Name: Muhammad Tayyab

ID # 7945

Section "B"

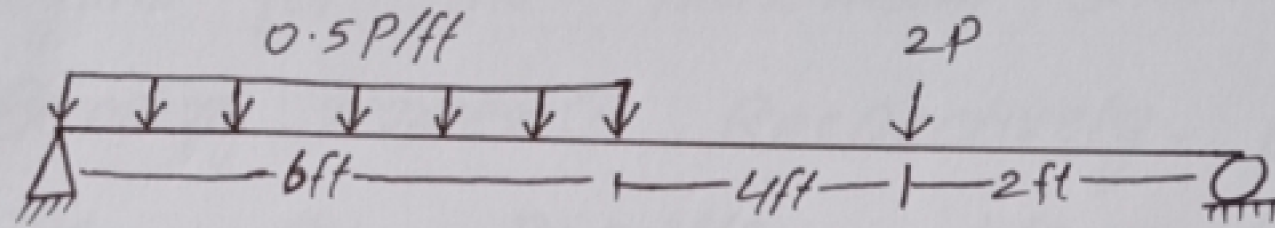
Fourth Semester.

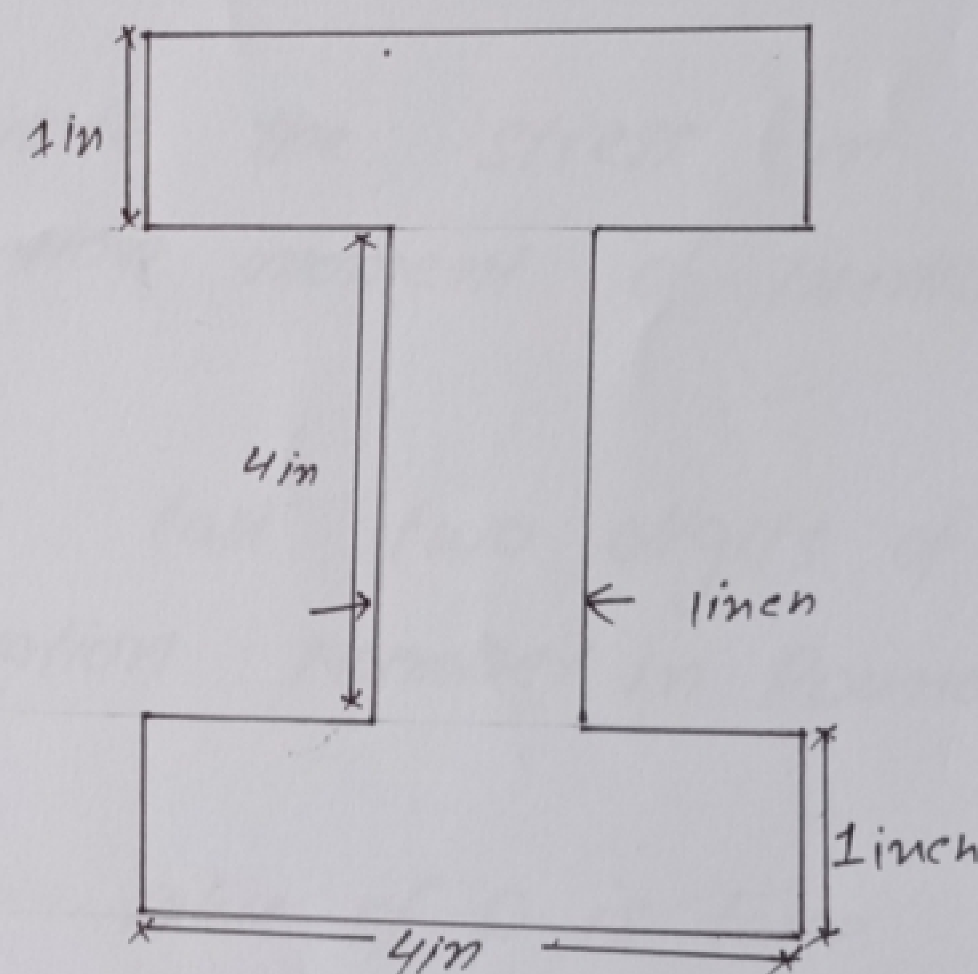
Sub: MOS II

Instructer: Muhammad Saqib

Department of civil Engineering

Q NO # 01





Construct the Mohar's Circle Diagram and find the Principle Stresses and Maximum in Plane Shear Stress at the Center stress of a Point 'C' Located at the center of the uniformly distributed load. And 1 inch below from Top fiber of the Beam. The cross-section of the Beam shown However, To Construct the Mohar's circal it is Necessary to draw the Shear Stress and flexure stress variation diagram for the maximum shear force and Bending moment. Respectively. Compare the the Results obtain from the Mohar's circle with the stress transformation equation.

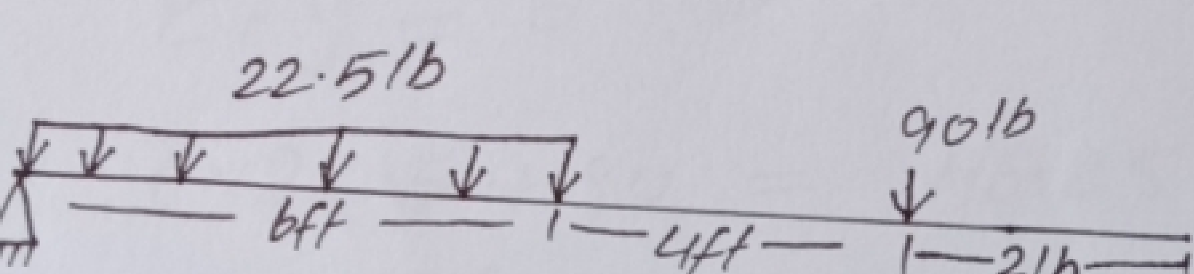
Hints:→ To calculate the stress in the Beam crossection moment of inertia must be know.

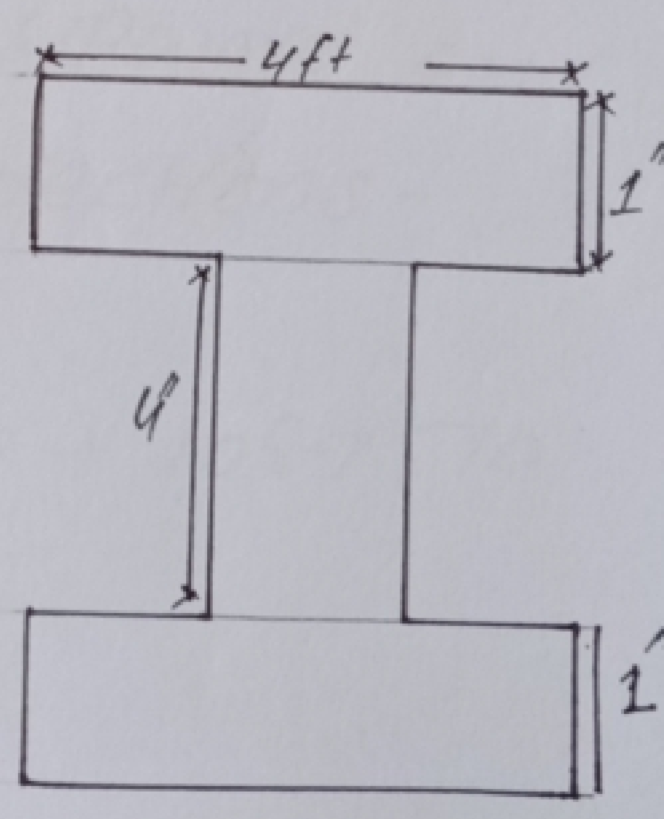
where P is the last two digits of your class Registration Number in Pound.

Solution:→ The value of P is take last two diclits of class Registration Number so 7945.

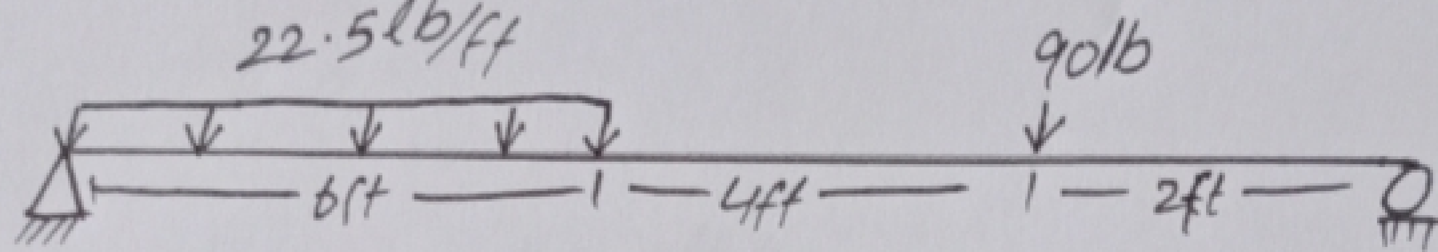
$2P = 2(45 = 90\,lb$

$0.5P = 0.5(45) = 22.5\,lb$





First of all we find Rection of Beam.



