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Subject :- Introduction to Structure Dynamics & Earthquick Engineering.

Date :- 29-06-2020.

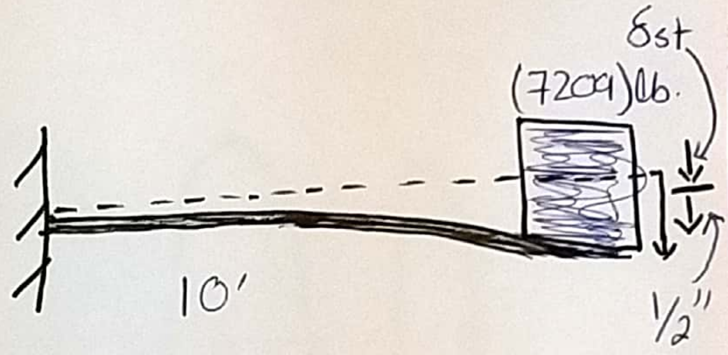
Time :- 9 AM - 3PM.

Ihtisham
29-06-20

Question # 01. Problem. (1)

Given data:-

- Downward Direction = 1/2 in.
- $E = 29{,}000$ ksi
- $I = 150$ in^4.
- $\delta_{st} = 7209$.



Solution:-

E.O.M. for damped free vibrations is;

$$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow (1).$$

It is know as that;

$K = 90625$ lb/ft & $m = 31.06$ lb.sec^2/ft.

$$C = \zeta \times 2m\omega_n$$

$$= 7209 \times 2 \times 31.06 \times 54.9 \times 0.025.$$

$$C = 85.26 \text{ lb.sec/ft.}$$

By substituting value of k, c & m in (1) we get,

$$90625u + 85.26\dot{u} + 31.06\ddot{u} = 0.$$

②

Solution to the EOM for damped free Vibration is:

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\cdot \sin(\omega_D t)\right].$$

- $\omega_D = 54.9$ rad/sec.

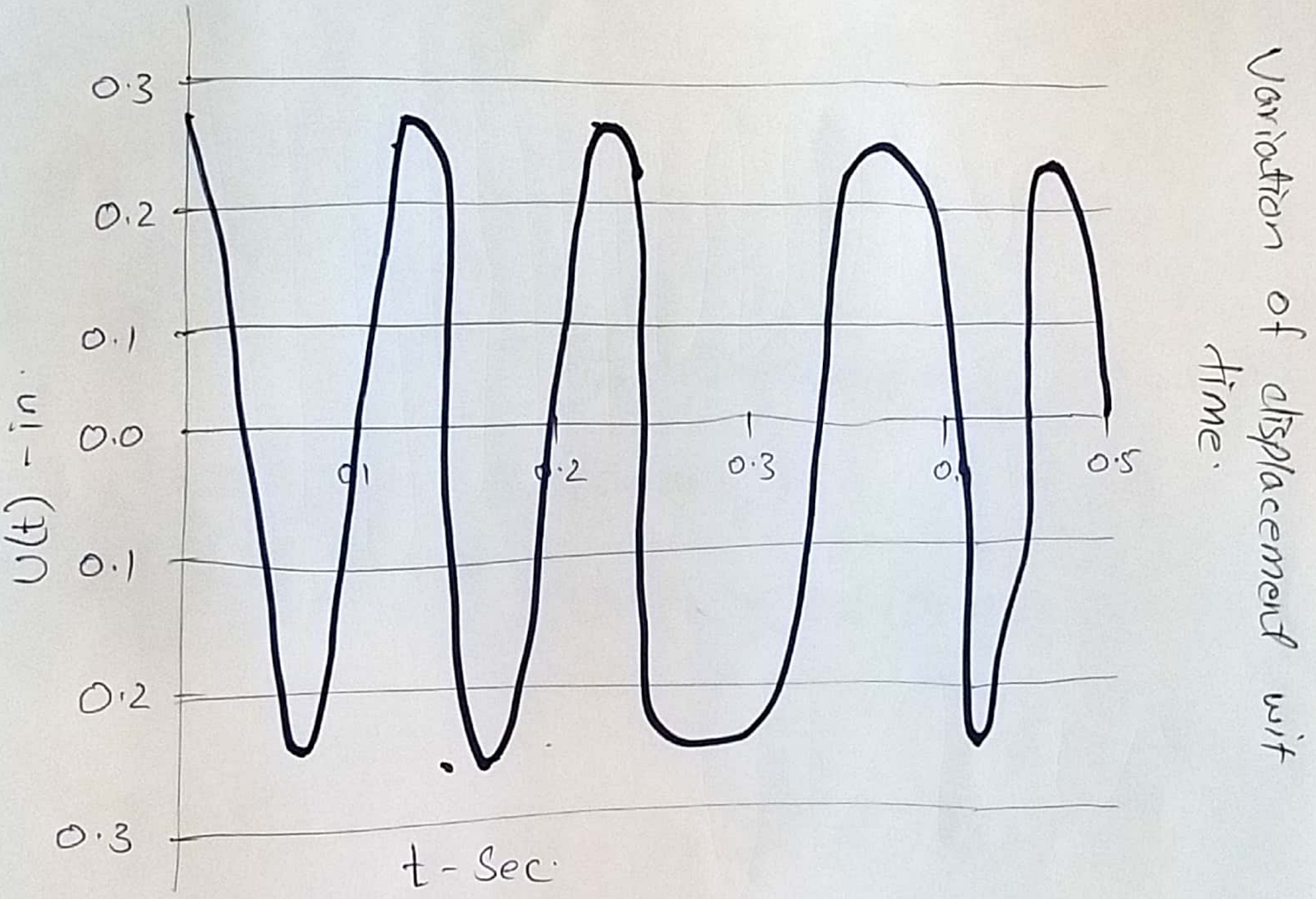
- $u(t) = = e^{-0.025 \times 54.9t}\left[\frac{1}{48} \times \cos(54.9t) + \frac{1}{54.9} \times \left[0 + \frac{1}{48} \times 0.025 \times 54.9 \times \sin(54.9t)\right]\right].$

