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ID NO : 7954

SUBJECT : MOS 2

DEPARTMENT : BE (CIVIL)

SECTION : "B"

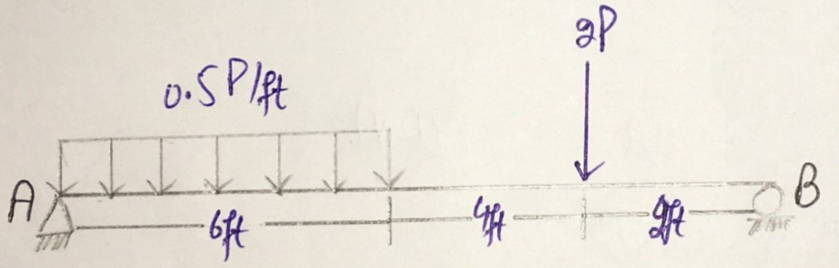
INSTRUCTOR : Engr. M. SAQIB

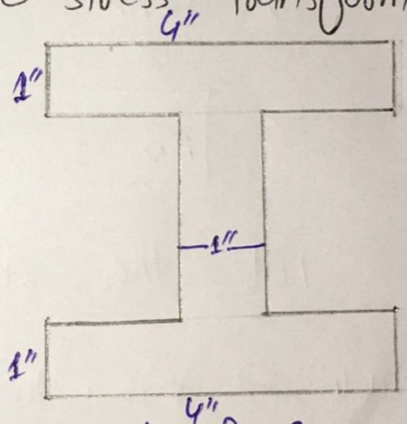
MID TERM EXAM

QUESTION

Construct the mohr's Circle diagram and find the Principle stress and maximum in plane shear stress for the stress state of Point C located at the Centre of uniformly distributed load and 1 inch below the top fiber of beam cross section shown in figure.

However to Construct the Mohr's circle it is necessary to draw the shear stress and flexure Stress variation diagram for maximum shear force and bending moment respectively. Compare the results obtained from the Mohr's Circle with the stress transformation equations.





Where is P is the last two digit of Class id.

i.e $P = 54$

SOLUTION:

PART A:

We have to find the the

→ Rections

→ Shear force

→ Bending Moment diagram.

$\Sigma F_y = 0$ $\quad +\uparrow$ upward Positive.

$\Rightarrow R_A + R_B - (0.5 \times 54 \times 6) - 2(54) = 0$

$\Rightarrow R_A + R_B - 162 - 108 = 0$

$\Rightarrow R_A + R_B = 270$ —— (A)

Now

$\Sigma M_A = 0$ $\quad \circlearrowleft +$ Anti-clockwise Positive

$= (R_A \times 12) - (108 \times 10) - (162 \times 3) = 0$

$\Rightarrow (12 R_B) - 1080 - 486 = 0$

$\Rightarrow \frac{\cancel{12} R_B}{\cancel{12}} = \frac{1566}{12}$

$\Rightarrow \boxed{R_B = 130.5 lb}$

Put the value of R_B in equ (A)

equ (A) $\Rightarrow R_A + \cancel{103}\,130.5 = 270$

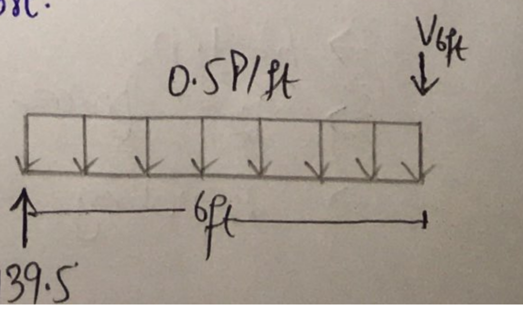
$\Rightarrow R_A = 270 - 130.5$

$\Rightarrow \boxed{R_A = 139.5 lb}$

SHEAR FORCE:

First we have to find the shear force (v) at 6ft from left support.

$\Sigma F_y = 0$ $\quad +\uparrow$ upward Positive

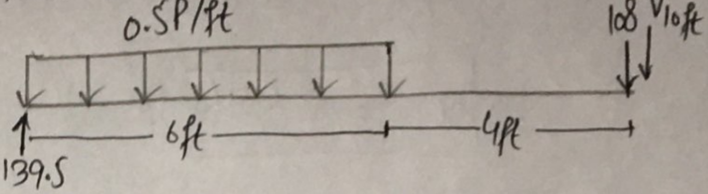


$\Rightarrow -V_{6ft} + 139.5 - 162 = 0$

$\Rightarrow -V_{6ft} - 22.5$

$\Rightarrow \boxed{V_{6ft} = -22.5 lb}$

Now we have to find the shear stress at 10ft from left support



$\Sigma F_y = 0$

$\Rightarrow 139.5 - 162 - 108 - V_{10ft} = 0$

$\Rightarrow -V_{10ft} - 130.5 = 0$

$\Rightarrow \boxed{V_{10ft} = -130.5}$

BENDING MOMENT:

$\Sigma M_{6ft} = -(139.5 \times 6) + (27 \times 6)(6/3)$

$\Rightarrow \Sigma M_{6ft} = -783 + 324$

$\Rightarrow \boxed{\Sigma M_{6ft} = 351 \text{ lbft}}$

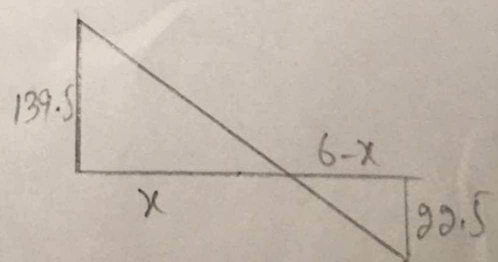
Now we have to find Moment at 3ft

$\Sigma M_{3ft} = -(139.5 \times 3) + (27 \times 6 \times 3)$

$\Rightarrow \Sigma M_{3ft} = -418.5 + 486$

$\Rightarrow \boxed{\Sigma M_{3ft} = 67.5 \text{ lbft}}$

Now we have to find the moment at Changing Point



$\frac{139.5}{x} = \frac{22.5}{6-x}$

$\Rightarrow 139.5 \times (6-x) = 22.5x$

$$\Rightarrow 837 - 139.5x = 22.5x$$

$$\Rightarrow 837 = 22.5x + 139.5x$$

$$\Rightarrow \frac{837}{162} = \frac{\cancel{162}x}{\cancel{162}}$$

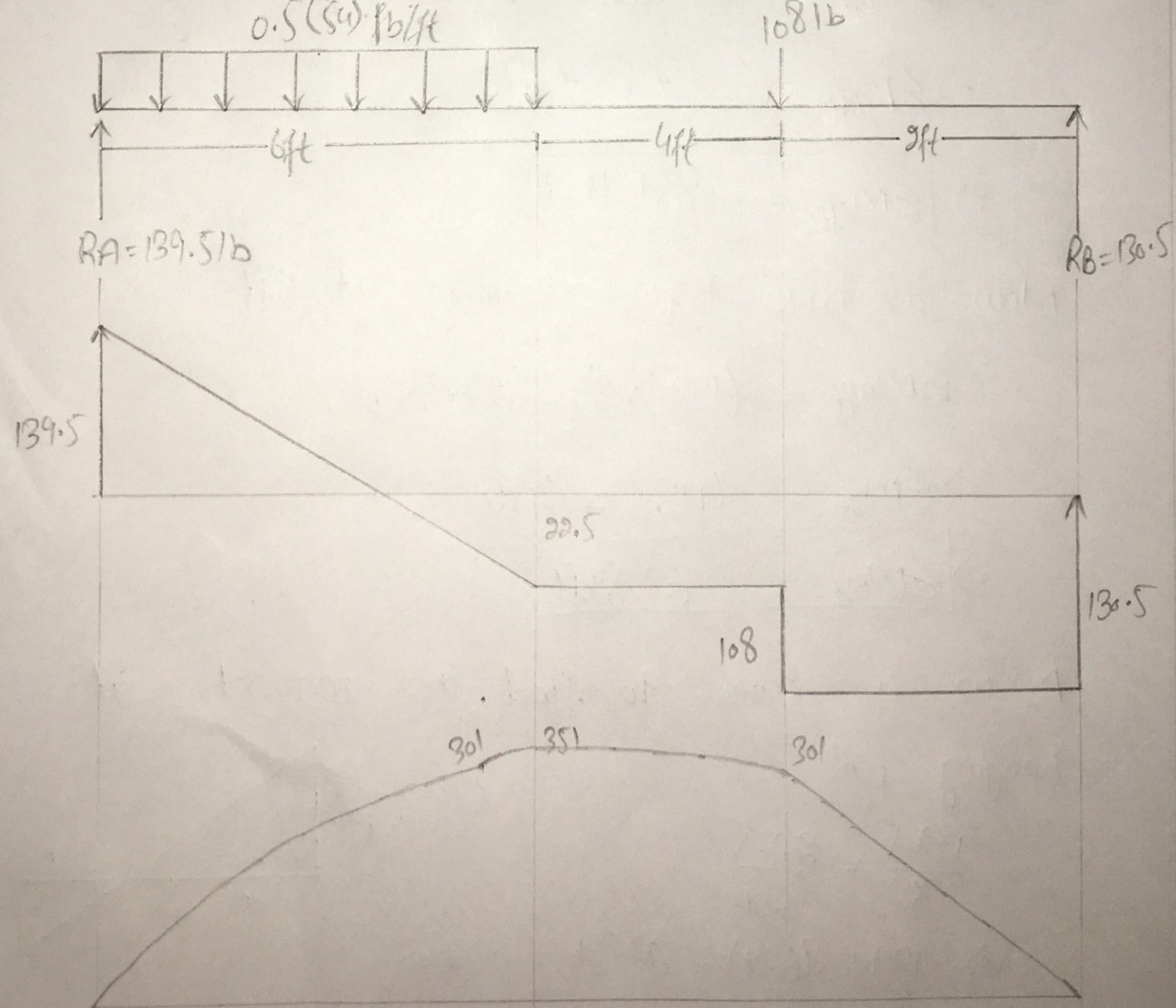
$$\Rightarrow \boxed{x = 5.16}\ ft$$

$$\Sigma M_{5.16ft} = 0$$

$$\Rightarrow M_{5.16ft} - (139.5 \times 5.16) + (27 \times 6 \times 5.16/2) = 0$$

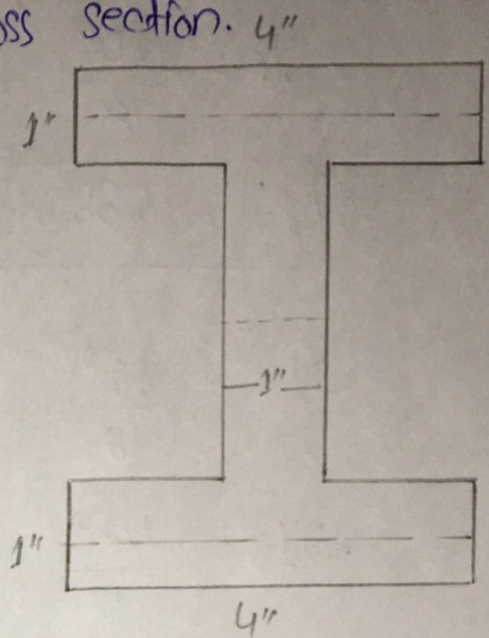
$$\Rightarrow \boxed{M_{5.16ft} = 301\ lbft}$$

SHEAR FORCE AND BENDING MOMENT DIAGRAM:



PART B:

Now we have to find the Moment of inertia of the beam cross section.



