Iqra National University, Peshawar
Department of Electrical Engineering

FINAL – ASSIGNMENT SPRING2020
Date:26/6/2020

Course Code: MTH 102 **Course Title:** Calculus and analytic geometry
Prerequisite: Abdullah **Instructor:** HIMAYATULLAH
Module: 3 **Program:** BEE **Total Marks:** 50 **:** 16194

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	a	. **Estimate** $\int \theta \sqrt[4]{1-\theta^2}\ d\theta$	Marks 7 PLO2 C2
	b	**Estimate** $\int_0^1 x^3(1+x^4)^3 dx$ using substitution method.	Marks 7 PLO2 C2
Q2	(a)	**Illustrate** the centre and radius of the sphere $x^2+y^2+z^2+3x-4z+1$.	Marks 5 PLO1 C3
	(b)	The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. **Apply** the integration find the volume of solid.	Marks 4 PLO1 C3
Q3		If A= $2i-4j+\sqrt{5}k$, $and\ B=-2i+4j-\sqrt{5}k$ then **illustrate** the vector $proje_A\, B$	Marks 9 PLO1 C3
Q4		Find the area of the region between the graph and the x-axis Where $y=-x^2+5x-4$, $[0,\ 2]$.	Marks 9 PLO1 C3
Q5	(a)	**Estimate** the angle between $A=i-2j-2k\ and\ \ B=6i+3j+2k$	Marks 5 PLO1 C3
	(b)	**Change** into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$	Marks 4 PLO1 C3

			PLO2 C2

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QNO 1 (Part a)

ANS:

Given:

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Solution:

Let

$$1-\theta^2 = u$$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta}u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta d\theta = -\frac{1}{2}du$$

Now

$$= \int (u)^{1/4} \cdot \left(-\frac{1}{2}\right) du$$

$$= -\frac{1}{2}\int u^{1/4}\, du$$

$$\therefore \frac{1}{4}+1 = \frac{5}{4}$$

$$= -\frac{1}{\cancel{2}} \cdot \frac{\overset{2}{\cancel{4}}}{5} u^{5/4} + C$$

$$= -\frac{2}{5} u^{5/4} + C$$

By back substitution

$$= -\frac{2}{5} (1-\theta^2)^{5/4} + C$$

Result:

$$= -\frac{2}{5} (1-\theta^2)^{5/4} + C$$

QNO 1 (Part b)
ANS:-

Given:-

$$\int_0^1 x^3 (1+x^4)^3 \, dx$$

Solution:-

let

$$1 + x^4 = u \quad \text{—— (i)}$$

$$\frac{d}{dx}(1+x^4) = \frac{d}{dx} u$$

$$4x^3 = \frac{du}{dx}$$

$x^3 dx = \frac{1}{4} du$

Now put $x = 0$ in eq 1

$1 + 0^4 = 1$

$u = 1$

& put $x = 1$ in eq 1

$1 + (1)^4 = 2$

$u = 2$

$= \int_1^2 (u)^3 \frac{1}{4} du$

$= \frac{1}{4} \int_1^2 u^3 du$

$= \frac{1}{4} \frac{u^4}{4} \Big|_1^2$

$= \frac{1}{4} \left(\frac{(2)^4}{4} - \frac{(1)^4}{4} \right)$

$= 3/8$

Result:-

$= 3/8$ ANS.

QNO 2 (Part A)

ANS:-

Given:-

$$x^2+y^2+z^2+3x-4z+1=0$$

Required:-

Center of sphere = ?

Radius of sphere = ?