- $u(t) = = e^{-1.373t}\left[0.0208 \times \cos(54.9t) + 0.00052 \times \sin(54.9t)\right].$

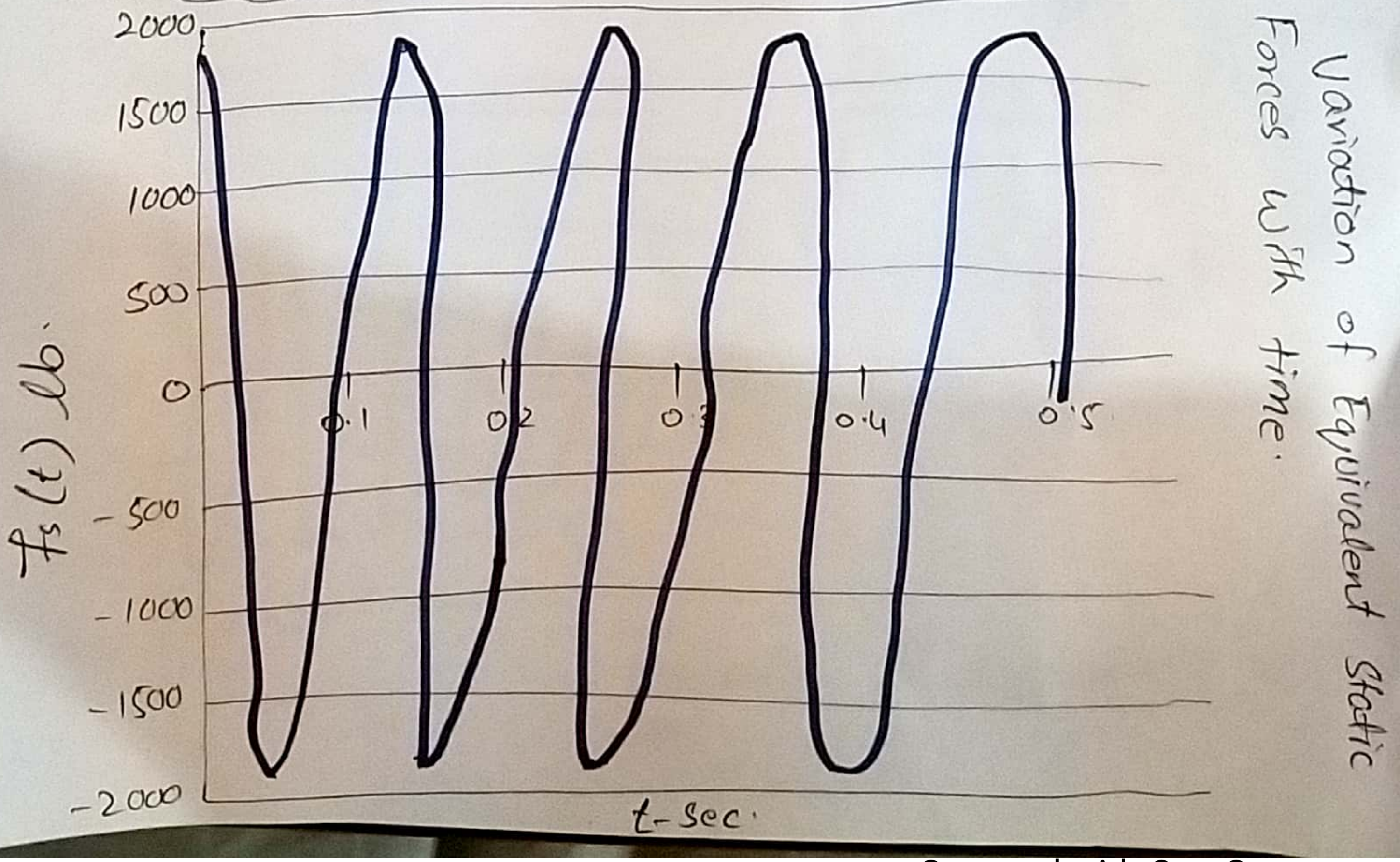
- $f_s(t) = k \cdot u(t) = 90625 \times u(t)$