$y_1 = 5.5$

$y_2 = 3$

$y_3 = 0.5$

$A_1 = 4\,inch^2$

$A_2 = 4\,inch^2$

$A_3 = 4\,inch^2$

$$\bar{y} = \frac{(A_1 \times y_1) + (A_2 \times y_2) + (A_3 \times y_3)}{A_1 + A_2 + A_3}$$

Putting values

$$\Rightarrow \bar{y} = \frac{(4 \times 5.5) + (4 \times 3) + (4 \times 0.5)}{4+4+4}$$

$$\Rightarrow \boxed{\bar{y} = 3''}$$

Now

$$I_1 = \frac{bh^3}{12}$$

$$\Rightarrow I_1 = \frac{4 \times 1^3}{12}$$

$$\Rightarrow \boxed{I_1 = 0.33\ inch^4}$$

$$I_2 = \frac{bh^3}{12}$$

$$\Rightarrow I_2 = \frac{1\times 4^3}{12}$$

$$\Rightarrow \boxed{I_2 = 5.33\ \text{inch}^4}$$

$$I_3 = \frac{bh^3}{12}$$

$$\Rightarrow I_3 = \frac{4\times 1^3}{12}$$

$$\Rightarrow I_3 = 0.33\ \text{inch}^4.$$

Now "d"

$$d_1 = \bar{y} - y_1$$

$$\Rightarrow d_1 = 3 - 5.5$$

$$\Rightarrow \boxed{d_1 = -2.5}$$

$$d_2 = \bar{y} - y_2$$

$$\Rightarrow d_2 = 3 - 3$$

$$\Rightarrow \boxed{d_2 = 0}$$

$$d_3 = \bar{y} - y_3$$

$$\Rightarrow d_3 = 3 - 0.5$$

$$\Rightarrow \boxed{d_3 = 2.5}$$

Ad^2

$$A_1 d_1^2$$

$$= 4\times(-2.5)^2$$

$$\Rightarrow 25\ \text{inch}^4$$

$$A_2 d_2^2$$

$$= 4\times(0)^2$$

$$= 0$$

$$A_3 d_3^2$$

$$= 4\times(2.5)^2$$

$$= 25\ \text{inch}^4$$

Now

$$I_{1x} = I_1 + A_1 d_1^2$$

$$\Rightarrow I_{1x} = 0.33 + 25$$

$$\Rightarrow \boxed{I_{1x} = 25.33 \text{ inch}^4}$$

$$I_{2x} = I_2 + A_2 d_2^2$$

$$\Rightarrow I_{2x} = 0 + 5.33$$

$$\Rightarrow \boxed{I_{2x} = 5.33 \text{ inch}^4}$$

$$I_{3x} = I_3 + A_3 d_3^2$$

$$\Rightarrow I_{3x} = 0.33 + 25$$

$$\Rightarrow \boxed{I_{3x} = 25.33 \text{ inch}^4}$$

Now

$$I_{xx} = I_{1x} + I_{2x} + I_{3x}$$

$$\Rightarrow I_{xx} = 25.33 + 5.33 + 25.33$$

$$\Rightarrow \boxed{I_{xx} = 55.99 \simeq 56} \text{ inch}^4$$

PART C:

We have to find the

→ Shear stress

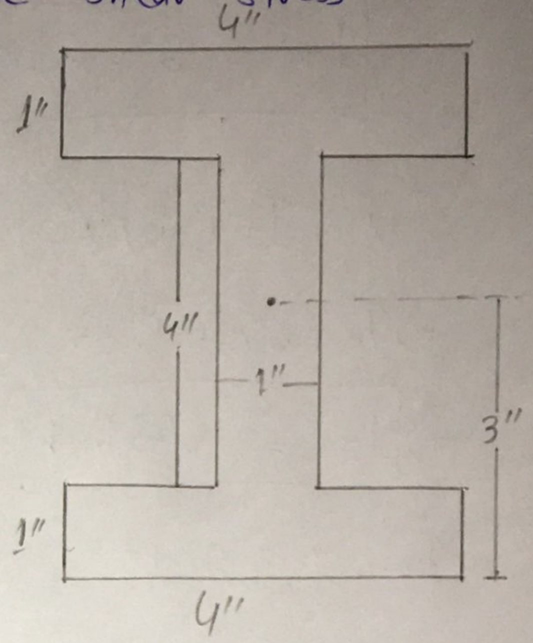
→ Fluxure stress

→ Variation diagram.

SHEAR STRESS:

CASE 1:

We have to find the shear stress at top fiber.



As we know that $\tau = \frac{VQ}{It}$

As top fiber $A = 0$

$Q = \bar{y}A \qquad \because \bar{y} = 3''$

$Q = 3'' \times 0 = 0$

$V = 130.5, \ I = 56, \ b = 4 = t$

$\Rightarrow \tau = \frac{130.5 \times 0}{56 \times 4}$

$\Rightarrow \boxed{\tau = 0}$ Psi

CASE 2:

Find stress 1 inch below the top fiber but here there is two cases.

$\because \bar{y} = 2 + 1/2 = 2.5$

$A = 1 \times 4 = 4$

$\tau_A = \frac{VQ_A}{I b_A}$

$\because Q_A = \bar{y}A$

$\Rightarrow Q_A = 2.5 \times 4 = 10$

$b_A = 4$ inch

$\Rightarrow \tau_A = \frac{(130.5)(10)}{56 \times 4}$

$\Rightarrow \boxed{\tau_A = 5.82}$ Psi

$$\tau_B = \frac{VQ_B}{I\, b_B}$$

$\because Q = 2.5 \times 4 = 10$

$b_B = 1$

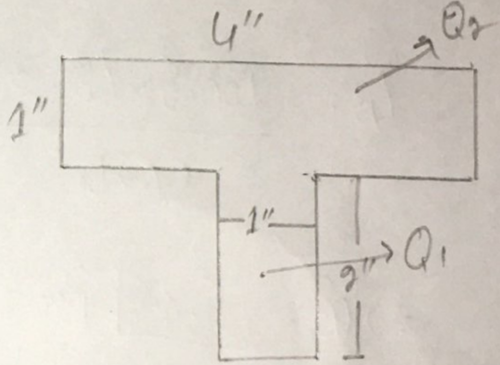
$$\Rightarrow \tau_B = \frac{(130.5)(10)}{56 \times 1}$$

$$\Rightarrow \boxed{\tau_B = 23.30} \text{ Psi}$$

CASE 3:

Find stress at centriodal axis.

