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Question = 1 :-

Answer :-

1) $w = \sin(x+ct) + \cos(2x+2ct)$.

Sol:-

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \quad \text{---(1)}$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct).$$

$$= [-\sin(x+ct) - 4.\cos(x+2ct)]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2[-\sin(x+ct) - 4(\cos(2x+2ct))]$$

$$= \boxed{c^2 \frac{\partial^2 w}{\partial w^2}} \text{ prove -}$$

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ii) $w = \tan(2x+ct)$

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$= c\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c\sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct)$$

$$= 2c^2\sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\Rightarrow \frac{\partial^2 w}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\Rightarrow \frac{\partial^2 w}{\partial x^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2\, 8\sec^2(2x+ct)\tan(2x+ct)$$

So it is not satisfied.

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Question "2":-

fourier Series:-

$$f(x) = x, \quad -\pi < x \leq 0$$
$$= 2x, \quad 0 \leq x \leq \pi$$

Sol:-

Given function is

$$f(x) = \begin{cases} x \,; & -\pi < x \leq 0 \\ 2x \,; & 0 < x \leq \pi \end{cases}$$

have to find fourier series co-efficient a_0, a_n & b_n

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx - \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

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$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \pi/2} \quad (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right].$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} \,; & \text{if } n \text{ is odd} \\ 0 \,; & \text{if } n \text{ is even} \end{cases}} \quad (2)$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\boxed{b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = -\frac{3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}}$$

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So the required fourier series is

$$f(x) = \frac{A_0}{2} + \sum_{n+1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{2n-1} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$$

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Question = 3:-

Given:-

$y'' - 4y' + 13y = 8\sin 3x,\ y(0) = 1\ \&\ y'(0) = 2$

Sol:- $y'' - 4y' + 13y = 8\sin 3x$ — ①

Associated Homogenous eq ①

13.---

$y'' - 4y' + 13y \cancel{- 8\sin 3x} = 0$ — ②

chang ② into Auxiliary equation

put $y = m$ in ②

$m^2 - 4m + 13 = 0$

use Quadratic formula

$a = 1,\ b = -4,\ c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - (4)(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

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$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 + 6i}{2}$$

$$= 2 \pm 3i$$

$$\boxed{\begin{aligned} m_1 &= 2+3i \\ m_2 &= 2-3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (A)$$

Let

$$y_p = A\cos 3x + B\sin 3x \rightarrow (x)$$

Diff w.r.t. "x"

$$y'p = -3A\sin 3x + 3B\cos 3x.$$

Again Diff w.r.t "x".

$$y_p'' - 9A\cos 3x - 9B\sin 3x$$

Put in (1)

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$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x) + 13(A\cos 3x + B\sin 3x) - 8\sin 3x$

$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x + 12A\sin 3a + 13B\sin 3x - 8\sin 3x$

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing coefficients.

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \Rightarrow (a)$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B.$

$\Rightarrow \boxed{A = 3B} \Rightarrow (b)$

Put (b) in (a)

$4B + 12(3B) - 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \rightarrow (c)$

Put (c) in (b)

$\Rightarrow \boxed{A = 3/5} \rightarrow (d)$

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Put (c) & (d) in (x)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin x \rightarrow (B)$$

The G. sol is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + \frac{3}{5}\cos x + \frac{1}{5}\sin 3x - (C)$$

Now we need to find the values of c_1 & c_2 for this

put

$x = 0$ & $y = 1$ in (C)

$$1 = e^{x(2)}(c_1\cos 3(0) + c_2\sin 3(0) + \frac{3}{5}\cos(0) + \frac{1}{5}\sin 3(0)$$

$$1 = (c^{(1)} + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + \frac{3}{5}$$

$$c_1 = 1 - \frac{3}{5}$$

$$\boxed{c_1 = \frac{2}{5}} \rightarrow (xx)$$

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Diff (C) w.r.t "x"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos)$

$= \frac{6}{5}\sin 3x + \frac{8}{5}\cos 3x \rightarrow$ (D)

Put $y' = 2,\ x = 0$ in (D)

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$

$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$

$- \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x.$

Put $y' = 2,\ x = 0.$

$2 = c_1(2e^{x(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)).$

$+ c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0))$

$- \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0))$

$2 = c_1(2) + c_2(3) - 0 + \frac{3}{5}$

$2 = 2c_1 + 3c_2 + \frac{3}{5}$

Put $c_1 = \frac{2}{5}$

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$2 = \frac{4}{15} + 3c_2 + \frac{3}{5}$

$2 - \frac{7}{5} + 3c_2$

$3c_2 = 2 - \frac{7}{5}$

$3c_2 = \frac{3}{5}$

$\boxed{c_2 = \frac{3}{15}} \rightarrow ***$

Put $**$ & $***$ in (c)

$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

Required general equal

Q No = 4:-

$(D^2 - DD')z = \cos x \cos 2y.$

Sol:-

It is already in symobolic form

$(D^2 - DD')z = \cos x \cos 2y \rightarrow (a)$

Put A.E $D^2 - DD' = 0$

We know that

$\frac{D}{D'} = m$ i-e $D = m. D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

Therefore C.F $= f_1(y) + f_2$

from eq (a)

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$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y.$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} \cos x \cos 2y.$$

$$= 2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

~~C.F = F₁(y-x) + xf~~

$$C.F = F_1(y, x) + x\, f_2(y-x).$$

$$P.I \frac{1}{D^2 + 2DD' + D'} \left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')} \left[2(y-x) + \sin(x-y)\right)$$

By genral method.

$$m = -1, \quad y - x = c$$

$$= \frac{1}{D+D'} \left[\int [2c + \sin(-c)]\right] dx$$

$$= \frac{1}{D+D'} \left[2cx - (\sin c) x\right]$$

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Replacing c by $y-x$.

$$\int (2xc - x\sin c)\,dx \Rightarrow cx^2 - \frac{x^2}{2}\sin c.$$

Replacing c by $y-x$.

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y).$$

Hence required solution is

$$\boxed{z = C.F + P.I = F_1(y-x) + xF_2(y-x) + x^2y - y^3 + \frac{1}{2}x^2\sin(x-y).}$$