- $f_s(t) = e^{-1.373t}\left[1885 \cos(54.9t) + 47.13 \times \sin(54.9t)\right]$

Undamped free vibration:- Question #01.



Variation of displacement with time.

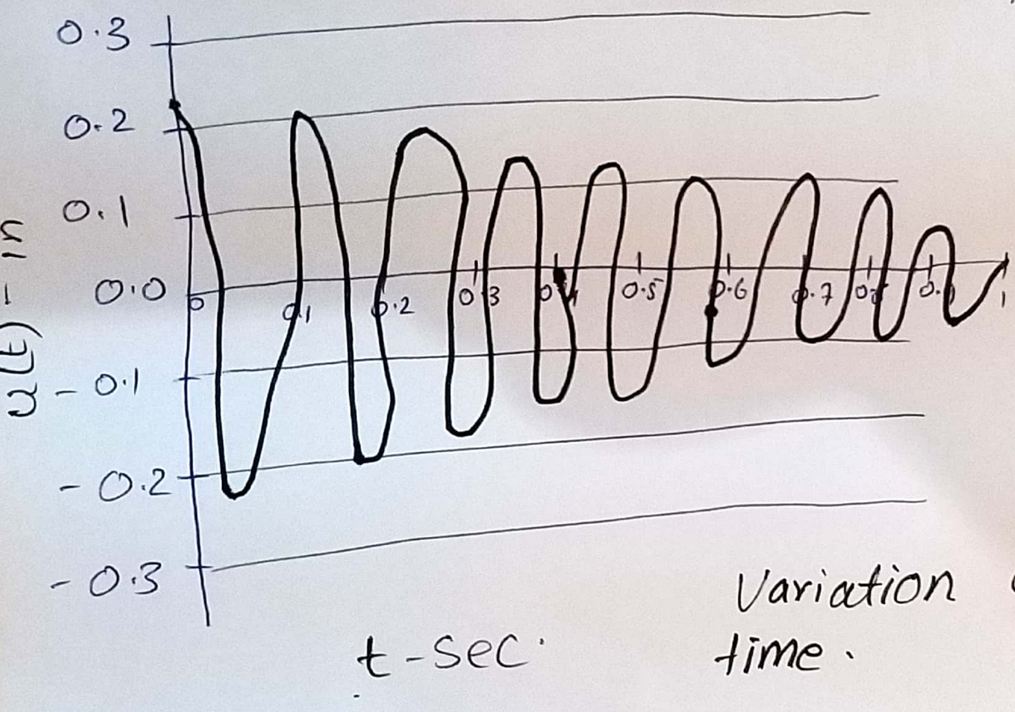


Damped Free Vibration:-

Question #2.