Now here



$$Q = Q_1 + Q_2 \quad \text{—(i)}$$

$$Q_1 = \bar{y}_1 A_1 = 2/2\,(1 \times 2) = 2$$

$$Q_2 = 2.5 \times 4 = 10$$

So

$$Q = 2 + 10$$

$$\Rightarrow Q = 12$$

Now

$$\tau_{max} = \frac{VQ}{I\, b}$$

$\because b = 1$ inch

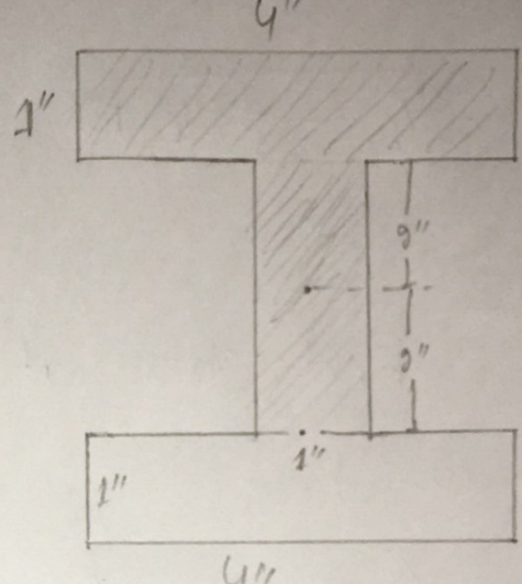
$$\Rightarrow \tau_{max} = \frac{130.5 \times 12}{56 \times 1}$$

$$\Rightarrow \boxed{\tau_{max} = 27.9642} \text{ Psi}$$

CASE 4:

Find shear stress 1 inch above the bottom fiber.

Here we have two cases.



$$\tau_A = \frac{130.5 \times (2.5 \times 4)}{56 \times 1}$$

$$\Rightarrow \boxed{\tau_A = 23.30} \text{ Psi}$$

$$\tau_B = \frac{130.5 \times (2.5 \times 4)}{56 \times 4}$$

$$\Rightarrow \boxed{\tau_B = 5.82} \text{ Psi}$$

CASE 5:

Find shear stress at bottom fiber

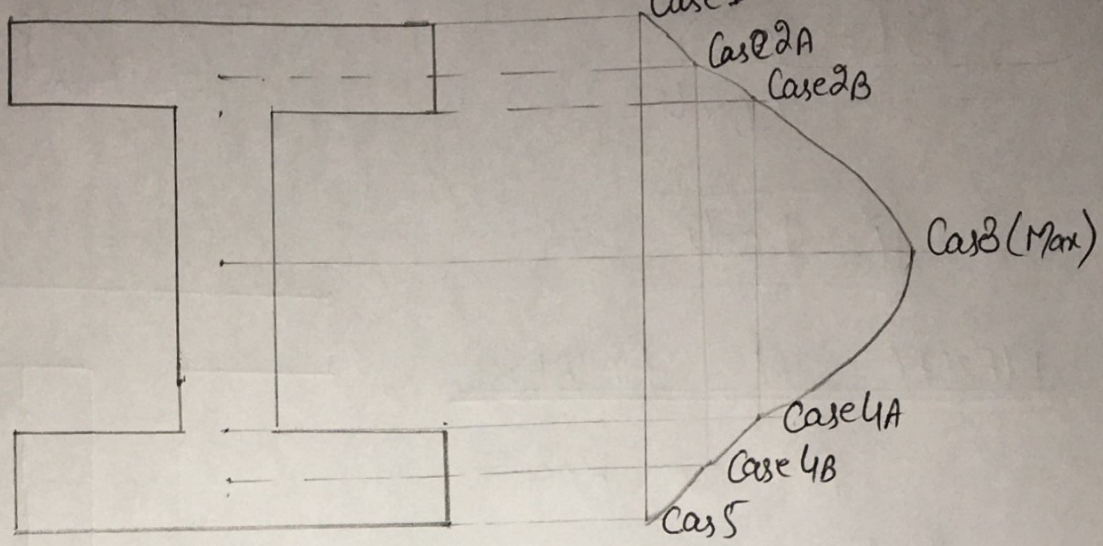
Here $\bar{y} = 0$ So $Q = 4 \times 0 = 0$

$$\tau = \frac{VQ}{Ib}$$

$$\Rightarrow \tau = \frac{130.5 \times 0}{56 \times 1}$$

$$\Rightarrow \boxed{\tau = 0} \text{ Psi}$$

SHEAR STRESS VARIATION DIAGRAM:



CASE 6:

Now we have to find the maximum shear stress at a distance of 6ft from left support of beam along its length.

As we know that shear force at 6ft is

$V = 22.5 lb$

$$\tau_{max} = \frac{VQ}{Ib} \qquad \because Q = 12 \text{ (From case 3)}$$

$$\Rightarrow \tau_{max} = \frac{22.5 \times 12}{56 \times 1}$$

$$\Rightarrow \boxed{\tau_{max} = 4.82 \text{ Psi}}$$

CASE 7:

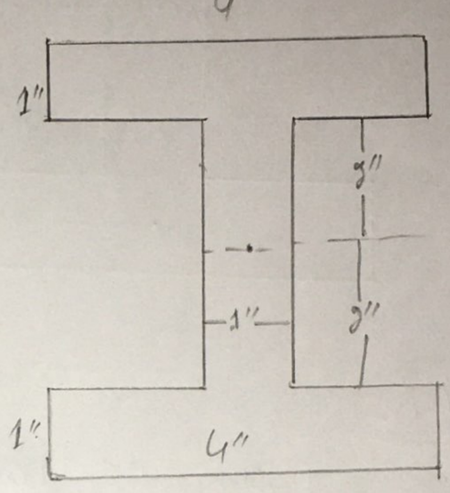
Now we have to find the shear stress at a distance of 6ft from left support and 1inch below the top fiber.

$$\Rightarrow \tau = \frac{VQ}{Ib}$$

$$\Rightarrow \tau = \frac{22.5 \times 10}{56 \times 4}$$

$\because Q = 10$
$b = 4\,inch$ (from case 2A)

$$\Rightarrow \boxed{\tau = 1.004}\ Psi$$



FLEXURE STRESS ANALYSIS:

As we know that

$$\sigma = \frac{My}{I}$$

The maximum bending Moment is 351 lb.ft.