$R_A = 116.25 lb$

$R_B = 108.75 lb$

$\circlearrowright + \sum M_B = 0$

$$R_A * 12 - (22.5 * 6)(6+3) - 90 * 2 = 0$$

$$R_A = 116.25 lb$$

$\circlearrowleft + \sum M_A = 0$

$$-R_B * 12 + (90 * 10) + (22.5 * 6) * 3 = 0$$

$$R_B = 108.75 lb$$

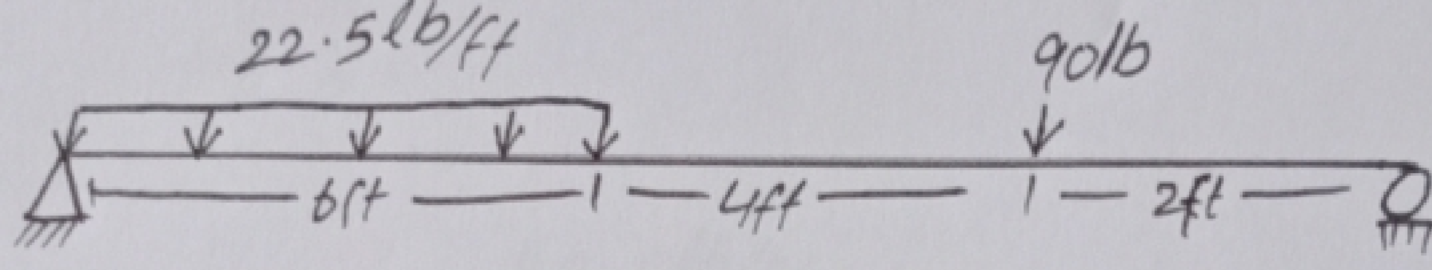
Now we check sum of upward and downward forces: Thier rections-

$\sum F_y = 0$

$$6 \times 22.5 + 90 = 116.25 lb + 108.75 lb$$

$$225 = 225$$

First of all we find Rection of Beam.



$R_A = 116.25 lb$

$R_B = 108.75 lb$

$\circlearrowright + \ \Sigma M_B = 0$

$$R_A * 12 - (22.5 * 6)(6+3) - 90 * 2 = 0$$

$$R_A = 116.25 lb$$

$\circlearrowright + \ \Sigma M_A = 0$

$$-R_B * 12 + (90 * 10) + (22.5 * 6) * 3 = 0$$

$$R_B = 108.75 lb$$

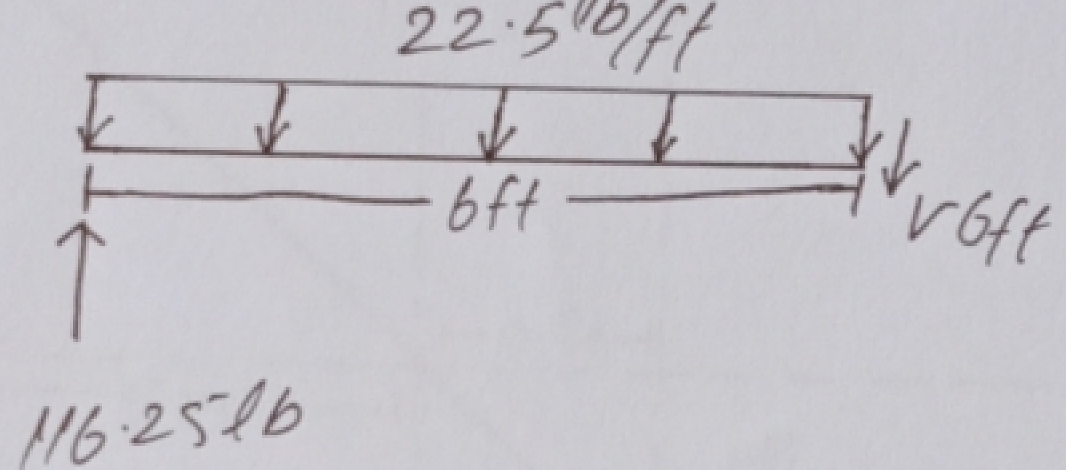
Now we check sum of upward and downward forces: Thier rections-

$\Sigma F_y = 0$

$$6 \times 22.5 + 90 = 116.25 lb + 108.75 lb$$

$$225 = 225$$

Now we find Shear force at point of chaing:



$\overset{+}{\uparrow} \Sigma F_y = 0$

$$116.25 - 22.5 * 6 - V_{6ft} = 0$$

$$V_{6ft} = 116.25 - (22.5 * 6)$$

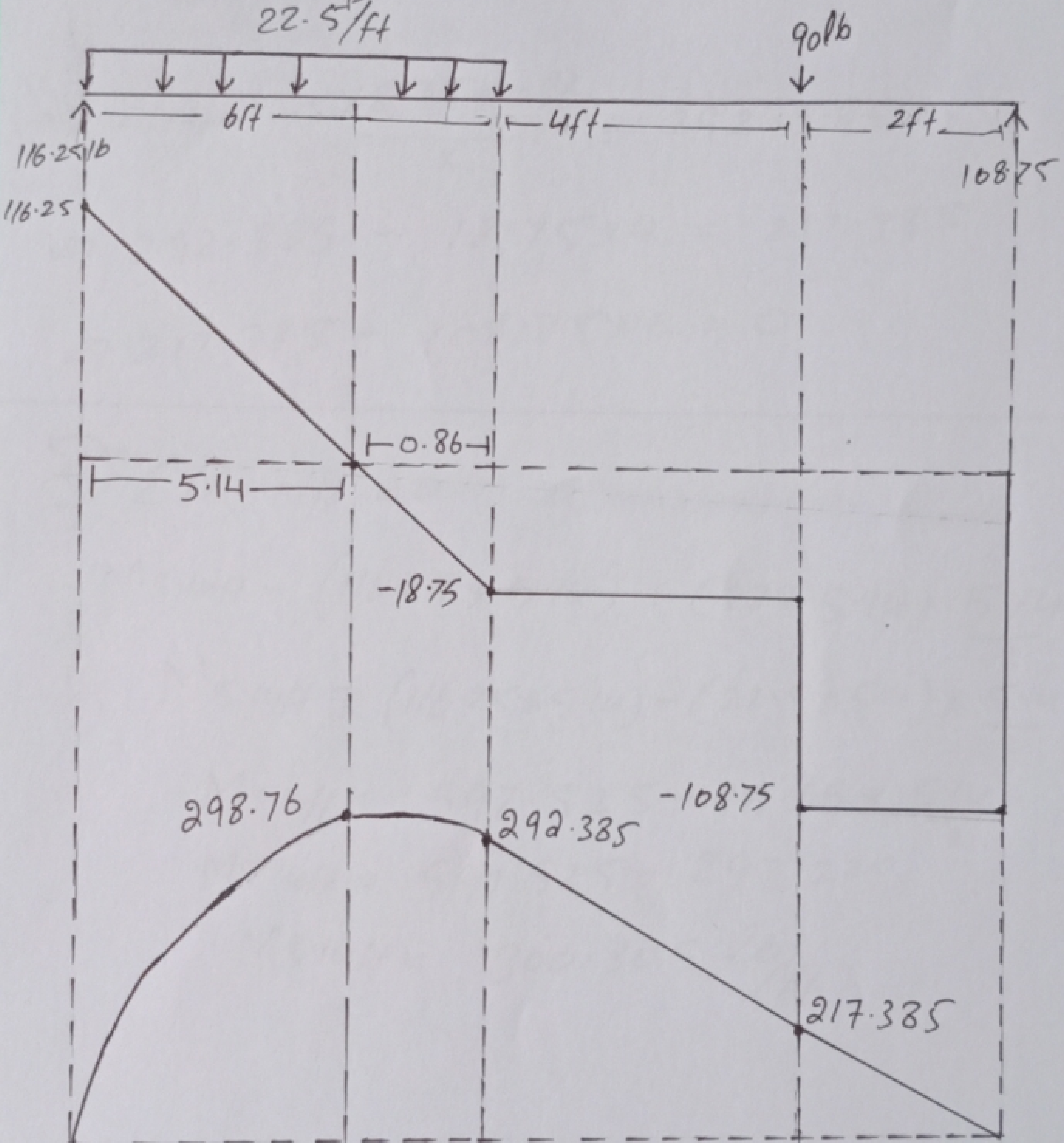
$$\boxed{V_{6ft} = -18.75 lb}$$

$\overset{+}{\uparrow} \Sigma F_y = 0$

$$-V_{10ft} - 90 - (22.5 * 6) + 116.25 lb = 0$$

$$V_{10ft} = -90 - (22.5 * 6) + 116.25 lb$$

$$V_{10ft} = -108.75 lb.$$



x = distance = ?

$$\frac{116.25}{x} = \frac{18.75}{6-x}$$

$$x\,18.75 = 116.25(6-x)$$

$$x\,(18.75+16.75) = 135.5$$

$$x = \frac{697.5}{135.5} = 5.14$$

$$x = 5.14$$

$$6-x$$

$$6-5.14 = 0.86$$

BM Detail:

$\Rightarrow \frac{116.25 * 5.14}{2} = 298.76$

$\Rightarrow 298.76 - \frac{18.75 * 086}{2} = 292.385$

$\Rightarrow 292.385 - 18.75 * 4 = 217.385$

$\Rightarrow 217.385 - 108.75 * 2 = 0$

$\overset{+}{\circlearrowright} \Sigma M_{5.14ft} = 0$

$M_{5.14ft} - (116.25 * 5.14) + (22.5 * 5.14) \frac{5.14}{2} = 0$

$M_{5.14ft} = (116.25 * 5.14) - (22.5 * 5.14) * \frac{5.14}{2} = 0$

$M_{5.14ft} = 597.525 - 115.65 * \frac{5.14}{2}$

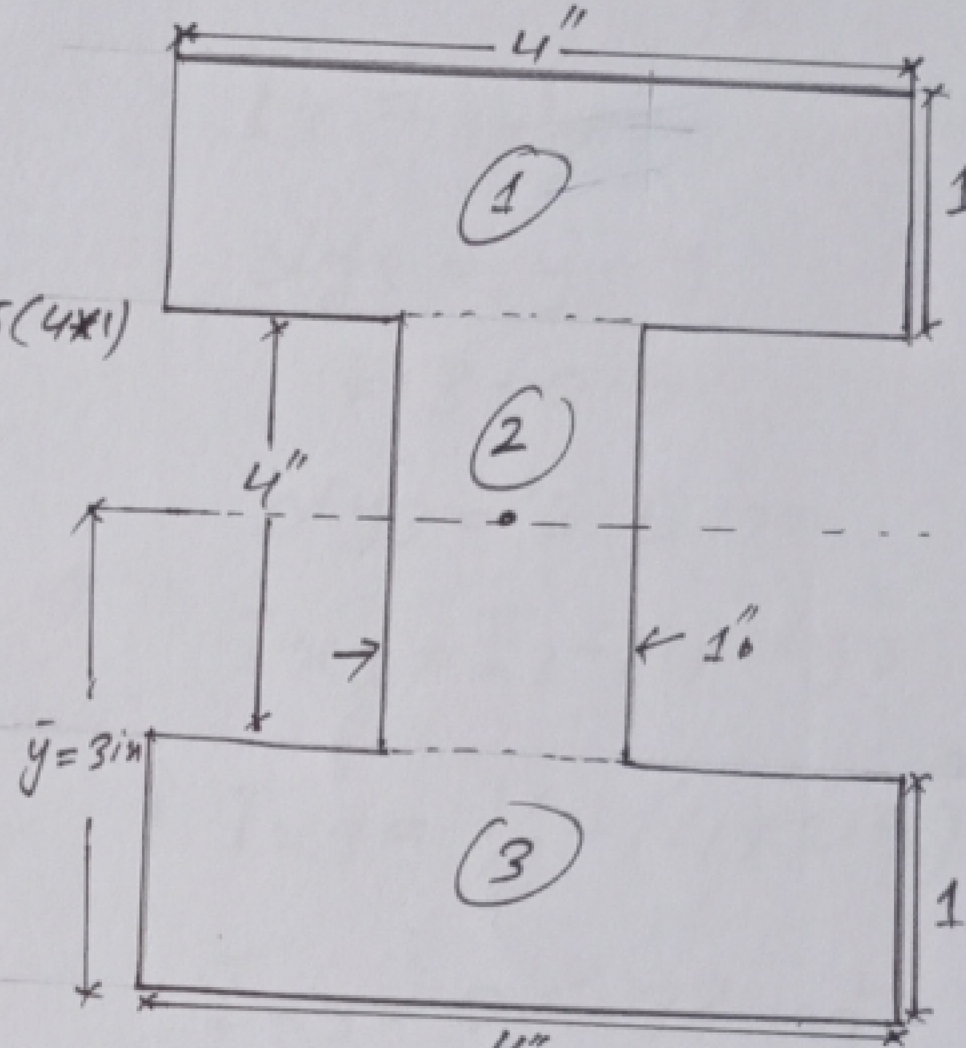
$M_{5.14ft} = 597.525 - 297.220$

$M_{5.14ft} = 300.305 \ lb/ft$

find the moment of inertia of the Beam cross-section.

⇒ Centroid of I-section:

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}$$



$$\bar{y} = \frac{(5.5)(4*1) + 3(4*1) + 0.5(4*1)}{4+4+4}$$

$$\boxed{\bar{y} = 3in}$$

$$I_x = I_{x1} + I_{x2} + I_{x3} \text{ --- (A)}$$

For $I_{x1} = I_1 + A_1 d^2 y_1$

$A_1 = 4*1 = 4in^2$

$$I = \frac{bh^2}{12} = \frac{4(1)^2}{12}$$

$$I_1 = \frac{1}{3} in^4$$

$y = y - \acute{y} = 5.5 - 3$

$dy_1 = 2.5in$

$I_{x1} = I_1 + A_1 d^2 y_1$

$$I_{x1} = \frac{1}{3} + 4*(2.5)^2$$

$I_{x1} = 25.33$

For I_{x2}

$A_2 = 4*1$

$A_2 = 4 in^2$

$I_2 = \frac{bh^2}{12}$

$I_2 = \frac{1*4^3}{12}$

$I_2 = 16/3 \ in^4$

$dy_2 = y - \bar{y}$

$= 3-0$

$dy_2 = 0$

$I_{x2} = I_2 + A_2 dy_2^2$

$I_{x2} = \frac{16}{3} + 0$

$I_{x2} = 5.333 in^4$

For I_{x3}

$A_3 = 4*1$ $A_3 = 4 in^2$

$I_3 = \frac{bh^3}{12} = \frac{4\times1^3}{12}$

$I_3 = \frac{1}{3}$

$dy_3 = \bar{y} - y$

$= 3 - 0.5$

$dy_3 = 2.5 \ in$

$I_{x3} = I_3 - A_3 dy_3^2$

$I_{x3} = 1/3 + (4*2.5)^2$

$I_{x3} = 25.33 \ in^4$

Now put the value of I_{x1}, I_{x2}, I_{x3} in equation (A) we get.

$I_x = 25.33 + 5.33 + 25.33$

$I_x = 56 in^4$

Shear Stress: As per the Question the maximum shear stress $\tau = \frac{VQ}{Ib}$ occurs where the maximum shear force lies. In this diagram the max shear force value is 116.75 lb.

Now we find the shear stress $T = \frac{VQ}{Ib}$ it is nesessary to find "I" moment of inertia of the Given crossection. which we already find in previous page.

As we evaluate the shear force and Bending moment diagram and moment of Inertia of the section it is possible to calculate the Shear stress and flenural stress at any point.

For Shear stress:

Shear stress along the depth of the section. $T = \frac{VQ}{Ib}$

where $V_{max} = 116.75\ lb$

$I_x = 56\ in^4$

b = Thickness / breath

$Q = \bar{y}A$

Case (1) T = at the top fiber

$$T = Q = \bar{y} * 0 = 0$$

$$\tau = \frac{116.75(0)}{56*1} = 0$$

Case (2) $\tau =$ in 0.5in below top fiber.