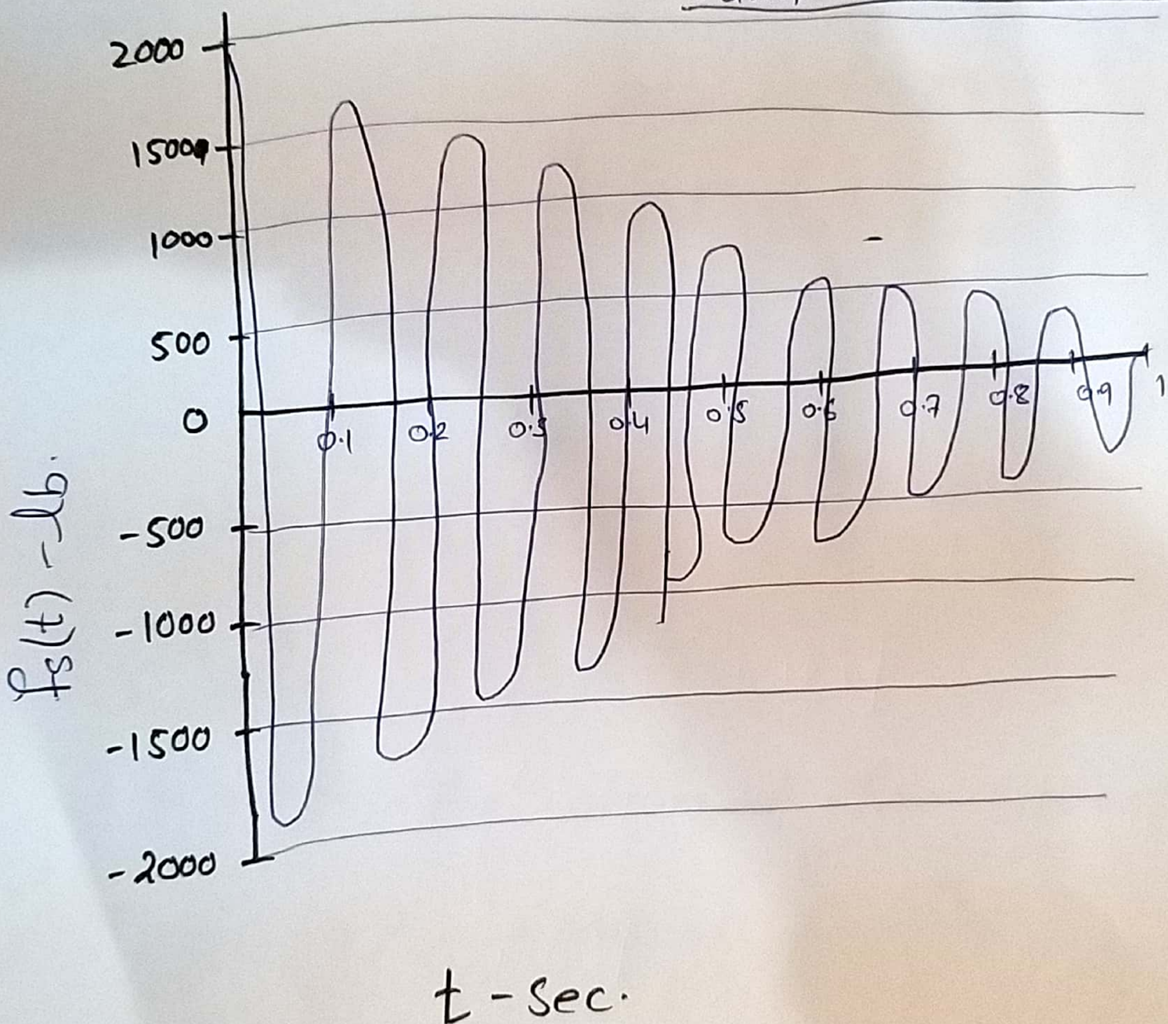
(4)

Ans:-



Variation of displacement with time.

Damped Free Vibration:-



Variation of Equivalent Static forces with time.

(6)

Damping Ratio of Reinforcement :-

Concrete with consideable creaking = 3-5 %

= 3%

Using the data of the beam given in Question # 01.

Solution:-

E.O.M. for deamped free vibration is

$Ku + c\dot{u} + m\ddot{u} = 0$ —— (1).

from Question # 1

$K = 90625$ lb/ft & $m = 240.15$ lbf.sec²/ft.

