COURSE TITLE :- ENA

MODULE :- 4th

INSTRUCTOR :- SIR SHAHRAYAR

-:STUDENT DETAILS:-

NAME :- NAVEED ALAM

STUDENT ID NO. :- 14965

STUDENT SIGNATURE :-

Q No. 1:- The switch in fig. 1 has been in position A for a long time. At $t = 0$ the switch moves to B. Determine $V(t)$ for $t > 0$ and calculate its value at $t = 2s$ and $8s$.

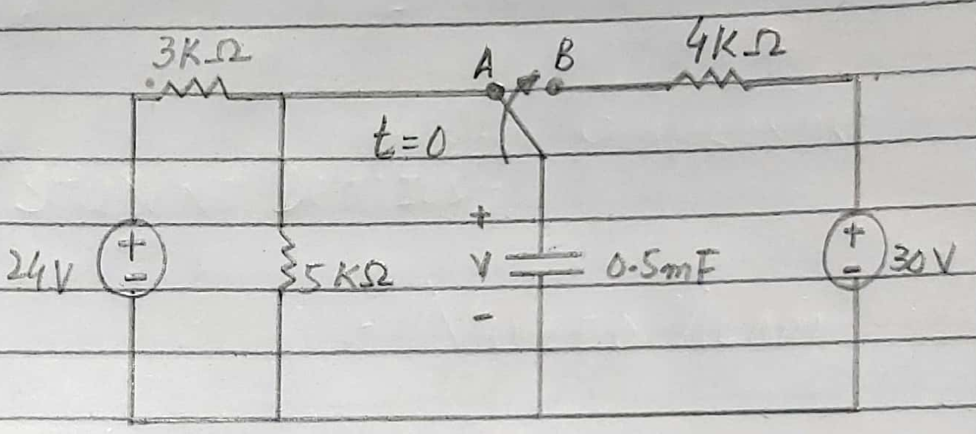


Figure 1

Solution:- For $t < 0$ the switch is at Position A capacitor act like an open circuit to DC. But V is same as the voltage across 5KΩ resistor. Hence the voltage across the capacitor just before $t=0$ is obtained by voltage division as.

$$V(0) = \frac{5}{5+3}(24)$$

$$V(0) = \frac{5}{8}(24)$$

$$V(0) = 15\ V$$

Scanned with CamScanner

Using the fact that capacitor voltage cannot change instantaneously-

$$V(0) = V(0^-) = V(0^+) = 15\text{ V}$$

$t > 0$ the switch is position B. Thevenin resistance connect to the capacitor $R_{TH} = 4K\Omega$ the time constant is

$$\tau = R_{Th}C = 4\times10^3 \times 0.5 \times 10^{-3}$$

$$\tau = 2s$$

Since the capacitor act like an open circuit to DC at steady state

$$V(\infty) = 30V$$

Thus

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$V(t) = 30 + (15-30)e^{-t/2}$$

$$V(t) = 30 + (-15)e^{-t/2}$$

$$V(t) = (30 - 15e^{-0.5})\text{ V}$$

At

$t=2$, $V(2) = 30 - 15e^{-2/2}$

$$V(2) = 30 - 15e^{-1}$$

$$V(2) = 24.48V$$

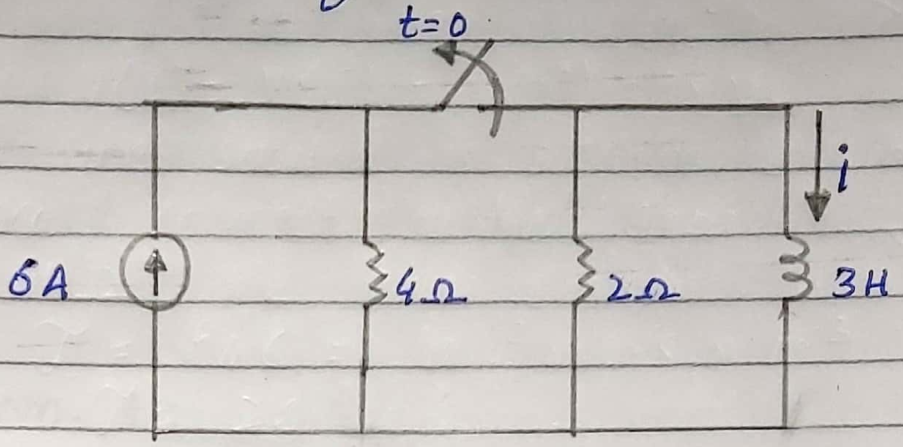
At

$t=8$, $V(8) = 30 - 15e^{-8/2}$

$$V(8) = 30 - 15e^{-4}$$

$$V(8) = 29.72V$$

QNO. 2:- Determine the inductor current for both $t>0$ and $t<0$ for the circuit in fig. 2.