CASE 1:

Find Stress at top fiber

$$\sigma_{TOP} = \frac{(351)(3)}{56} \qquad \because y = 3$$

$$\Rightarrow \boxed{\sigma_{TOP} = 18.80\ Psi}$$

CASE 2:

Find Stress at 1inch below top fiber

$$\sigma_{1inch} = \frac{(351)(2)}{56} \qquad \because y = 2$$

$$\Rightarrow \boxed{\sigma_{inch} = 12.53}\ Psi$$

CASE 3:

Find fluxure stress at centriodal axis.

$$\sigma_{centr} = \frac{1351 \times 0}{56} \quad \because y = 0$$

$$\boxed{\sigma_{centre} = 0 \, Psi}$$

CASE 4:

Find flexure stress at 1 inch above the bottom fiber

$$\sigma = \frac{My}{I}$$

$$\Rightarrow \sigma = \frac{351 \times 2}{56} \quad \because y = 2$$

$$\Rightarrow \boxed{\sigma = 12.53 \, Psi}$$

CASE 5:

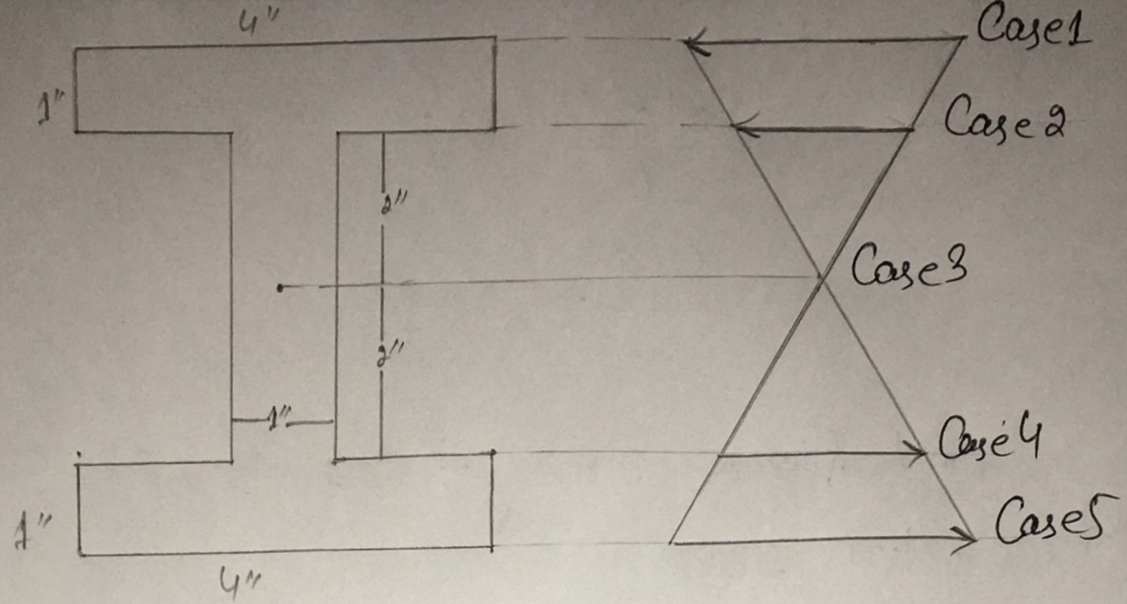
Find flexural stress at the bottom fiber

$$\sigma_{Bottom} = \frac{My}{I}$$

$$\Rightarrow \sigma_{Bottom} = \frac{351 \times 3}{56} \quad \because y = 3$$

$$\Rightarrow \boxed{\sigma_{Bottom} = 18.80 \, Psi}$$

FLEXURE STRESS VARIATION DIAGRAM:



STRESS STATE OF A POINT ELEMENT:

Now we have to find stress state of a Point element located at a distance of 3ft from left support and 1 inch below the top fiber.

As to find the Condition of stressed element at Point C in this given I section. It require to find all the Stresses at this Point.

As in the given Problem the stresses acting on Point C is flexural and shear stresses. There is no torsional Stress force acting on this beam. due to load Symmetry along the beam axis. (longitudinal axis)

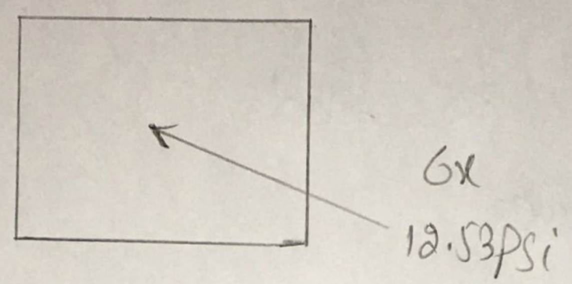
Flexure stress at Point C

$\sigma = 12.53\ Psi$ (From Flexural stress Case-2)

Shear stress at Point "C"

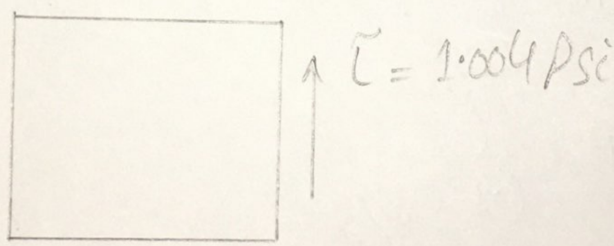
$\tau = 1.004$ (From Case 7 of Shear Stress)

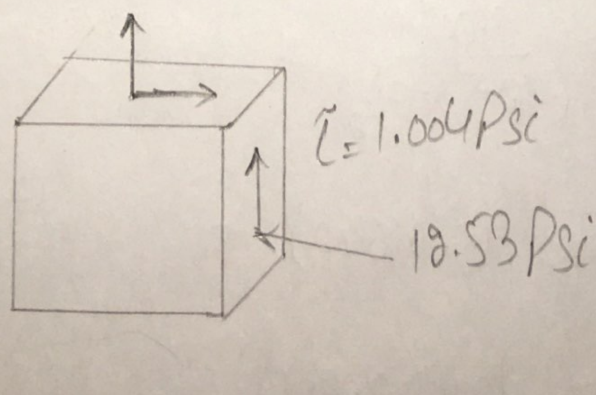
Consider this Point "C" is a Planer element.



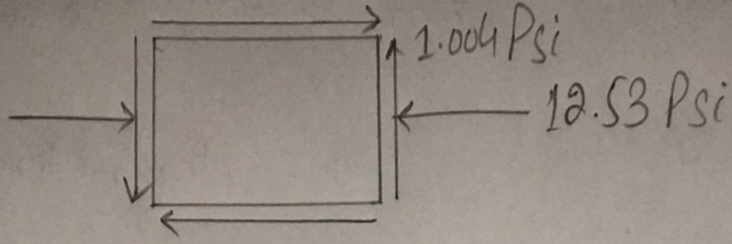
12.53 Psi is a Compressive because Point C lies in Compression zone of beam Crosss Section.

