,dx + c\right) = c\,x + \frac{1}{2}x^2$$

Back substitution of $z = y^2$ of $z = y^2$

$$y^2 = cx + \frac{1}{2}x^3$$

HOMOGENEOUS EQUATION:- For equation of the following type, where all the coeffecient are constant it, can be evaluated according to different

Laplace EQUATION:-

Laplace equation forms an important governing condition for many types of problems. Some of the more common forms are given by.

Three dimensional laplace equation

$$U_{xx} + U_{yy} + U_{zz} = U$$

Two dimensional heat conduction.

$$a^2 (u_{xx} + u_{yy})\,u_t$$

Two dimensional seepage problem

$$(K_x U_{x}x + K_y U_{yy}) = 0$$

There a two major types of boundary conditions to this problem

Drichlet Problem:- Boundary conditions prescribed as u.