Sol.:- 2

For $t<0$, the switch is closed and inductor act as short circuit. Therefore inductor current $i = 6A$.

For $t>0$, the switch is opened and time constant

$$\tau = \frac{L}{R}$$

$$\tau = \frac{3}{2}$$

Now the inductor current $i(t) = -\frac{t}{\tau}$

$$i(t) = 6e^{-t/\frac{3}{2}}$$

$$i(t) = 6e^{-\frac{2t}{3}} u(t) \text{ A.}$$

QNo. 3:- A series RLC circuit is described by

$$L\frac{d^2i}{dt} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5H$, $R = 4\Omega$ and $C = 0.2F$. Let $i(0) = 1$, $di(0)/dt = 0$.

Sol:- STEP NO. 1:-

The step respone of branch voltage of the given RLC circuit is describe by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divide by L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10}{L}$$

Right hand side of equation multiply by $\frac{C}{C}$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10C}{LC}$$

And $C = 0.2$ F thus

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \ldots\ldots (1)$$

Scanned with CamScanner

G- Equation for Source-Free series RLC circuit is given By-

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{I_s}{LC} \quad \ldots\ldots (2).$$

STEP NO. 2 :-

Compare 1 and 2

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

STEP NO. 3 :-

From (3) α is given by

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/s} \rightarrow (6)$$

Natural frequency ω_0 is given by

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From (4)

$$\omega_0 = \sqrt{10} \text{ rad/s} \rightarrow (7)$$

From 6 and 7

$\therefore$ $\alpha > \omega_0$

$\therefore$ The circuit is overdamped

Root of characteristic equ are given

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$S_1 = -4 + \sqrt{4^2 - \sqrt{10}^2}$$
$$S_1 = -4 + \sqrt{6} \text{ rad/s}$$

and,

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$S_2 = -4 - \sqrt{4^2 - \sqrt{10}^2}$$
$$S_2 = -4 - \sqrt{6} \text{ rad/s.}$$

As from 5 steady state current is given by

$$I_s = 20 \times LC$$
$$20 \times 0.5 \times 0.2 = 2A \longrightarrow (8)$$

STEP NO. 4:-

Current for overdamped case is given by

$$i(t) = I_s + A_1 e^{S_1 t} + A_2 e^{S_2 t} \quad t > 0 \rightarrow (9)$$

Substitute $t = 0$,

$$i(0) = I_s + A_1 + A_2$$

Scanned with CamScanner

Substitute,

Thus $1 = 2 + A_1 + A_2$

$$A_1 + A_2 = -1 \rightarrow (10)$$

STEP NO. 5 :-

From (9) find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 S_1 + A_2 S_2$$

Substitute the value

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \rightarrow (11)$$

Solve (10) and (11) simultaneously

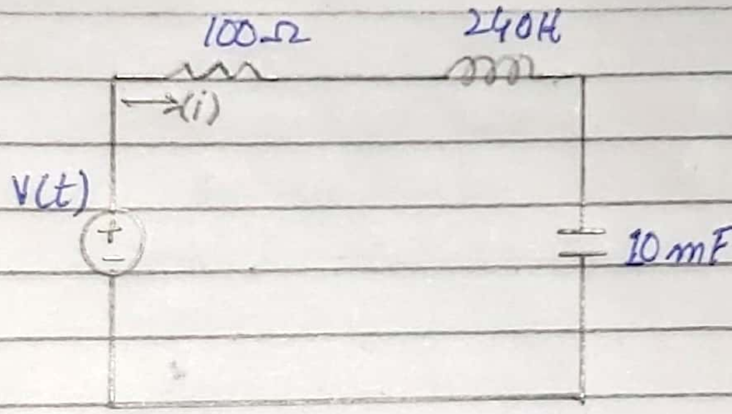
$$A_1 = -1.316$$
$$A_2 = 0.316$$

STEP NO. 6 :-

$$i(t) = 2 - 1.316e^{(-4+\sqrt{6})t} + 0.316e^{(-4-\sqrt{6})t} \text{ A}$$

Q No. 4:- A Series RLC circuit has $R = 100\,\Omega$, $L = 240H$ and $C = 10mF$. If the input voltage is $V(t) = 10\cos 2t$, find the current flowing through the circuit.