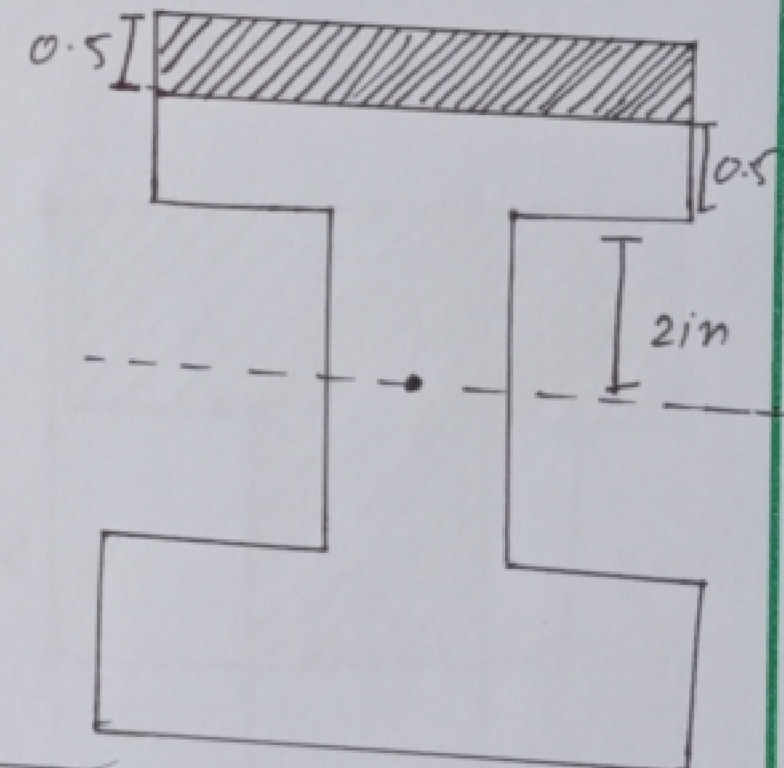
$Q = \bar{y}A$

$Q = \left(2+0.5+\frac{0.5}{2}\right)(0.5*4)$

$Q = 5.5$

$\tau = \frac{VQ}{Ib} = \frac{116.75(5.5)}{56*4}$

$\boxed{\tau = 2.86 \text{ psi}}$



Case (3) $\tau =$ 1in below top fibe

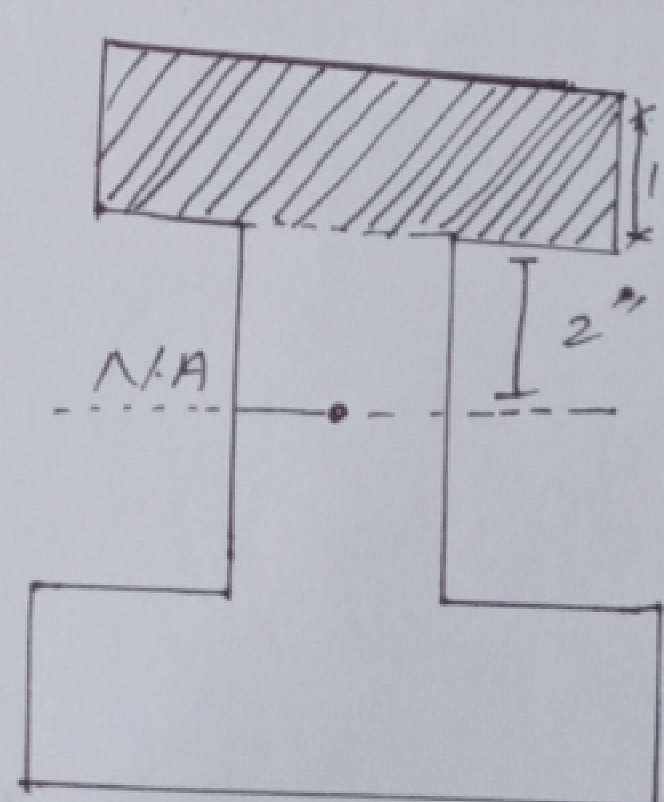
(A) $Q = \bar{y}A$, $b = 4''$

$Q = \left(2+\frac{1}{2}\right)(1*4)$

$Q = 10in$

$\tau = \frac{VQ}{Ib} = \frac{116.75(10)}{56*4}$

$\boxed{\tau = 5.21}$



(B) $Q = \bar{y}A$, $b = 1''$

$Q = 10 in$

$$\tau = \frac{116.75(10)}{56 * 1}$$

$$\boxed{\tau = 20.84 psi}$$

Case (4) τ - at centeroid

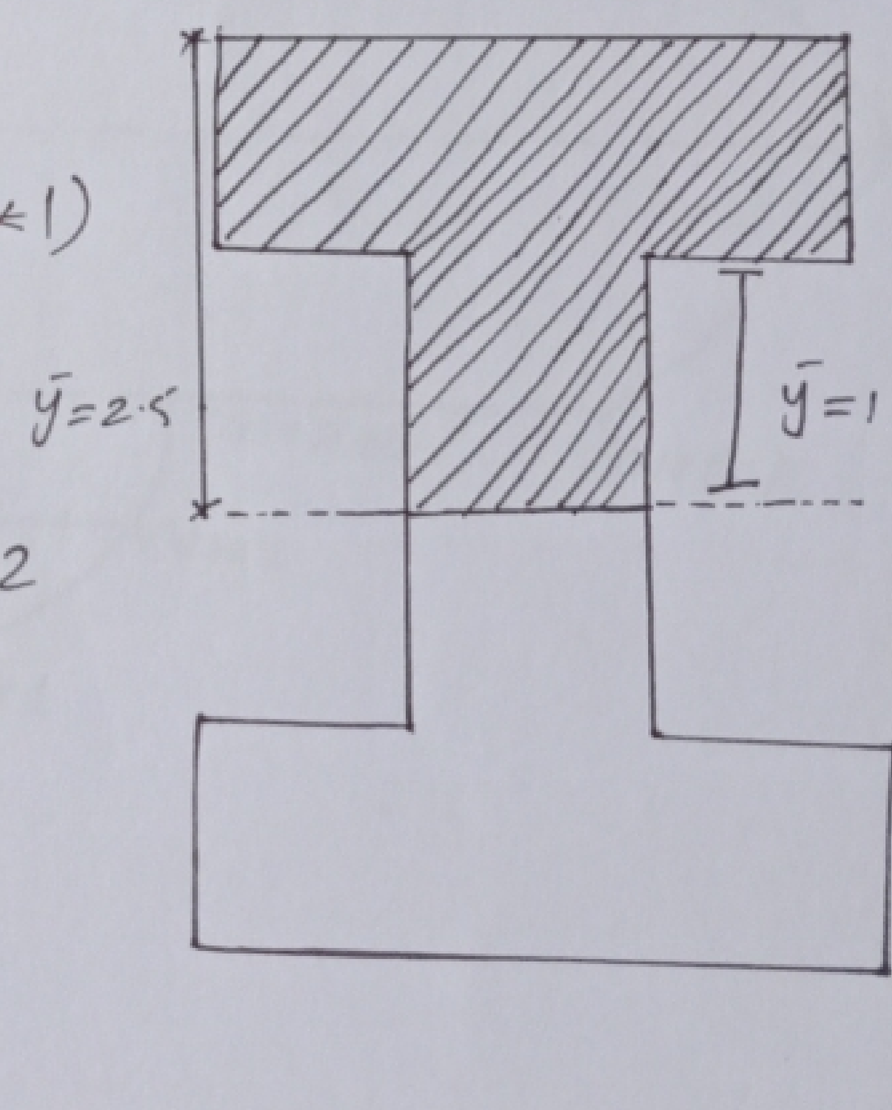
$Q = Q_1 + Q_2$

$Q = \bar{y}_1 A_1 + \bar{y}_2 A_2$

$Q = 2.5 * (4*1) + 1(2*1)$

$Q = 12 in^3$

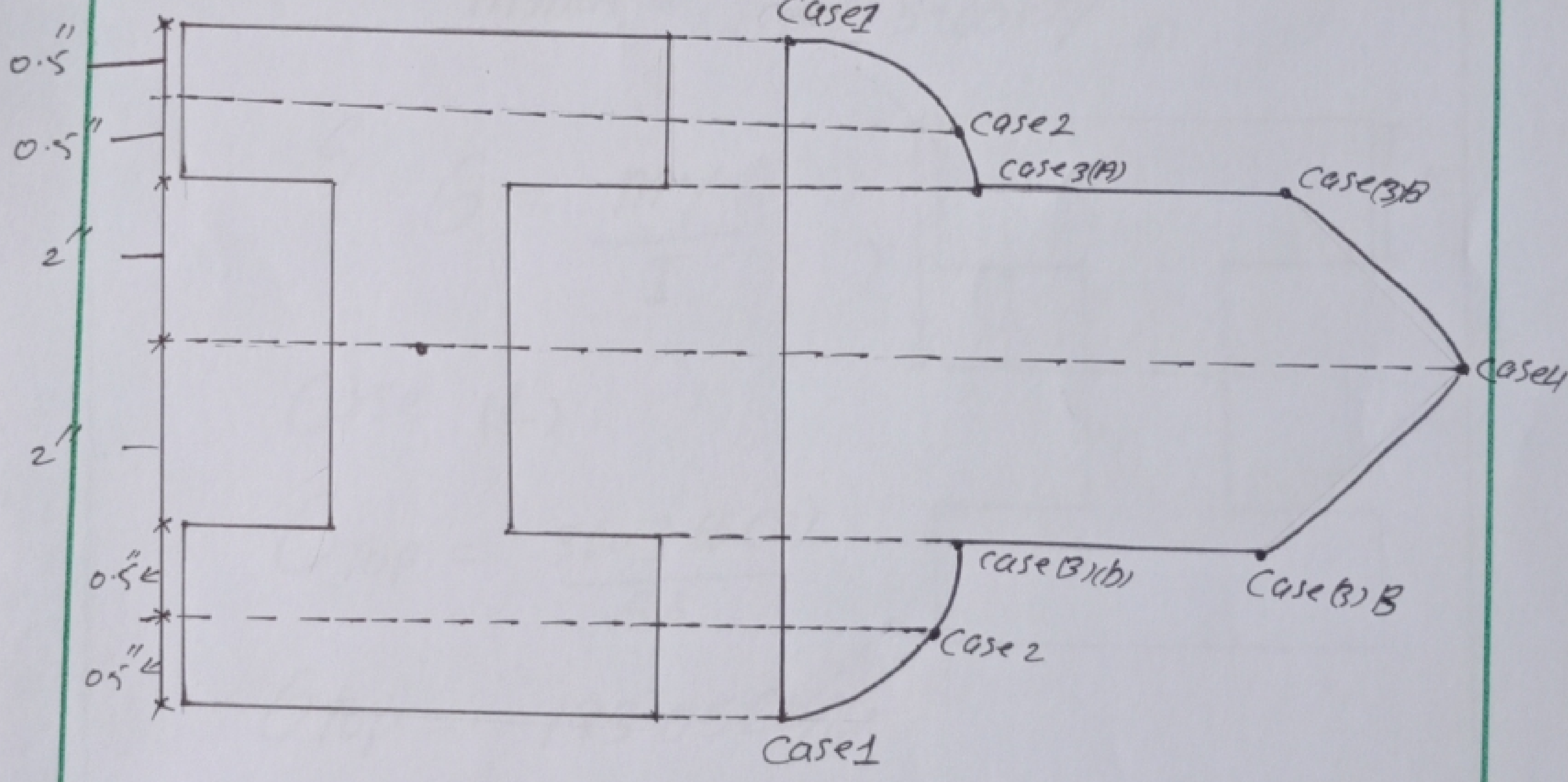
$$\tau = \frac{VQ}{Ib} = \frac{116.75 * 12}{56 * 1}$$



$$\boxed{\tau = 25.01 psi}$$

As the I-section is symmetrical x & y axis will occur below of the centroidal axis as occured above.

Shear Stress veration Diagram.