$\omega_n = 19.425$ rad/sec.

$c = \xi \times 2 m \omega_n$

$c = (0.03) \times 2 (240.15) (19.425)$

$c = 279.89$ lbf.sec/ft

Put value in eq - (1).

$90625u + 279.89\dot{u} + 240.15\ddot{u} = 0$.

Solution to the EOM for damied free vibration is, 7

$\Rightarrow u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\ddot{u}(0) + \dot{u}(0)\xi\omega_n\right]\sin\omega_D t\right]$

$\Rightarrow \omega_D = 19.425$ rad/sec.

$\Rightarrow u(t) = e^{-0.03 \times 19.425} + \left\{\frac{1}{24} \times \cos(19.425t) + \frac{1}{19.425} \times \left[\frac{0+1}{24} \times 0.03 \times 19.425\right] \times \sin(19.425t)\right.$.

$\Rightarrow u(t) = e^{-0.582t}\left[0.041 \times \cos(19.425t) + 0.001825 \times \sin(19.425t)\right]$.

$\Rightarrow f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t)$.

$\Rightarrow f_s(t) = e^{-0.582t}\left[(90625 \times 0.041)\cos(19.425t) + (90625 \times 0.00125)\sin(19.425t)\right]$.

$f_s(t) = e^{-0.582t}\left[3715.62\cos(19.425t) + 113.28\sin(19.425t)\right]$

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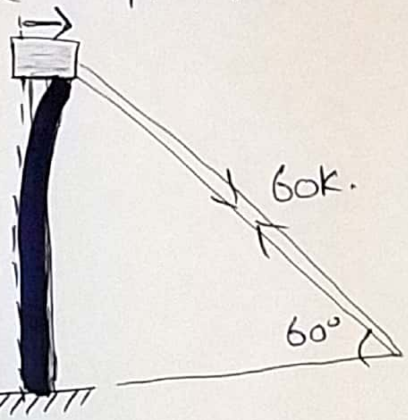
Question #03:-

(7209/1000)in = 7.2 in.



Solution:-

$u_1 = 7.2$ in.

After

J = , $u_{j+1} = u_6 =$

a) Damping Ratio:-

$$\zeta = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{i+1}}\right]$$

$$= \frac{1}{2\pi\xi} \ln(7.2/2.286)$$

$= 0.0254 = 2.54\%$.

b) Tn = ?

7 cycle of vibration are completed in 3.75 sec

- Time required to complete one cycle =

3.75 / 7 = TD.

= TD = 0.53 sec

Now,

$$\omega_D = \omega_n \sqrt{(1-\xi^2)}$$

$$2\pi/\omega_D = 2\pi/(\omega_n\sqrt{1-\xi^2}$$

$$\Rightarrow T_D = T_n/\sqrt{1-\xi^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$\Rightarrow T_n = 0.53 \times \sqrt{1-(0.0254)^2}$$

$$\Rightarrow T_n = 0.5098 = \boxed{0.51 \text{ sec}}$$

c): $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{2} = 15 K/in$$

$$K = 18000 \, lb/ft$$

d): weight of the tank, $w = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(\frac{w}{g})}} = \sqrt{\frac{K \times g}{w}}$$

$$\Rightarrow \omega_n^2 = K \times g / w$$

$$w = K \times g / \omega_n^2$$

also $\omega_n = 2\pi / T_n$

$\Rightarrow W = kg\left(\frac{4\pi^2}{T_n^2}\right) = kg \times \frac{T_n^2}{4\pi^2}$

$W = \frac{18000 \text{ lb}}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2} \times \frac{(0.51 \text{ sec})^2}{4\pi^2}$

$W = 74875 \text{ lb} = \boxed{74.9 \text{ K}}$

e) $c = ?$

It is know that $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi / T_n)$

$\Rightarrow c = \frac{0.0254 \times 4 \times \pi \times \left(\frac{74875}{32.2}\right)}{0.51}$

$\Rightarrow c = 1455.3 \text{ lb.sec/ft}$

f) No. of cycle to reduce displacement amplitude from 2 in. to 0.5 in, $j = ?$

$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_i}{u_{j+1}}\right)$

$\Rightarrow j = \frac{1}{2 \times \pi \times 0.0254} \ln\left(\frac{2}{0.5}\right)$

$\Rightarrow j = 8.69$ or 9 cycles.