Solution 4:-



Consider the following voltage is applied the series RLC circuit -

$$V(t) = 10\cos 2t \text{ V}$$

Here

Ampilitude $V_m = 10V$

Angular frequency $\omega = 2$ rad/s

Phase Angle $\phi = 0°$

So phase for voltage, $V(t)$.

$$V(t) = 10\angle 0° \text{ V}$$

Inductive Reactance of the circuit.

$$X_L = \omega L$$

$\omega = 2$ rad/s , $L = 240$ H

$$X_L = (2 \text{ rad/s})(240 \text{ H}).$$

$$X_L = (480\ \Omega).$$

Now for Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

$\omega = 2\ rad/s$, $C = 10\,mF$

$$X_C = \frac{1}{2(10 \times 10^{-3})}$$

$$X_C = \frac{1}{2(10^{1-3})}$$

$$X_C = \frac{1}{2(10^{-2})}$$

$$X_C = \frac{1 \times 10^{2}}{2} = \frac{100}{2} = 50$$

$$X_C = 50\ \Omega$$

Now for impedence of the circuit -

$$Z = R + jX_L - jX_C$$

$R = 100\ \Omega$, $X_L = 480\ \Omega$, $X_C = 50\ \Omega$

$$Z = (100 + 480 - 50)$$

$$Z = (100 + j430)\ \Omega$$

Represent the impedance, Z is Phasor from

$$Z = (100 + j430)\,\Omega$$

$$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$$

$$Z = \sqrt{10000 + 184900} \angle \tan^{-1}(4.3)$$

$$Z = \sqrt{194900} \angle \tan^{-1}(4.3).$$

$$Z = 441.47 \angle 76.90^\circ\,\Omega$$

Now the current flowing "i".

$$i = \frac{V(t)}{Z}$$

$$V(t) = 10\angle 0,\ Z = 441.47 \angle 76.9^\circ$$

$$i = \frac{10\angle 0^\circ \ V}{441.47\angle 76.9^\circ\,\Omega}$$

$$i = \frac{10}{441.47} \angle [0 - 76.9]\,A$$

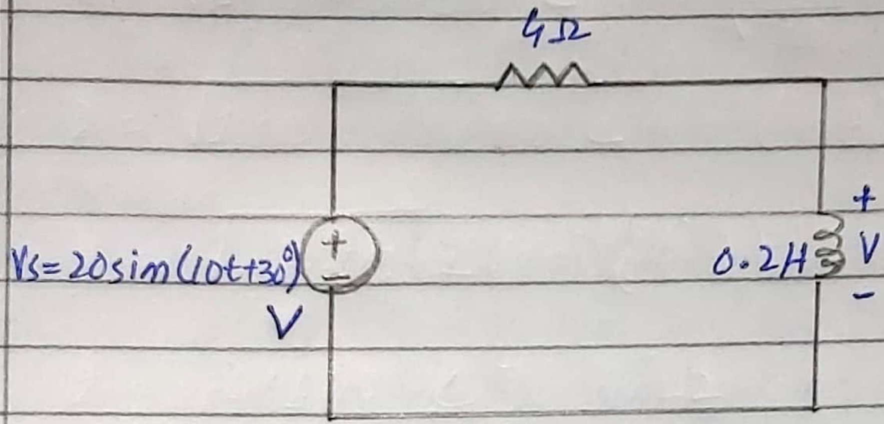
$$i = 22.6 \times 10^{-3} \angle -76.9^\circ A.$$

$$i = 22.6 \angle -76.9^\circ mA$$

So the General expression for current "i" -

$$i = 22.6 \cos(2t - 76.9^\circ)\ mA.$$

Q No. 5:- Find v(t) and i(t) in the circuit shown in figure 3.



Solution 5:-

$Vs = 20 \sin(10t + 30^\circ) V$, $R = 4\Omega$, $L = 0.2H$.

$Vs = 20 \sin(10t + 30^\circ) V$

$Vs = 20 \cos(10t + 30^\circ - 90^\circ) v$

$Vs = 20 \cos(10t - 60^\circ) V$

$Vs = 20 \angle -60^\circ V$

$\omega = 10 \text{ rad/sec}$

$X_L = i\omega L$

$0.2H = i \times 10 \times 0.2$

Now

$z = 4 + i2\Omega$

$$i = \frac{20\angle -60^\circ}{4 + i2}$$

$$I = \frac{20\angle -60^\circ}{\sqrt{4^2 + 2^2} \angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$I = \frac{20\angle -60^\circ}{4.72\angle 26.57^\circ}$$

$$I = 4.472\angle -86.57.$$

Now

$$i(t) = 4.472\cos(10t - 86.57)$$
$$i(t) = 4.472\sin(10 - 86.57 + 90)$$
$$i(t) = 4.472\sin(10t + 3.43^\circ)\,A.$$

As

$$V = j2 \times (4.47\angle -86.57).$$
$$V = j2 \times (0.2675 - j4.464)$$

$$V = 8.92 + j0.53512$$

from Rectangular to polar from

$$V = \sqrt{(8.926)^2 + (0.53512)^2}\angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$$

$$V = 8.944\angle 3.4V$$

Now

$$V(t) = 8.944\cos(10t + 3.4)$$
$$V(t) = 8.944\sin(10t + 3.4^\circ + 90^\circ)$$

$$V(t) = 8.944\sin(10t + 93.4)V.$$