Flexural Stresses Analysis:

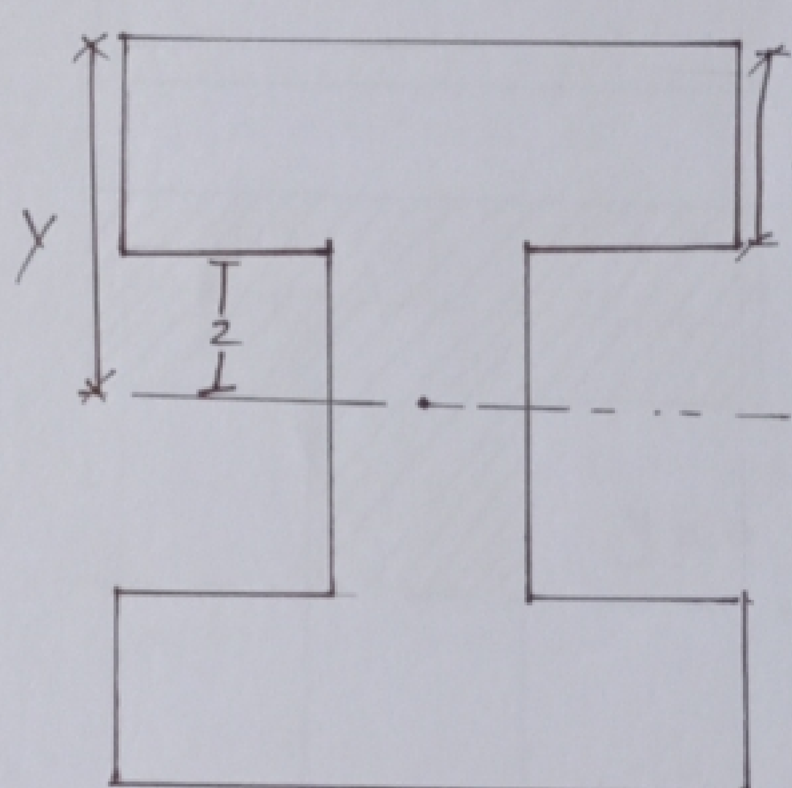
For the flexural stress Analysis we consider Maximum Moment which as given below

$$M_{max} = 300.305\, lb/ft$$

$$M_{max} = 300.305 \times 12$$

$$M_{max} = 3603.66\, lb/in$$

& $$\sigma = \frac{MY}{I}$$

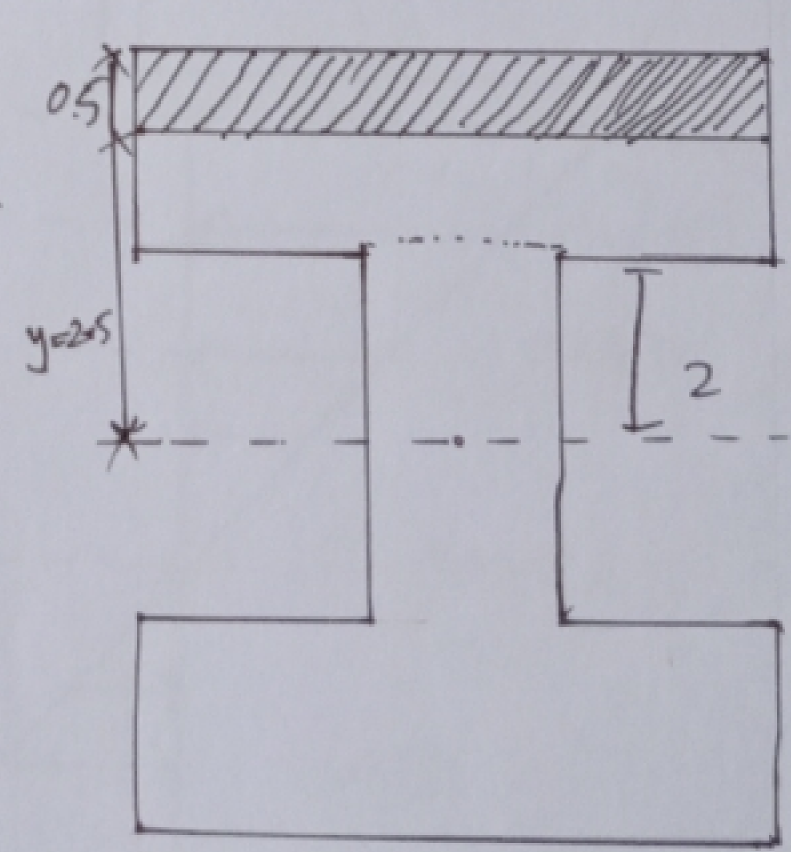


Case (1)

$$\sigma_{Top} = \frac{3603.66(3)}{56}$$

$$\sigma_{Top} = 193.05\, lb/in^2$$

Case (2)



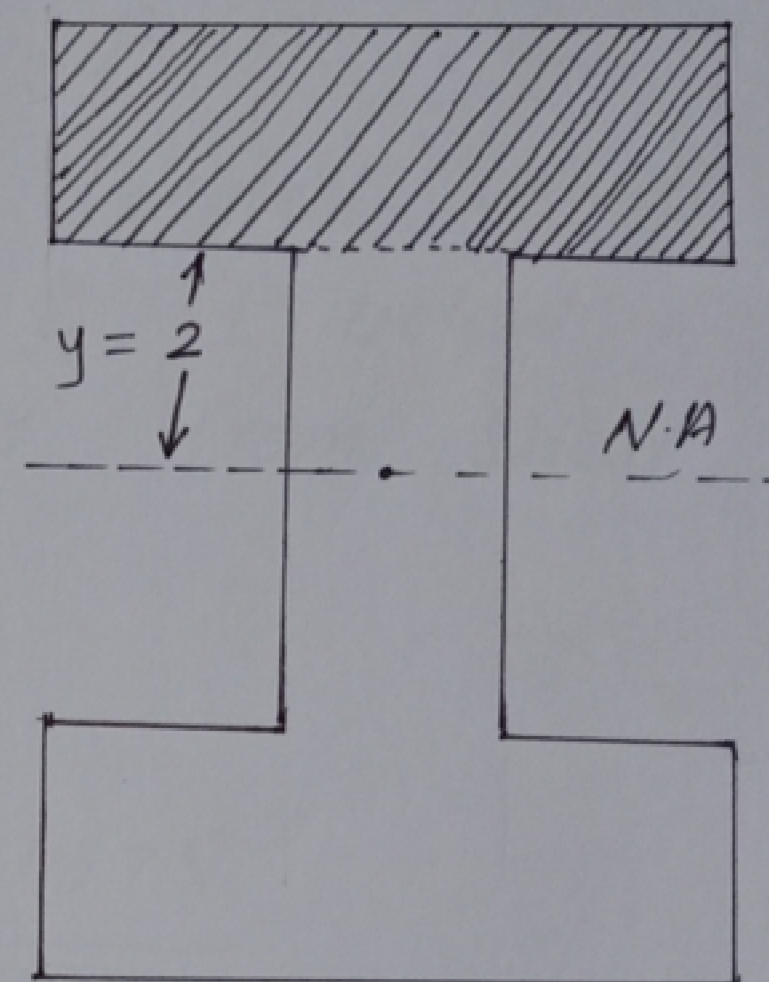
$$\sigma_{0.5} = \frac{3603.66(2.5}{56}$$

