NAME # SHAHKAR SALEEM

ID # 7943

SUBJECT # Numerical Analysis

SECTION # "B"

SEMESTER # 4th

DATE # 23-Sep-2020.

Q1:- Apply both euler's method and the improved euler's method to the Solution of

$$\frac{dy}{dx} = 2x \quad , \quad y(0) = 1$$

for $0 \leq x \leq 0.5$ Using $h = 0.1$ Compare your answer with the analytic solution work throughout three decimal places.

Sol

$$f(x,y) = 2x$$

$$x_0 = 0, \quad y_0 = 1$$

$$h = 0.1$$

$$x_{n+1} = x_n + h$$

put $n = 0$

$$x_1 = x_0 + h$$

$$= 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

$$x_2 = x_1 + h$$

$$= 0.1 + 0.1$$

$$\boxed{x_2 = 0.2}$$

$$\boxed{x_3 = 0.3}$$

$$\boxed{x_4 = 0.4}$$

$$\boxed{x_5 = 0.5}$$

1^{st} iteration

Euler's formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$n = 0$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1\left[(0)+(1)\right]$$

$$= 1.1$$

Modified euler's formula.

$$y_{n+1} = y_n + h/2\left[f(x_n, y_n) + f\left(x_{n+1}, y^*_{n+1}\right)\right]$$

$$y_1 = y_0 + h/2\left[(x_0, y_0) + (x_1, y_1^*)\right]$$

$$y_1 = 1 + 0.1/2\left[(0)+(1)+(0.1)+(1.1)\right]$$

$$y_1 = 1 + 0.05\left[1 + 1.2\right]$$

$= 1 + 0.05\,(2.2)$

$\boxed{= 1.11}$

2nd iteration

Euler's formula.

$y_2^* = y_1 + hf\,(x_1, y_1)$

$y_2^* = 1.11 + 0.1\,[(0.1) + (1.11)]$

$y_2^* = 1.11 + 0.1\,(1.21)$

$y_2^* = 1.111 + 0.121$

$y_2^* = 1.231$

Modified Euler's formula.

$y_2 = y_1 + 0.1/2\,[x_1, y_1] + [x_2 + y_2^*]$

$= 1.11 + 0.05\,[(0.1) + (1.11) + (0.2) + (1.231)]$

$= 1.11 + 0.05\,[1.21 + 1.431]$

$= 1.11 + 0.05\,(2.641)$

$= \boxed{1.242}$

3rd iteration

$y_3^* = y_2 + hf(x_2, y_2)$

$y_3^* = 1.242 + 0.1(0.2) + (1.242)$

$y_3^* = 1.242 + 0.1442$

$y_3^* = 1.3862$

Modified Euler's Method.

$y_3 = y_2 + 0.1/2 \left[x_2, y_2\right] + \left[x_3, y_3^*\right]$

$= 1.242 + 0.05 \left[(0.2) + (1.241) + (0.3) + (1.3862)\right]$

$= 1.242 + 0.05 \left[1.442 + 1.6862\right]$

$= 1.398$

4th iteration

$y_4^* = y_3 + hf(x_3, y_3)$

$y_4^* = 1.398 + 0.05(0.3 + (1.398))$

$\boxed{y_4^* = 1.483}$

Modified euler's Method.

$y_4 = y_3 + 0.05\left[x_3, y_3\right] + \left[xy, y_4^*\right]$

$= 1.398 + 0.05 [0.3 + 1.398] + [0.4 + 1.483]$

$= 1.398 + 0.05 [1.698 + 1.883]$

$= \boxed{1.577}$

5th Iteration

$y_5^* = y_4 + hf(x_4, y_4)$

$= 1.577 + 0.05 (0.4 + 1.577)$

$= 1.675$

Modified Euler's Method.

$y_5 = y_4 + 0.05 [x_4, y_4] + [x_5, y_5^*]$

$= 1.577 + 0.05 [0.4 + 1.577] + (0.5 + 1.675)$

$= 1.577 + 0.05 [4.152]$

$= \boxed{1.785}$

Q2:- Use the fourth-order Runge Kutta method to obtained a Solution of

$$dy/dx = x^2+x-y$$

Subject to $y=0$ when $x=0$, for $0 \leq x \leq 0.6$ with $h=0.2$ work throughout to four decimal places.

Sol Given data:-

$$y=0, \quad x=0, \quad h=0.2 \quad 0 \leq x \leq 0.6$$

$$y_{n+1} = y_n + K$$

1st Iteration

$$n=0$$

$$y_1 = y_0 + K \quad , \quad K = 1/6\,(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = hf(x_n, y_n)$$

$$K_1 = h(x_0^2 - x_0 - y_0)$$

$$K_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{K_1 = 0}$$

$$K_2 = hf\left(x_n + h/2,\ y_n + h/2\right)$$

$$= 0.2f\left(x_0 + h/2,\ y_0 + h/2\right)$$

$$= 0.2f\left(\frac{0 + 0.2}{2},\ \frac{0 + 0.2}{2}\right)$$

$$= 0.2f\left(0.1,\ 0.1\right)$$

$$= 0.2\left(0.1^2 + \cancel{0.1} - \cancel{0.1}\right)$$

$$= \boxed{K_2 = 0.0020}$$

$$K_3 = hf\left(0 + 0.2/2,\ \frac{0 + 0.002}{2}\right)$$

$$= 0.2f\left(0.1,\ 0.001\right)$$

$$= 0.2\left(0.1^2 + 0.1 - 0.001\right)$$

$$\boxed{K_3 = 0.0218}$$

$$K_4 = hf\left(x_n + h,\ y_n + K_3\right)$$

$$= 0.2f\left(0 + 0.2,\ 0 + 0.0218\right)$$

$$= 0.2f\left(0.2,\ 0.0218\right)$$

$$= 0.2\left(0.2^2 + 0.2 - 0.0218\right)$$

$$\boxed{K_4 = 0.0436}$$

$$K = 1/6\left(0 + 2(0.00\cancel{0}2) + 2(0.0218) + 0.0436\right)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$y_1 = 0.0152$$

Q3:-

Given data :-

$$a = 0, \; b = 10, \; n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Sol

x	0	1	2	3	4	5	6	7	8	9	10.
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula.

$$\int f(x)dx = h/2\left[f(x_0)+2(f(x_1)+f(x_2)+f(x_3) \cdots f(x_9))+f(x_{10})\right]$$

$$= 1/2\left[10.1+2(17.2+24.4+29.9+34.6+41.2+50.9+57.8+62.1)\right]$$

$$= \boxed{412.9}$$ ANSWER.

Q4:- Estimate the values of the following integral using Simpson's Rule.

$$\int_2^3 \ln(x^3+1)\,dx.$$

Use 10 strips.

Sol

$n = 10$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$f(x)$	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now Using formula.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4\left(f(x_1)+f(x_3)\cdots\right]+2\left[f(x_2\cdots)+f(x_n)\right]\right.$$

$$= 0.1/3\left[0.693+4\left(0.846+1.162+1.476+1.777\right)+2\left(1.003+1.320+1.628+1.922\right)+2.062\right]$$

(11).

$= \boxed{1.184}$ ANSWER.