If the Point C lies below the Centroidal than stress would be tensile.





Combine Stress on 2D element.



PART D:

We have to find.

- → Principle Stresses
- → Stresses transformation
- → Mohr's circle
- → Comparision of Mohr's Circle result with Principle stress equations.

PRINCIPLE STRESSES:

We have to find θP

$$\tan 2\theta P = \frac{\tau}{(\sigma x - \sigma y)/2}$$

$$\Rightarrow \tan 2\theta_P = \frac{1.004}{(-12.53-0)/2}$$

$$\Rightarrow \tan 2\theta_P = (-0.1642)$$

$$\Rightarrow 2\theta P = \tan^{-1}(0.1642)$$

$$\Rightarrow \theta P = -9.104/2$$

$\Rightarrow \theta p = -4.552$

Now for $\sigma x'$

$$\sigma x' = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta + \tau xy \sin 2\theta$$

$$\Rightarrow \sigma x' = \frac{(-12.53+0)}{2} + \frac{(-12.53-0)}{2} \cos 2(-4.552) + (1.004 \times \sin 2(-4.552))$$

$$\Rightarrow \boxed{\sigma x' = -12.6047} \text{ Psi (compression)}$$

Now for $\sigma y'$

$$\sigma y' = \frac{\sigma x + \sigma y}{2} - \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau xy \sin 2\theta$$

$$\Rightarrow \sigma y' = \frac{(-12.53+0)}{2} - \frac{(-12.53-0)}{2} \cos 2(-4.552) - (1.004 \times \sin 2(-4.552))$$

$$\Rightarrow \boxed{\sigma y' = -0.0799 \text{ Psi}} \text{ (compression)}$$

Shear Plain

$$\because \tan 2\theta p = \frac{-(\sigma x - \sigma y)/2}{\tau xy}$$

$$\Rightarrow \tan 2\theta_p = \frac{-(-12.53-0)/2}{1.004}$$

$$\Rightarrow \theta = 40.447$$

$$\tau x'y' = -\frac{\sigma x - \sigma y}{2} \sin 2\theta + \tau xy \cos 2\theta$$

$$\Rightarrow \tau x'y' = -\frac{(-12.53-0)}{2} \sin 2(40.47) + (1.004 \cos 2(40.47))$$

$$\Rightarrow \boxed{\tau x'y' = 6.34}$$

STRESSES TRASFORMATION:

Now find Stress State Condition of Point C at a Clockwise orientation $\theta = -30^\circ$ (assumed)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma_{x'} = \frac{(-12.53+0)}{2} + \frac{(-12.53-0)}{2}\cos 2(-30) + 1.004\sin 2(-30)$$

$$\Rightarrow \boxed{\sigma_{x'} = -10.266 \text{ Psi}}$$ (compression)

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma_{y'} = \frac{(-12.53+0)}{2} - \frac{(-12.53-0)}{2}\cos 2(-30) - 1.004\sin 2(-30)$$

$$\Rightarrow \boxed{\sigma_{y'} = -2.26 \text{ Psi}}$$ (compression.

Now

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Rightarrow \tau_{x'y'} = \frac{12.53-0}{2}\sin 2(-30) + 1.004\cos 2(-30)$$

$$\Rightarrow \boxed{\tau_{x'y'} = -4.9236 \text{ Psi}}$$

Mohr's CIRCLE:

Cooridinates:

$$(h,k) = \left(-12.53/2 , 0\right)$$

$$\Rightarrow (h,k) = (-6.265, 0)$$

$$\Rightarrow \boxed{(h,k) = (-6.3, 0)}$$

Radius:

$$r = \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow r = \sqrt{\left(\frac{-12.53 - 0}{2}\right) + (1.004)^2}$$

$$\Rightarrow r = 6.344$$

$$\Rightarrow \boxed{r = 6.3}$$

$V = 12.53$ Psi

Scale:

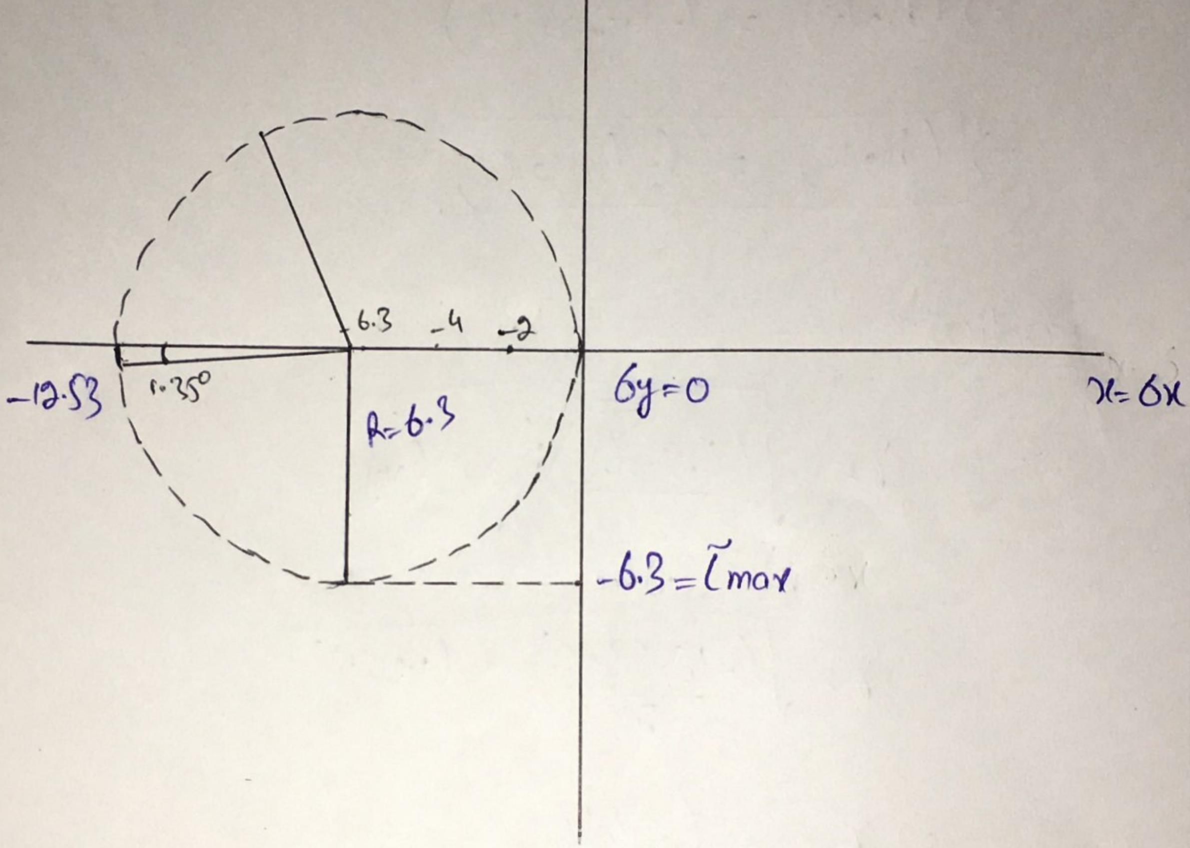
$2\,psi = 1cm$

Now

$\theta = \tan^{-1}\left(\frac{1.004}{12.53+30}\right)$

$\Rightarrow \theta = 1.35^\circ$

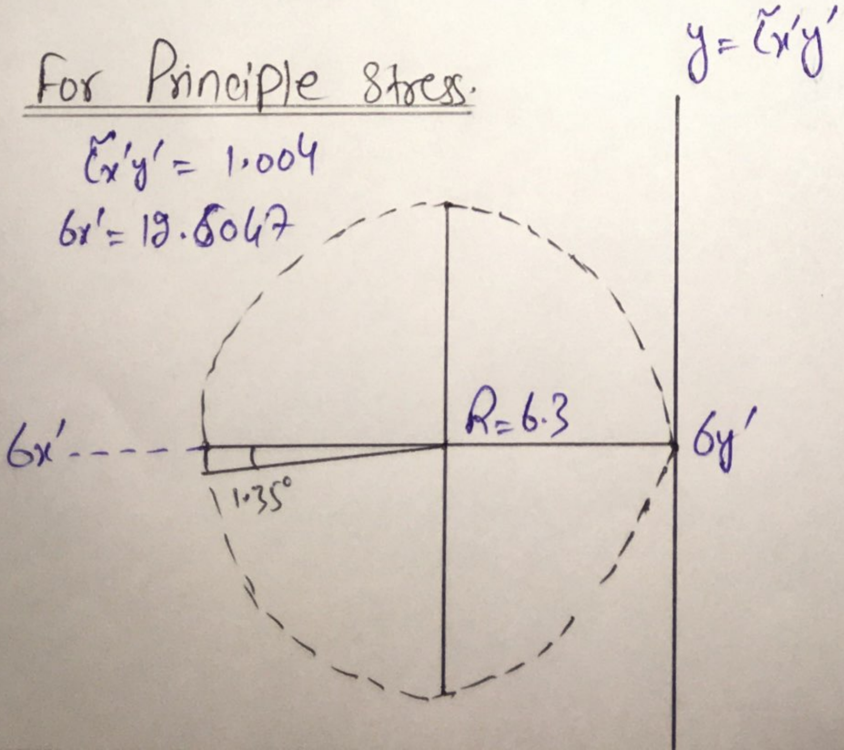
$\tau_{xy} = 1.004$



For Principle Stress.

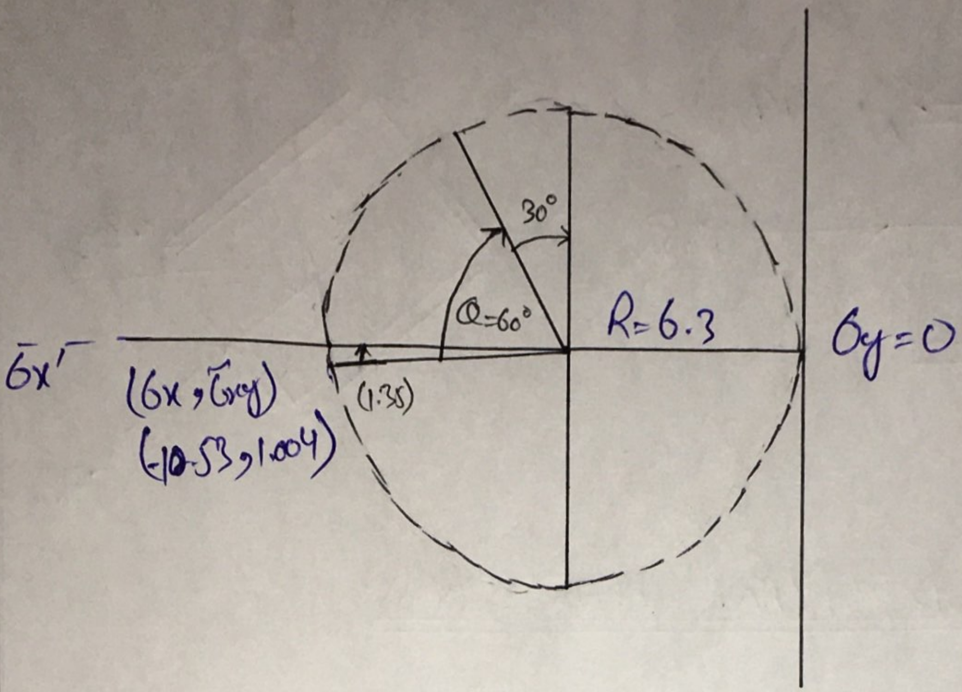
$\tau_{x'y'} = 1.004$

$\sigma_{x'} = 12.8047$



for New Orientation (-30°) clockwise.