$$\sigma_{0.5} = 160.87\, psi$$

Case 3

$$\sigma_1'' = \frac{360\cdot 3.66(2)}{56}$$

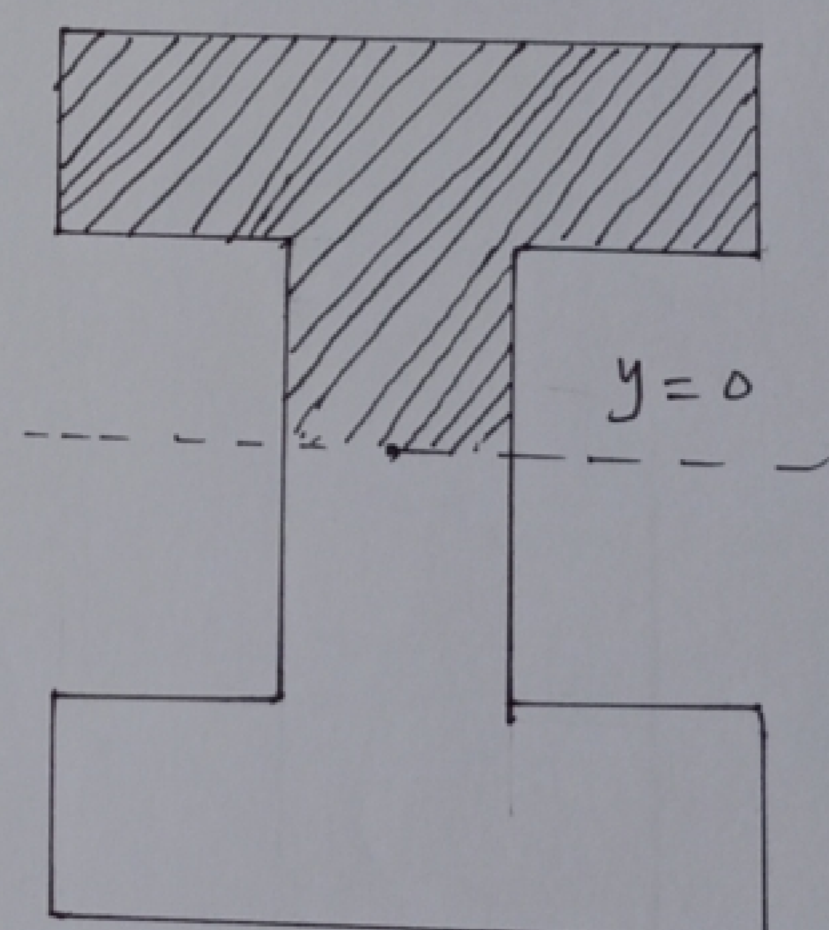
$$\sigma_1'' = 128.70 \text{ psi}$$



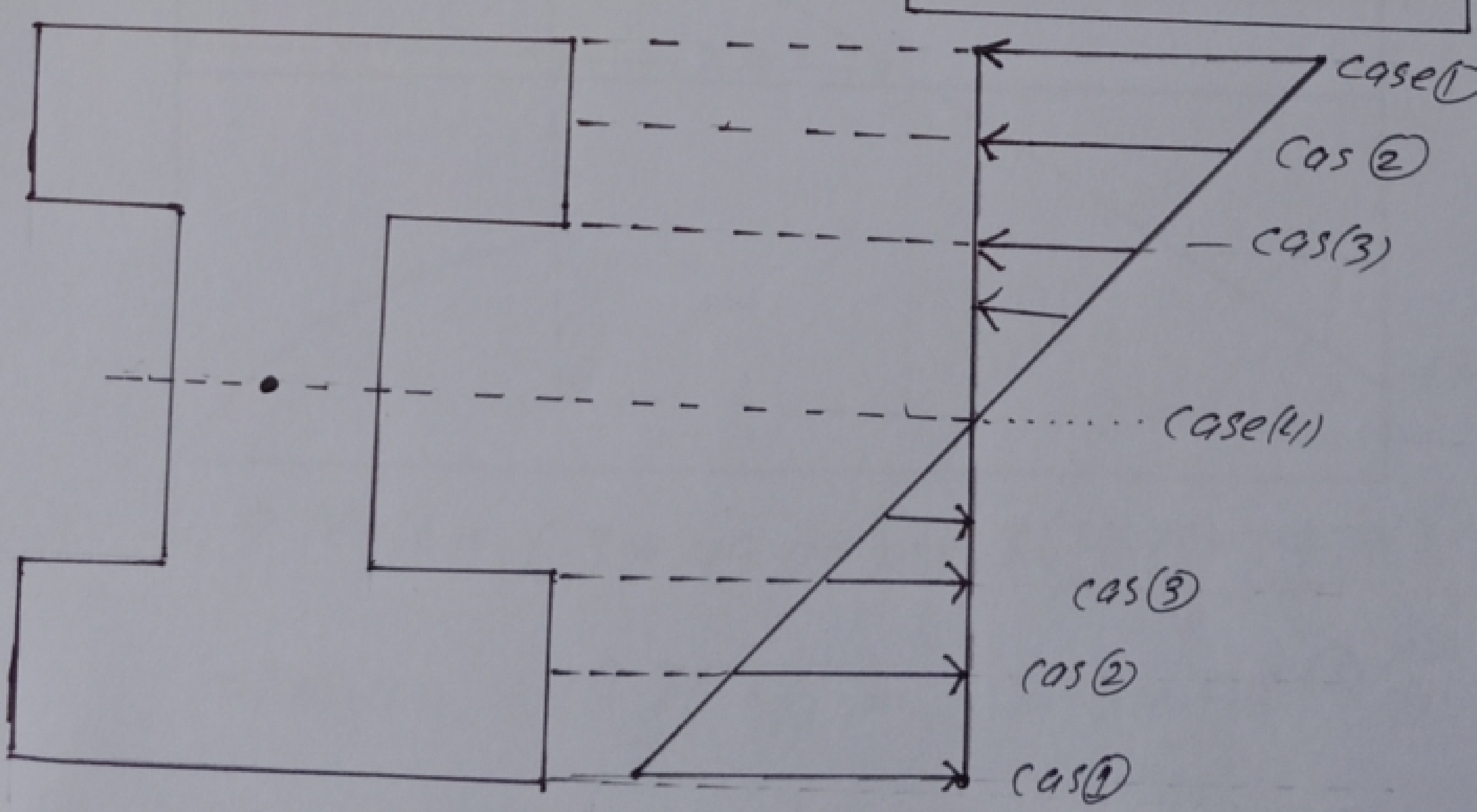
Case (4)

$$\sigma_{N.A} = \frac{3603.66(0)}{56}$$

$$\sigma_{N.A} = 0$$



Note: Because of the Symmetrical Shape of I-Section Flexural Streasses below The N.A Will be Same as above.

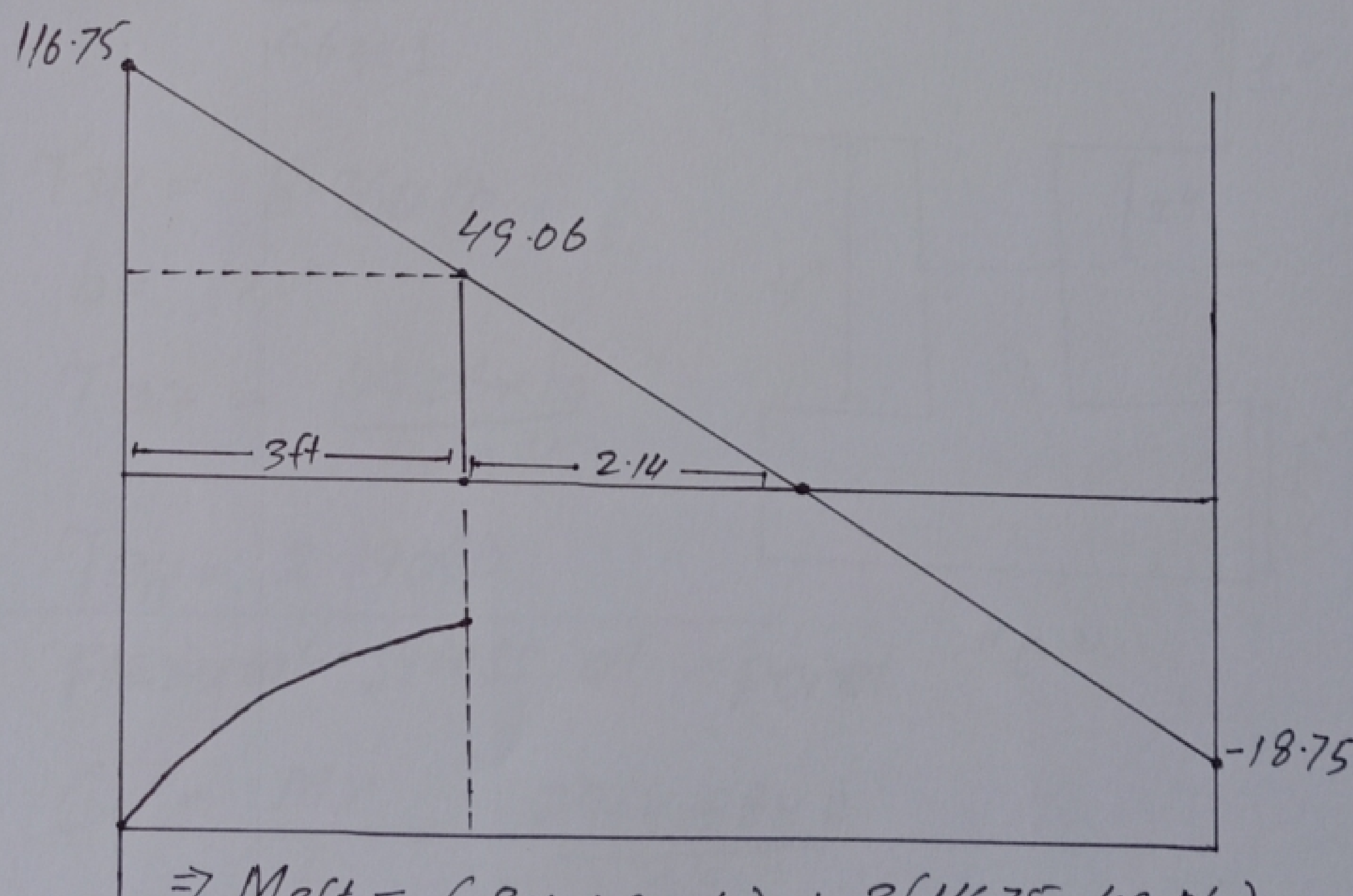


Stress State at Point "C"

Stress at point "C" located at 3ft from ~~left~~ left support & 1" below from the top fiber from shear force diagram.