Solution:-

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)+y^2+(z^2-4z) = -1$$

Adding $\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$ to both side

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y)^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right)=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y)^2+(z-2)^2=\frac{-4+9+16}{4}$$

$$\left(x+\frac{3}{2}\right)^2+(y)^2+(z-2)^2=\frac{21}{4}$$

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So $(x_0, y_0, z_0) = \text{Centre}$

Centre $= \left(-\frac{3}{2}, 0, 2\right)$

Now finding radius

Radius $a = \sqrt{\frac{21}{4}}$

Result:

Centre $= \left(-\frac{3}{2}, 0, 2\right)$

Radius $= \sqrt{\frac{21}{4}}$

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QNo2 (part b)

ANS:

Given:

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x \leq b$$

Solution:

We know that

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \left. \frac{x^2}{2} \right|_0^4$$

$$V = \frac{\pi}{2}(4^2 - 0) \Rightarrow \frac{\pi}{\cancel{2}} \cancel{16}\,8$$

Result:

$$V = 8\pi \quad \text{ANS}$$

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QNO 3

ANS:- Given :-

$$A = 2i - 4j + \sqrt{5}k$$

$$B = -2i + 4j - \sqrt{5}k$$

Required:-

$$\text{project}_A B = ?$$

Solution:-

$$\text{Project}_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

$$(B \cdot A) = (-2i + 4j - \sqrt{5}k)(2i - 4j + \sqrt{5}k)$$

$$= -4 - 16 - (\sqrt{5})^2$$

$$= -4 - 16 - 5$$

$$= -25$$

$$(A \cdot A) = (2i - 4j + \sqrt{5})(2i - 4j + \sqrt{5})$$

$$= 4 + 16 + (\sqrt{5})^2$$

$$= 25$$

$$\text{Project}_A B = \left(\frac{\cancel{-25}}{\cancel{25}}\right)(2i - 4j + \sqrt{5}k)$$

$$\text{Project}_A B = -2i + 4j - \sqrt{5}k \quad \text{ANS.}$$

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QNO 4

ANS:

Given:

$$y = -x^2 + 5x - 4 \quad [0, 2]$$

Required:

Area = ?

Solution:

As $a = 0$

$b = 2$

$$A = \int_a^b f(x)\,dx$$

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$A = \left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x\right)\Big|_0^2$$

$$A = -\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) - (0)$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right)$$

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$$A = -\frac{8}{3} + \frac{\cancel{20}^{10}}{\cancel{2}} - 8$$

$$A = -\frac{2}{3} = -0.6$$

As area is Never in Negative so we take the value positive.

<u>Result</u>

$A = 0.6$

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QNO 5 (Part A)

ANSn **Givenn**

$A = i - 2j - 2k$

$B = 6i + 3j + 2k$

Requiredn

Angle Cosine = $\cos\theta = ?$

Soln

$A = i - 2j - 2k$

$B = 6i + 3j + 2k$

$$\cos\theta = \left(\frac{A \cdot B}{|A| \cdot |B|}\right)$$

$A \cdot B = (i - 2j - 2k)(6i + 3j + 2k)$

$= 6 - 6 - 4$

$A \cdot B = -4$

Now

$|A| = \sqrt{1 + 4 + 4}$

$= \sqrt{9}$

$|A| = 3$

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$$|B| = \sqrt{(6)^2+(3)^2+(2)^2}$$
$$= \sqrt{36+9+4}$$
$$= \sqrt{49}$$
$$|B| = 7$$

Putting value

$$\cos\theta = \frac{-4}{3\times 7}$$

$$\theta = \cos^{-1}\frac{-4}{21}$$

Result:-

$$\theta = \cos^{-1}\frac{-4}{21}$$

$$\theta = 100.98^\circ$$

QNO 5 Part (b)

ANS:-

Given:-

$$x^2+y^2+(z-1)^2=1$$

Required:-

Sperical Coordination equation = ?

Solution:-

$$x^2+y^2+(z-1)^2=1$$

$$(\rho \sin\phi \cos\theta)^2+(\rho \sin\phi \sin\theta)^2$$

$$+(\rho\cos\phi-1)^2=1$$

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\theta+1$$

$$-2\rho\cos\phi=1$$

$$\rho^2\sin^2\phi(1)+\rho^2\cos^2\phi-2\rho\cos\phi=1-1$$

$$\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

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$$\rho^2(2) - 2\rho\cos\phi = 0$$

$$\rho^2 - 2\rho\cos\phi = 0$$

$$\frac{\rho^{\cancel{2}}}{\cancel{\rho}} = \frac{2\cancel{\rho}\cos\phi}{\cancel{\rho}}$$

$$\rho = 2\cos\phi$$

Results

$$\rho = 2\cos\phi$$