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Q no:- 01 (a):-

Find the Polynomials of degree 3 or less that interpolates the Point (0,2), (1,1), (2,0) & (3,-1)

Sol:

According to Given Condition

The Lagrange form is,

$$P(x) = 2\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 1\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} +$$

$$0\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} - 1\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$\Rightarrow -\frac{1}{3}(x^3-6x^2+11x-6) + \frac{1}{2}(x^3-5x^2+6x) - \frac{1}{6}(x^3-3x^2+2x)$$

$$\Rightarrow -\frac{1}{3}(x^3-5x-6) + \frac{1}{2}(x^3-x) - \frac{1}{6}(x^3-x)$$

$$\Rightarrow \boxed{-x+2} \rightarrow \text{Ans}$$

Hence proved

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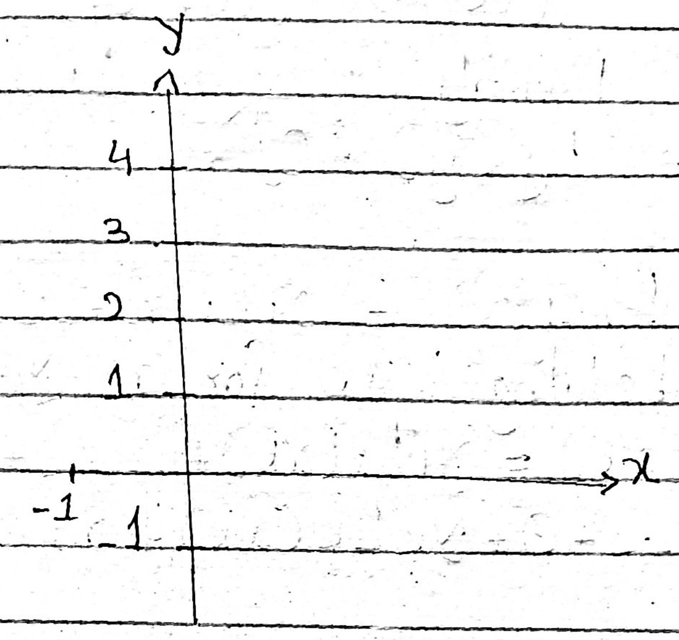
Q.No:-1(b):-

Find the Interporating Polynomial for the data Points (0,1), (2,2) & (3,4) fore the fig Given below

Sol:-



Substduting into Lagrange formula Jields

$$P_2(x) = 1\frac{(x-2)(x-3)}{(0-2)(0-3)} + 2\frac{(x-0)(x-3)}{(2-0)(2-3)} + 4\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$$\Rightarrow \frac{1}{6}(x^2-5x+6)+2\left(-\frac{1}{2}\right)(x^2-3x)+4\left(\frac{1}{3}\right)(x^2$$

$$\Rightarrow \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

So $P_2(0) = 1$,

$P_2(3) = 4$

for General PurPose n point (x_i, y_i)
for the degree polynomial n-1

~~$P_{n-1}(x) = y_1 L_1(x) + \ldots + y_n L_n$~~

$$L_K(x) = \frac{(x-x_1)\ldots(x-x_{K-1})(x-x_{K+1})\ldots(x-x_n)}{(x_K-x_1)\ldots(x_K-x_{K-1})(x_K-x_{K+1})\ldots(x_K-x_n)}$$

The ProPerty of L_K & $L_K(x_K) = 1$
While $L_K(x_j) = 0$

$$P_{n-1}(x) = y_1 L_1(x) + \ldots + y_n L_n(x)$$

Subsituting x_K for x Yields

$$P_{n-1}(x_K) = y_1 L_1(x_K) + \ldots + y_n L_n(x_K) = 0 + \ldots + 0 + y_K L_K(x_K) + 0 + 0 \ldots + 0 = y_K$$

Hence Prooved

Q.No:- 2(a):-

Use the two Point forward difference formula with $h=0.1$ approximate the derivative of $f(x)=\frac{1}{x}$ at $x=2$

Sol:- As we Know that
The two Point forward difference formula is:

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

$$f'(x) \approx \frac{1/2.1 - 1/2}{0.1}$$

$$\boxed{f'(x) = -0.2381} \rightarrow (1)$$

The difference between this approximation and the correct derivative

$f'(x) = -x^{-2}$ at $x=2$ is the error

$$0.2381-(-0.2500)$$

$$\boxed{\Rightarrow 0.0119} \rightarrow (2)$$

Compare error predicted formula which is given

$h f''(2)/C$ for some c b/w 2 & 2.1

Since

$f''(x) = 2x^{-3}$, the error

$(0.1)^{-3} \approx 0.0125$ &

$(0.1)(2.1)^{-3}$

≈ 0.0108

However the Information is Usually in not available

According to Tylor Theorm

Now In Second formula

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_1)$$

&

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(c_2)$$

Where $x-h < c_2 < x < c_1 < x+h$

Substracting the two equation give the Three Points formula with an explicit error term

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2)$$

Hence we will get

$$\boxed{f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2)}$$

Required Ans

Q.No:-2(b):-

Use Newton divided difference to find the Interpolating polynomials Passing through the Centres points (0,1),(2,2)&(3,4)

Sol:-

$$\begin{array}{c|cccc} 0 & 1 & & & \\ & & 1/2 & & \\ 2 & 2 & & \frac{1}{2} & \\ & & 2 & & \\ 3 & 3 & & & \end{array}$$

After down x & y Co.ordinates in Seperate Coloumn,

Calculate the next Coloumn,

$$\frac{2-1}{2-0} = \frac{1}{2}$$

$$\Rightarrow \frac{2-\cancel{0}\,1/2}{3-0}$$

$$\boxed{\Rightarrow \frac{1}{2}} \longrightarrow (1)$$

then

$$\Rightarrow \frac{4-2}{3-2}$$

$$\Rightarrow \boxed{2} \rightarrow (2)$$

after Completing the divided difference table, the Co-efficient of the Polynomial are 1, 1/2, 1/2 are as written

$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}(x-0)(x-2)$$

or

In nested form

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + (x-2)\cdot\frac{1}{2}\right)$$

The base point for the nested form $r_1 = 0$ & $r_2 = 2$, then polynomial as

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2)$$

$$\boxed{P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1}$$

Ans is Prooved

Qno:- 3(a):-

Solve the least square Problem

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

Sol:- According to Given Condition

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

But we know that

The normal equation is

$A^T A x = A^T b$ are

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}$$

The Solution of the normal equation are

$\bar{x}_1 = 3.8$ & $\bar{x}_2 = 1.8$ the residual Vector is

$$r = b - A\bar{x} = \begin{bmatrix} 3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}$$

which has Euclidean norm $\|e\|_2 =$

$$\sqrt{(0.4)^2 + 2^2 + (-2.2)^2} \Rightarrow \boxed{3 \text{ Ans}}$$

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Q.No: 3 (b)

Find the line that best fits the three data points $(t, y) = (1, 2), (-1, 1)$ & $(1, 3)$ in figure

Sol:-

According to given figure

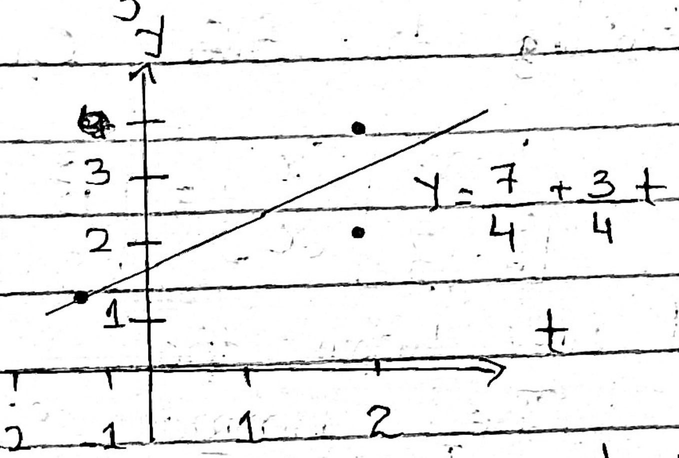

Fig 4.3 Best line example 4.3 One each of the data point lies above on, and below the best line

The model is $y = C_1 + C_2 t$ & the goal is to find the best C_1 & C_2. Substitution of the point into the model yields

$$C_1 + C_2(1) = 2$$
$$C_1 + C_2(-1) = 1$$
$$C_1 + C_2(1) = 3$$

Or In Matrix form is given below

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Ans is Proved