$$\frac{V_{3ft}}{(5.14-3)} = \frac{2.14 * 116.75}{5.14}$$

$$V_{3ft} = 49.06 lb$$

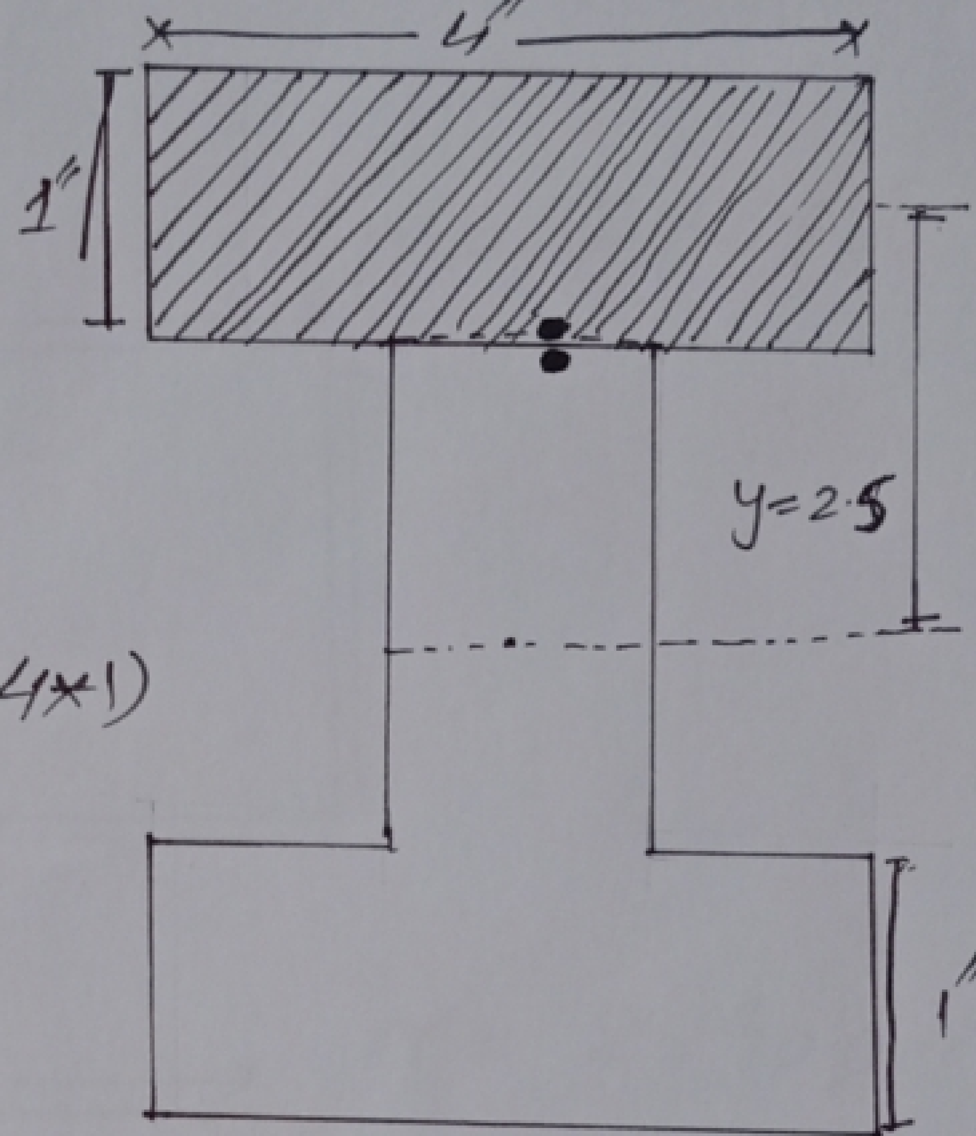


$$\Rightarrow M_{3ft} = (3 * 49.06) + 3\frac{(116.75 - 49.06)}{2}$$

$$\Rightarrow M_{3ft} = 248.71\ lb/ft = 2984.58\ lb/in$$

two values i-e b=1", b=4"

for b=1"



$I = 56\ in^4$

$$\tau_{3ft} = \frac{VQ}{Ib}$$

here $Q = \bar{y}A = (2.5)(4*1)$

$Q = 10\ in^3$

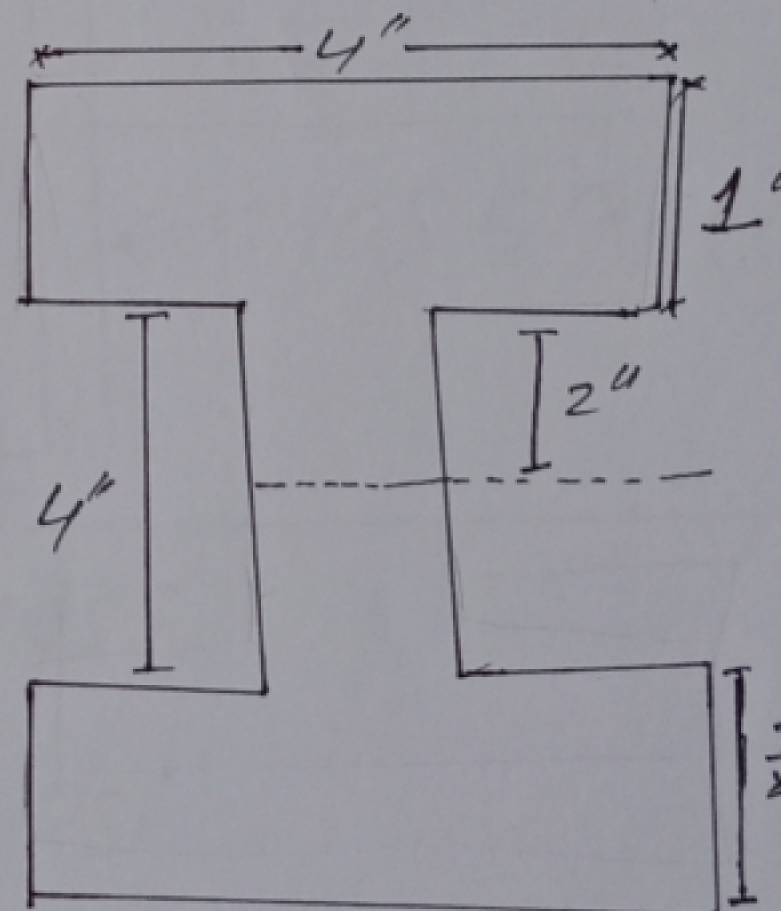
$$\tau_{3ft} = \frac{49.06*10}{56*1}$$

$\tau_{3ft} = 8.760\ lb$

$b = 4"$

$$\tau_{3ft} = \frac{49.06*10}{56*4}$$

$\tau_{3ft} = 2.190\ lb$



Flexural stress at point "c"

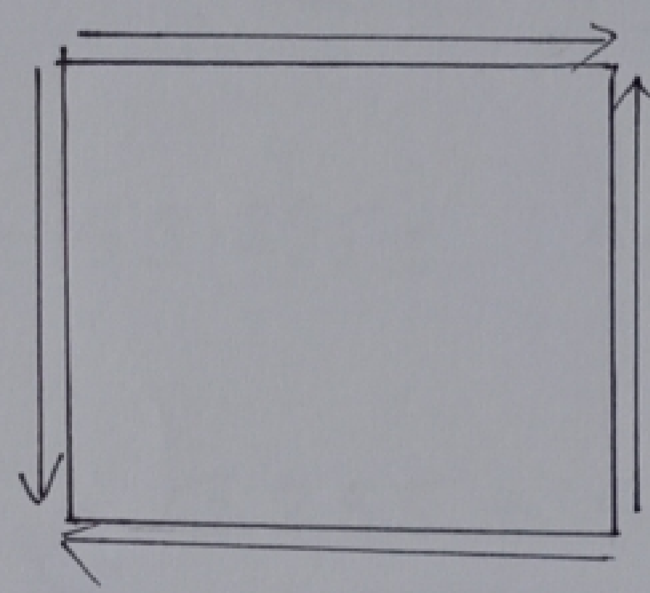
$$\sigma_c = \frac{My}{I} = \frac{2984.58*2}{56}$$

$\sigma_c = 106.59\ psi$

2D Representation of Point "C"

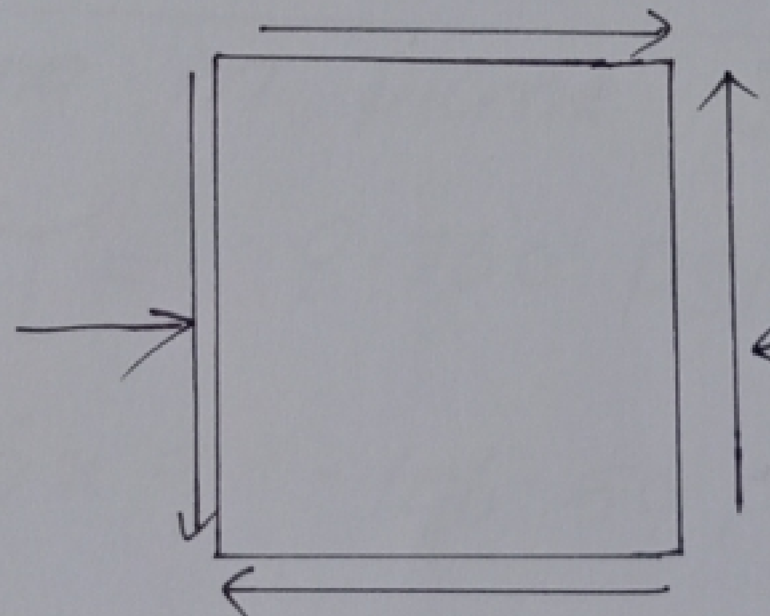
For $b = 1''$

$\tau = 8.760$ psi



For $b = 4''$

$\tau = 2.190$ psi



106.59 psi

Principle Stresses:

For $\tau = 8.760$ psi

$$\sigma_x = -106.59 \text{ psi}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{8.760}{\frac{-106.59 - 0}{2}}$$

$$\tan 2\theta_p = \frac{8.760}{-53.295} = \theta_p = \frac{\tan^{-1}(-0.164)}{2} = \boxed{\theta_p = -4.66}$$

(16)

$$\sigma_{1,2} \quad \frac{\sigma x+\sigma y}{2} \pm \sqrt{\frac{\sigma x+\sigma y}{2}+\tau xy^2}$$

$$\sigma_{1,2} = \frac{-106.59+0}{2} \pm \sqrt{\left(\frac{-106.59-0}{2}\right)^2+(8.760)^2}$$

$$\sigma_{1,2} = -53.295 \pm 54.01$$

$$\sigma y = \sigma_1 = -53.295+54.01 = -0.715\ psi$$

$$\sigma x = \sigma_2 = -53.295-54.01 = -107.305\ psi$$

Maximum in plane Shear Stress.

