Name: AbdulBasit Khan

ID: 14564

Subject: Signal & System

Instructor: Sir Mujtabah Ihsan

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Signature: Basit

Q 1 (a)

Show with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.

Ans

Fourier Transform of Differention Integration of Continous - time

Let $x(t)$ be a Continuous - time Signal with a fourier transform of $X(jw)$

i.e

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \frac{d}{dt} \left\{ e^{jwt} \right\} dw$$

Differentiating both sides with respect to (t)

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\frac{d}{dt}\{e^{j\omega t}\}\,d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\{e^{j\omega t}\cdot j\omega\}\cdot d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{j\omega X(j\omega)\}e^{j\omega t}\,d\omega$$

$$F\left\{\frac{d}{dt}x(t)\right\} = j\omega X(j\omega)$$

Result

we concluded that if a function is differentiated in time domain It is multiplied by jw in frequency domain.

Q1 (b)

if $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Produce $y(z)$ and $y(n)$

Ans

$$X[z] = 2 - 4z^{-2} + 2z^{-3}$$

$$H[z] = 3 + z^{-1} + 2z^{-2}$$

Now $Y(z) = H(z) * X(z)$

$$= (3 + z^{-1} + 2z^{-1}) * (2 - 4z^{-2} + 2z^{-3})$$

$$Y(z) = 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4} + 4z^{-2} - 8z^{-4} + 4z^{-5}$$

$$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$$

To find $y(n)$ use the delay Property

$$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3] - 6\delta[n-4] + 4\delta[n-5]$$

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Q2

$$f(x) = \begin{cases} -\frac{\pi}{2} & -\pi \le x \le 0 \\ \frac{\pi}{2} & \le x \le x \end{cases}$$

Ans

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx \right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\pi/2 \,dx + \int_{0}^{\pi} f(x)\,dx \,]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi} \frac{\pi}{2}\,dx$$

$$= \frac{1}{2\pi} \left[-\frac{\pi}{2} \int_{-\pi}^{0} 1\,dx + \frac{\pi}{2} \int_{0}^{\pi} 1\,dx \right.$$

$$= \frac{1}{2\pi} \left[-\frac{\pi}{2} x \Big|_{-\pi}^{0} + \frac{\pi}{2} x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2}\left[0-(-\pi)\right)+\frac{\pi}{2}\left[\pi-0\right]\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}(\pi)+\frac{\pi}{2}[\cdot\pi]\right]$$

$$= \frac{1}{2\pi}\left[\cancel{\frac{-\pi^2}{2}}+\cancel{\frac{\pi^2}{2}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{0}{2}\right] = 0$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\,\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\,\frac{\sin nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\sin n(0) - \sin n(-\pi)\right]$$

$$+ \frac{\pi}{2}\left[\sin n(\pi) - \sin n(0)\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$= \frac{1}{n\pi}(0)$$

$$a_n = 0$$

Now

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx \, dx + \int_{0}^{\pi} f(x)\sin nx \, dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx \, dx + \int_{0}^{\pi} \frac{\pi}{2}\sin nx \, dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} \sin nx\, dx + \frac{\pi}{2}\int_{0}^{\pi} \sin nx\, dx \right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\; -\frac{\cos nx}{n}\Bigg|_{-\pi}^{0} + \frac{\pi}{2}\; -\frac{\cos nx}{n}\Bigg|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-\cos n(0) + \cos n(-\pi)\right] + \frac{\pi}{2}\left[-\cos n(\pi) + \cos n(0)\right] \right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-1 + \cos n(-\pi)\right] + \frac{\pi}{2}\left[-\cos n\pi + \cos n(0)\right] \right]$$

it became -1

$$\frac{\cancel{\pi}}{\cancel{2}} = \frac{1}{n\pi}\left[\frac{-\cancel{\pi}}{\cancel{2}} - 1\left[-1 + \cos n(-\pi)\right] + 1\left[-\cos n\pi + 1\right] \right]$$

$$= \frac{1}{2n}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd} \end{cases}$$

Q2, 2

$$\left\{ b_n = \frac{4}{2n} \right\}$$

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \dots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \dots$$

$$f(x) = (0) + (0)\cos x + 0\cos(2x) + 0\cos 3x + \dots$$

$$= \frac{4}{2}\sin x + (0)\sin^2 x + \frac{4}{3(2)}\sin 3x + \dots$$

$$= \left\{ \frac{4}{2}\sin x + \frac{4}{0}\sin 3x + \dots \right\}$$

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Q3

if $x(z) = \dfrac{2z^2+2z}{z^2+2z-3}$

Retrieve $x[n]$ using Inverse Z-Transform

Ans

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$X(z) = \frac{2z(z+1)}{z^2+3z-z-3}$$

$$X(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$$

$$\frac{X(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

Or

$$\frac{2(z+1)}{z^2+2z-3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

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P

$$2(z+1) = A(z-1) + B(z+3) \quad \text{— ①}$$

Put $z = 1$ in eq 1

$$2(1+1) = B(1+3)$$

$$\cancel{4} = \cancel{4}B$$

$$B = 1$$

Put $z = -3$ in eq ①

$$2(-3+1) = A(-3-1)$$

$$\cancel{-4} = -\cancel{4}A$$

$$A = 1$$

Now Put A and B in eq 1

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Invers Z - Transform

$$X[n] = u[3] + 1(-1)^k$$

Q4

Express the transfer function using the given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [0]$$

Ans

We know that

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1} B + D$$

Putting the values

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = (s+2)s + 1 = s^2 + 2s + 1$$
$$= s^2 + 2s + 1$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2+2s+2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+2s+1}$$

$$H[s] = \frac{s+2}{s^2+2s+1}$$

Q5

Apply Fourier transform on the signal $x(t) = e^{-a|t|}\, u(t)$ where $u(t)$ is a unite step function

Ans

$$x(t) = e^{-a|t|} \qquad a > 0$$

$$x(j\omega) = ?$$

Solution

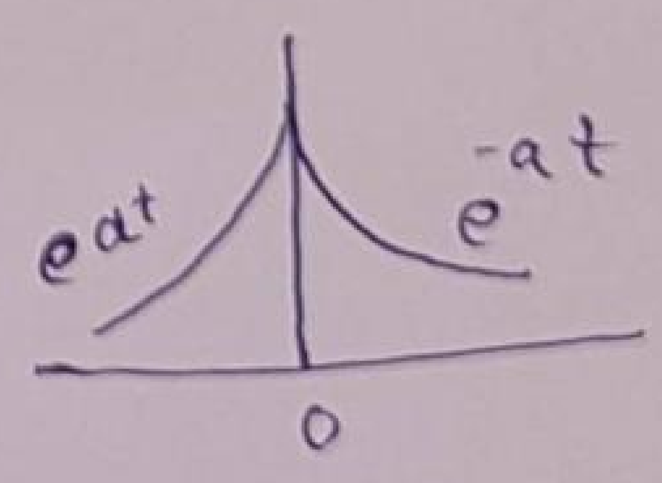
The Fourier transform of the given function $x(t)$ is given by.

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & t < 0 \end{cases}$$



$$x(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} \, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} \, dt$$

$$= \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_{-\infty}^{0} + \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)} \left[e^{0} - e^{-\infty} \right] - \frac{1}{(a+j\omega)} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{1}{(a-j\omega)} [1-0] - \frac{1}{(a+j\omega)} [0-1]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a + \cancel{j\omega} + a - \cancel{j\omega}}{a^2 - (j\omega)^2}$$

$$(j\omega) = \frac{2a}{a^2 + \omega^2}$$