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Module	Summer
Subject	Linear Algebra
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Q. No. 1:-

(a):-

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

Identify the $(3,2)$ entry of AB.

Sol:

We will multiply A with B.

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-6-6 & 4+2+6 \\ 4+6-2 & 16-2+2 \\ 0+3+4 & 0-1-4 \end{bmatrix}.$$

$$AB = \begin{bmatrix} -11 & 12 \\ 8 & 16 \\ 7 & -5 \end{bmatrix}.$$

The entry $(3,2)$ of AB is $= -5$.

---x---

Q1(b) Quadratic polynomial

points (1,3), (2,4) & (3,4)

Solution :-

Since there are three points, So we will use Quadratic Interpolating polynomial.

$$P_2(x) = a_0 + a_1 x^0 + a_2 x^2 \rightarrow (A)$$ we have to find a_0, a_1 & a_2?

As we have given the points where

$x_1 = 1, x_2 = 2,$ & $x_3 = 4$

Similarly

$y_1 = 3, y_2 = 4$ & $y_3 = 4$

So By Augmented matrix

$$\begin{bmatrix} 1 & x_1 & x_1^2 & y_1 \\ 1 & x_2 & x_2^2 & y_2 \\ 1 & x_3 & x_3^2 & y_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & : 3 \\ 1 & 2 & 4 & 4 \\ 1 & 4 & 16 & 4 \end{bmatrix}$$

So Convert it into Reduced echelon form.

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 1 & 3 & : & 1 \\ 0 & 3 & 15 & : & 1 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 1 & 3 & : & 1 \\ 0 & 0 & 6 & : & -2 \end{bmatrix} R_3 - 3R_2$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & -\frac{1}{3} \end{array}\right] \frac{R_3}{6} \Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} \end{array}\right] R_2 - 3R_3$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 0 & 10/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1/3 \end{array}\right] R_1 - R_3 \Rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1/3 \end{array}\right]$$

So $a_0 = 4/3$, $a_1 = 2$ & $a_2 = -1/3$

Putt in (A)

$$\boxed{P_2(x) = 4/3 + 2x - \frac{1}{3}x^2}$$

2 (a) $|A| = 2$ & $|B| = -3$

So

$|A^{-1} B^{T}| = |A^{-1}| |B^{T}|$

$\Rightarrow \det(A^{-1}) |B^{T}|$

$\Rightarrow \dfrac{1}{|A|} |B^{T}|$

$\Rightarrow \dfrac{1}{2}(-3)$

$\Rightarrow -3/2$

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Q: 02. (b)

$x+y+2z=1$
$x-2y+z=-5$
$3x+y+z=3$

Sol

$$= \left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} \\ R_2-R_1 \\ R_3-3R_1 \end{array}$$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & 0 & -\frac{13}{3} & 4 \end{array}\right] R_3 - \left(\frac{-2}{-3}\right) R_2$$

Using Backward Substitution

$-\frac{13}{3} z = 4$

$\boxed{z = \frac{-12}{13}}$

$-3y - z = -6$

$-3y + \frac{12}{13} = -6 \Rightarrow -3y = -6 - \frac{12}{13}$

$-3y = -\frac{78-12}{13}$

$-\cancel{3}y = \frac{\cancel{90}\ 30}{\cancel{13}}$

$\boxed{y = -\frac{30}{13}}$

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$$x + y + 2z = 1$$

$$x + \left(-\frac{30}{13}\right) + 2\left(-\frac{12}{13}\right) = 1$$

$$x = 1 + \frac{30}{13} + \frac{24}{13}$$

$$x = \frac{13 + 30 + 24}{13}$$

$$\boxed{x = 67/13}$$

Q.No.3: $A^{-1} = ?$ where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$.

Sol. We know that:

$$A^{-1} = \frac{adj A}{|A|} \rightarrow (*)$$

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}.$$

Expand by R_1:

$$|A| = 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} - (-2)\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$|A| = 3(-18) + 2(-15-2) + 1(0-6).$$

$$|A| = -54 + 2(-17) - 6.$$

$$|A| = -54 - 34 - 6.$$

$$|A| = -94 \rightarrow (i).$$

Now we need to find out

adj A

For this:

$$a_{11} = \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18.$$

$$a_{12} = -\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = -(-15-2) = +17$$

$$a_{13} = +\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = 0-6 = -6.$$

$$a_{21} = -\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -(6) = -6.$$

$$a_{22} = +\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -9-1 = -10.$$

$$a_{23} = -\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -(0+2) = -2.$$

$$a_{31} = +\begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -4-6 = -10$$

$$a_{32} = -\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -(6-5) = -1$$

$$a_{33} = +\begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 18+10 = 28.$$

$$= \begin{bmatrix} -18 & +17 & -6 \\ -6 & -10 & -2 \\ -10 & -1 & 28 \end{bmatrix}$$

Take Transpose.

$$\text{adj } A = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & +28 \end{bmatrix}. \rightarrow \text{(ii)}$$

put (i) & (ii) in (*).

$$\boxed{A^{-1} = -\frac{1}{94}\begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & +28 \end{bmatrix}.} \rightarrow \text{Required Sol.}$$