QNO1

Solve the following objective type questions.

1 The order of matrix A is m×P and the order B is P×n then the order of matrix AB is?

Sol: The order of matrix is equal to the no of its row multiply by no of column

So,

A = m×P has "m" no of rows and "p" no of column.

Similarly.

B = P×n

Then its "p" no of Rows and n has no of column. Also the number of colomn in A is equal to the number of rows in B so there matrix are comfortable for multiplication and there order will be

$A \times B = m \times n$

(2) The number of non-zero rows in Echelon from?

Sol The number of non-zero row in echelon form is (one).

(3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sols for singular matrix $|R| = 0$

So, $|B| = 1 \times a - 4 \times 2 = 0$

$= a - 8 = 0$

So value of $a = 8$

(4) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:

$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= (2i)(-i) - (i)(i)$

$= -2i^2 - i^2$

we know that $i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

(5) The matrix (3) $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is ?

Sol:

If each element of a principle diagonal of matrix is some non-zero scalar and all of other elements are zero then it is scalar matrix.

$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is a scalar matrix.

(6) Solution: $\frac{dy}{dx} + 2xy = y = ?$

Sol:

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{1dy}{y} = (1-2x)\,dx$$

$$\int \frac{1dy}{y} = \int (1-2x)\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

(7) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Sol: The order of differential equation is the order of height derivatives known as differential co-efficient and degree is the power of highest derivative so

order = 1

degree = 3

(8) The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx}\right) \text{ is}$$

Sol The this ques the order is

order = 2

& degree = 1

⑤ The different equation

$$2\frac{dy}{dx} + x^2 y = 2x + 3, \; y(0) = 5 \text{ is?}$$

$$2\frac{dy}{dx} + x^2 y = 2x+3$$

$$\int 2dy = \int (2x + 3 - x^2 y)\,dx$$

$$2y = \underline{2x^2} + 3x - y\,\frac{x^2}{3} + c$$

$$y = \frac{\cancel{2}x^2}{\cancel{2}\times 2} + \frac{3x}{2} - \frac{y\,x^3}{3\times 2} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + c$$

Put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

Then

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$$

(10) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ Expand by R_1

$|A| = +1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a \begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2 \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$

$= 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c - b)$

$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

Q NO 2 (Part A) ①

Express the determinent

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Sol: As the product of factors which are linear in a, b, c.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^3cb^3 - a^3b^2c$$

common abc

$$\Rightarrow abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)]$$

② Q NO 2 B (Part)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:
$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -2 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equ $(A - \lambda I) = 0$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow (3-\lambda)\{((3-\lambda)(2\lambda) - (-1) - (-1) + 1((-1)(2-\lambda)$$

(3) $-(-1)(-1)-1\,(-1)-(1)-(-1)(3-\lambda)))$

$= (3-\lambda)\,(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)\,(1+3-\lambda)$

$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$

$= \boxed{-\lambda^3+8\lambda^2-18-8} \longrightarrow a$

$\Rightarrow +1\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}$ —

Expand by C_1

$-1\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix}$

$-1(6-3\lambda-2\lambda+\lambda^2-1)+1(2+\lambda-1)$

$-\lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8}$ — b

$\Rightarrow -1\begin{vmatrix}-1 & 3-\lambda & -1\\ -1 & -1 & -1\\ 0 & -1 & 3-\lambda\end{vmatrix}$

Expand by C_1

$\Rightarrow -\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}+0\right]$

$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$

$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$

$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$

$\Rightarrow \boxed{-\lambda^2+5\lambda-8}$ — c

Put (a) (b) and (c) in (B)

$(2-\lambda\ [-\lambda^3-8\lambda^2-18\lambda+8]-\lambda^2-6\lambda-8-\lambda^2+6\lambda-8$

(4)

$-2\lambda^3 - 16\lambda^2 + 16 + \lambda^3 - 8\lambda^3 + 18\lambda^2 - 8\lambda$

$-\lambda + 6\lambda - 8 - \lambda^2 - 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 - 16\lambda^2 + 16\lambda^2 - \lambda^2 - 38\lambda - 8\lambda - 6\lambda$

$-6\lambda + 16 - 16$

$\lambda^4 - 10\lambda^3 + 32\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By Synthatic division

we get $\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \lambda = 2$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4) = 0$

$\lambda = 4, \lambda = 4$

$\Rightarrow \lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4$

Ans.

Q NO 3

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$x=2,\ y=6$$

Sol

$$(x^3+3y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

Divide both sides by $2xy\,dx$ we get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{— (2)}$$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (a)}$$

Put in a (1)

$$v + \frac{x\,dv}{2x} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}dx$$

$\Rightarrow$ Multiply both sides by $\frac{dy}{dv}$ we get.

$$2x\,dv = \frac{1+v^2}{v}dx$$

$\Rightarrow$ Multiplying both sides by $\frac{v}{x(1+v^2)}$ we get

$$\frac{v}{(1+v^2)}dv = \frac{1}{x}dx$$

Take $\int$ on b/s

$$\int\frac{2v}{1+v^2}dv = \int\frac{1}{x}dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$= 1 + v^2 = xC$$

Put $v = y/x$

$$1 + (y/x) = xC$$

$$\frac{x^2 + y^2}{x^2} = xC$$

$$x^2 + y^2 = x^3 C \text{ --- } ①$$

Put $x = 2$, $y = 6$ in equ ①

$$4 + 36 = 8c$$

$$c = \frac{40}{8} = 5 \text{ --- Put in eq } ①$$

So,

$$x^2 + y^2 = 5x^2$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2 (5x - 1)$$

Taking "$\sqrt{\ }$" on b/s

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

$$\Longrightarrow \boxed{y = \pm x\sqrt{5x-1}}$$