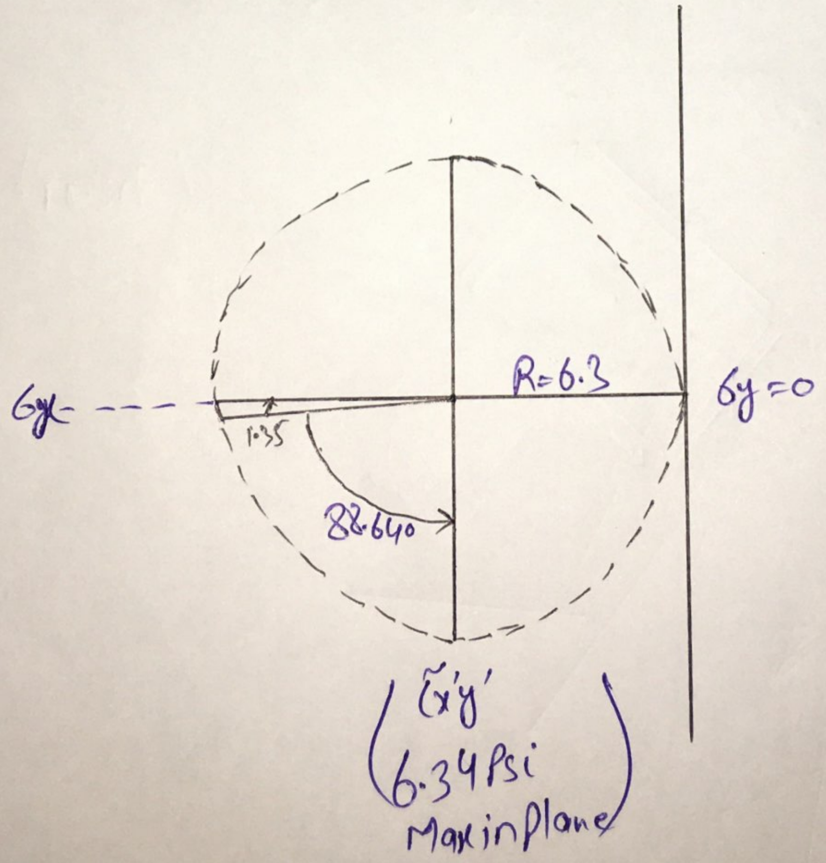
$\sigma_x = -12.53$, $\tau_{x'y'} = 1.004$ psi , $\sigma_{y'} = 0$



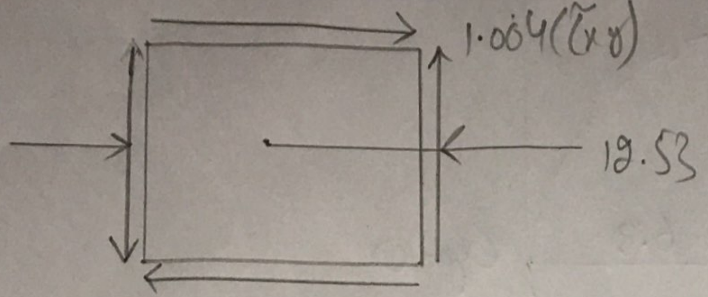
FOR ~~PRINCIPLE~~ SHEAR STRESS:

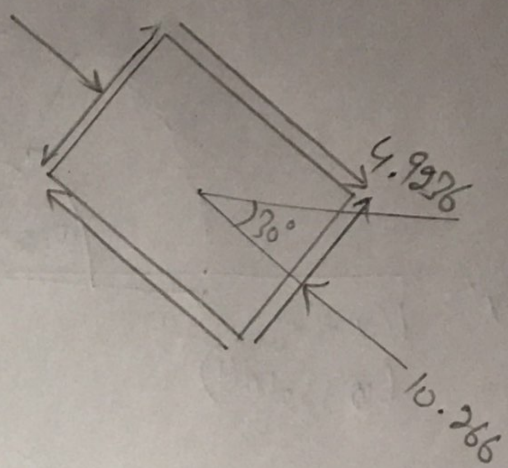
$\sigma_{x'} = \cancel{-12.6047}\ -12.53$

$\sigma_{y'} = 0$

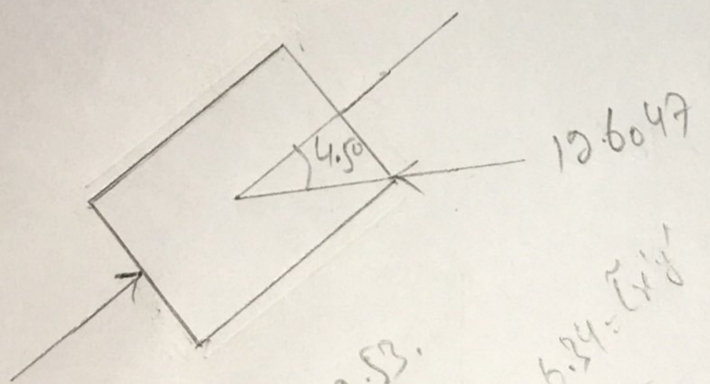


COMPARISON OF MOHR'S CIRCLE WITH SHEAR STRESS TRANSFORMATION:

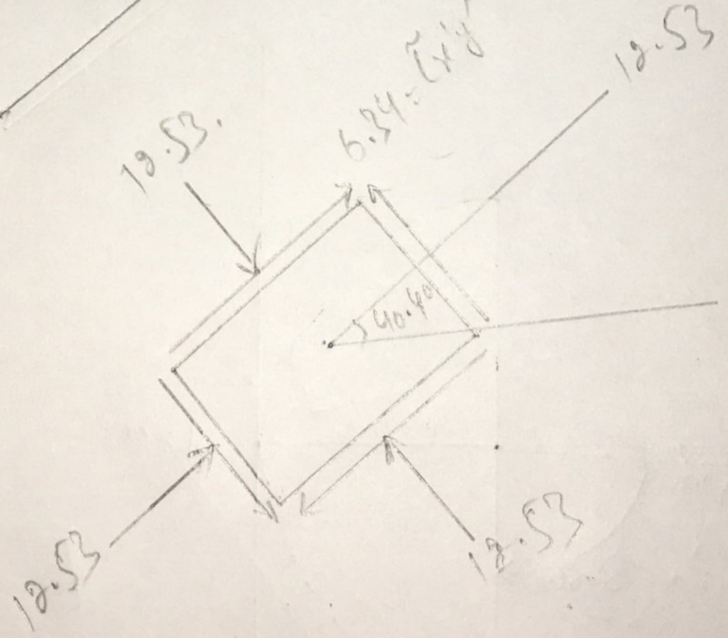




PRINCIPLE AND PLAN SHEAR STRESS:



(Principle)



(shear)