$$\tau = 8.760\ psi$$

$$\sigma x = -106.59\ psi$$

$$\tan 2\theta_s = \frac{-(\sigma x - \sigma y/2)}{\tau xy}$$

$$= \frac{-(106.59 - 0/2)}{8.760}$$

$$= \frac{53.295}{8.760} = \frac{\tan^{-1} 6.08}{2}$$

$$\boxed{\theta s = 40.32}$$

$\theta_s = 40.32$

τ_{max} in plane

$$\sqrt{\frac{\sigma_x - \sigma_y}{2} + \tau_{xy}^2}$$

$$\sqrt{\left(\frac{-106.59 - 0}{2}\right)^2 + 8.760^2}$$

$\tau_{max \text{ in plane}} = 54.01 \, lb$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

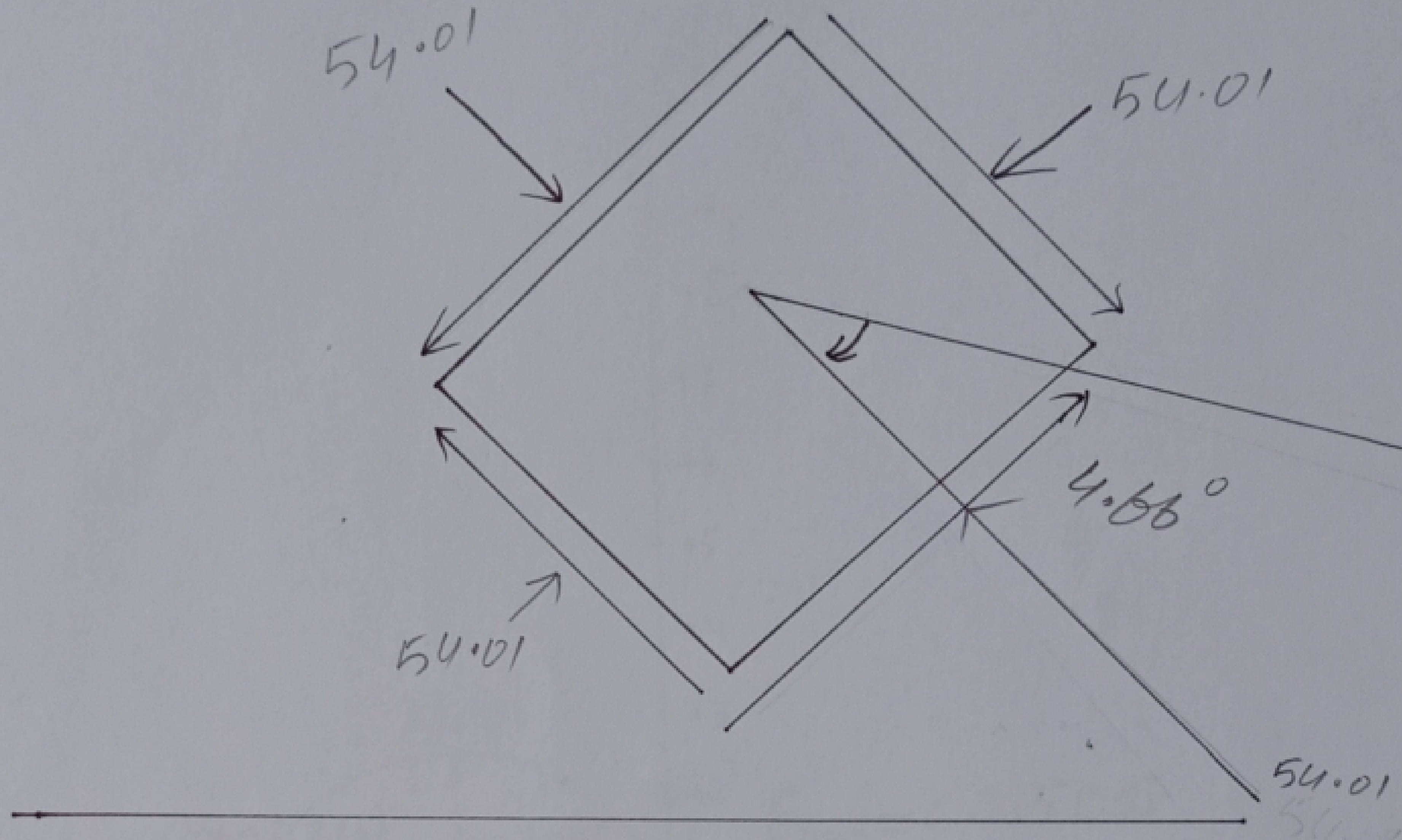
$$= \frac{-107.305 + 0.715}{2}$$

$= -53.295 \, lb$

& $\theta_s = -4.66$

$4.66 + 40.32 = 45^\circ$

The difference between principle shear stress is 45° degree.



Mohar's Circle

$$\sigma_x = 106.59 \text{ psi}$$

$$\tau_{xy} = 8.760 \text{ psi}$$

So we get

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{-106.59 - 0}{2}\right)^2 + 8.760^2}$$

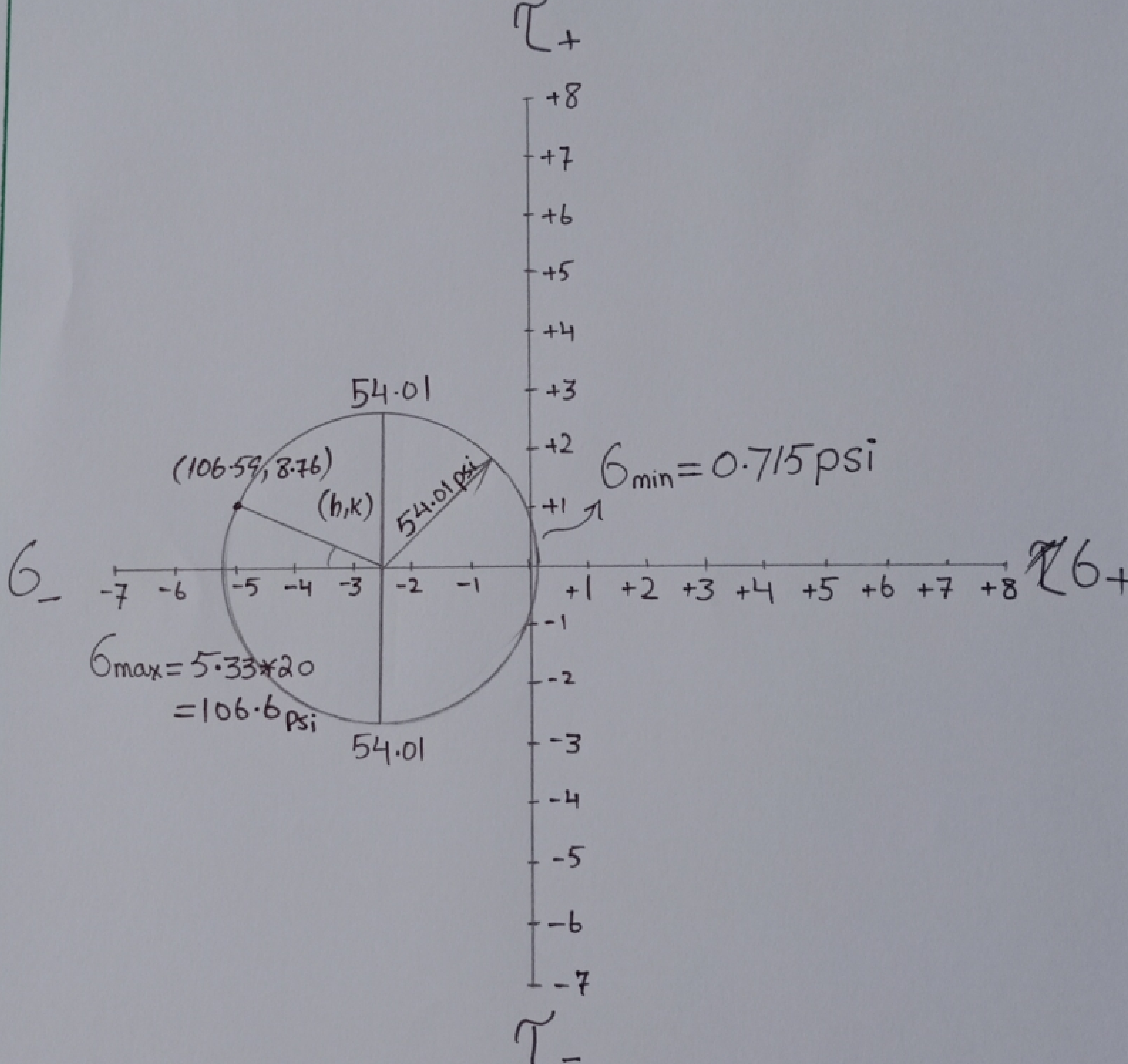
$$R = 54.01 \text{ psi}$$

Center co-orinates

$$= (h, k) = \frac{\sigma_x + \sigma_y}{2}, 0 \quad \left(\frac{-106.59 + 0}{2}, 0\right)$$

$$(h, k) = (-53.38, 0)$$

Assumption: for scale $1cm = 20 psi$



Similarly shear stress occur at $90°$ to the principle stresses in mohar's circle so here it is equal to radius of the circle which is $R = 54.01 psi$