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subjet = linear algebra.

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①

Question 1 (A)

Sol:

The non-parallel vectors.

$\overrightarrow{P_1P_2} = (-3, 2, 2)$ $\rightarrow (-1, 0, 3) - (2, -2, 1)$

$(-1, 0, 3\ (-2, +2, -1)$

$\overrightarrow{P_1P_3} = (3, -1, 3)$

$(-3, +2, 2$

the perpendicular vector is

$m = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

$\overrightarrow{p_1p_2} = \sqrt{(x_2-x_1+(y_2-y_1)^2}$

$\overline{p_1p_2} = \sqrt{(-1,-2)^2+}$

$$m = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$m = i(6+2) - j(-9-6) + k(3-6)$

$n = 8\hat{i} + 15\hat{j} - 3\hat{k}$

$m = (8, 15, -3)$

Now

$P_1(x_0, y_0, z_0) = (2, -3, 1)$

$n(a, b, c) = (8, 15-3)$

So equation of plane is

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$8(x-2) + 15(y+2) - 3(z-1) = 0$

$8x + 15y - 3z - 16 + 30 + 3 = 0$

$8x + 15y - 3z + 17 = 0$

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Question No (4 b)

Sol:→

$x-2 = -3t$

$\Rightarrow t = \frac{x-2}{3}$

$y-3 = t$

$\Rightarrow t = \frac{y-3}{1}$

$z-2 = 4t$

$\Rightarrow t = \frac{z-2}{-4}$

So

$$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

For 1st plane takes 1st & 2nd term.

$\frac{x-2}{-3} = \frac{y-3}{1} \Rightarrow (x-2) - 3(y-3)$

$x-2+3y-2-3=0 \Rightarrow \boxed{x+3y-1=0}$

Taking 1st & 3rd terms.

$\frac{x-2}{-3} = \frac{z-2}{-4}$

$\Rightarrow 1-4(x-2) = -3(z-2).$

$-4x+8 = -3z+6$

$-4x+12+8-6=0$

$\boxed{-4x+3z+7=0}$

$x+3y-11=0$

$-4x+3z+2=0$

are 2n plane.

Question No # 2

$L(x,y) = (x+1, y, x+y)$.

Sol.

$L(x,y) = (x+1, y, x+y)$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$u+v = (x_1, y_1) + (x_2, y_2)$.

$u+v = (x_1 + x_2, y_1 + y_2)$.

$L(u+v) = L(x_1 + x_2, y_1 + y_2)$.

$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2$

given that

$u = (x + y)$

$L(u) = L(x_1, y_1) = (x+1 + y + x_1 + y - 1)$

$L(u) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$

$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

Since $1 \neq 2$

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Question No. 3

Sol:-

a) Code the message "send money".

b) decode the message 67 44 41 49 39 113 76 62 164 69 55

(3) send him money.
19 5 14 4 .89 15 13 15 14 5 .25

4) Decode the message.
77 54 38 71 49 29 68 51 33 76 48 40 86, 53 52

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$ for decoding A^{-1} is must

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$X_5 = \begin{bmatrix} 88 \\ 53 \\ 52 \end{bmatrix} \quad X_1 = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}, X_2 \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}, X_3 \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} X_4 \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}$$

$$A^{-1}X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$A^{-1} X_2 = \begin{bmatrix} '' \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$\hookrightarrow$

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$A^{-1}x_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 73 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 15 \end{bmatrix}$

$Ax_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$

$Ax_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$

A B C D E f — 1 2 3 4 5 6 w, x y z — 23 24 25 26

Talk → Umar

$A = \begin{bmatrix} 1 & 23 \\ 1 & 12 \\ 0 & 12 \end{bmatrix}$

m = 15, e = 5, e = 5, t = 20, t = 20

o = 15, m = 13, o = 15, r = 18, r = 18.

o = 15, w = 23

$x_1 = \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix}, x_2 = \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix}, x_3 = \begin{bmatrix} 13 \\ 15 \\ 18 \end{bmatrix}, x_4 \begin{bmatrix} 18 \\ 15 \\ 23 \end{bmatrix}$

$Ax_1 = \begin{bmatrix} 1 & 23 \\ 1 & 12 \\ 0 & 12 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 38 \\ 28 \\ 15 \end{bmatrix}$

$Ax_2 = \begin{bmatrix} 1 & 23 \\ 1 & 12 \\ 0 & 12 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix} = \begin{bmatrix} 105 \\ 70 \\ 50 \end{bmatrix}$

$Ax_3 = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} 13 \\ 15 \\ 28 \end{bmatrix} = \begin{bmatrix} 97 \\ 64 \\ 57 \end{bmatrix}$

$Ax_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 18 \\ 15 \\ 28 \end{bmatrix} = \begin{bmatrix} 119 \\ 79 \\ 61 \end{bmatrix}$

So Bilal send message.

38, 28, 15, 105, 70, 50, 97, 64, 51, 119, 74

Question No #4.

sol:→

Equation of plane.

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Given that

$p=(x_0, y_0, z_0)=(-1, 3, 2)$

$n=(a, b, c)=(0, 1, -3)$

so

$0(x-(-1))+1(y-3)-3(z-2)$

$\underline{0(x+1)}+1(y-3)-3(z-2)$.

$\downarrow$

$0+y-3-3z+6$

$\Rightarrow y-3z-3+6$

$\Rightarrow y-3z+3$ Ans.

Question No # 5

Sol:-

Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

since we know that

$$Ax = \lambda x$$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ +2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 - \lambda x_1 + x_2 = 0$$
$$-2x_1 + 4x_2 - \lambda x_2 = 0$$
$$(1-\lambda) x_1 + x_2 = 0$$
$$2x_1 + (4-\lambda) x_2 = 0$$

$$\begin{bmatrix} 1-\lambda & + \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} = 0$$

$$((1-\lambda)(4-\lambda)) + 2 = 0$$
$$4-\lambda - 4\lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 - 5\lambda + 6 = 0$$
$$\lambda^2 - 3\lambda - 2\lambda - 6 = 0$$
$$\lambda(\lambda - 3) - 2(\lambda - 3) = 0$$
$$(\lambda-3)(\lambda-2) = 0 \Rightarrow \boxed{\lambda = 3}, \boxed{\lambda_2 = 2}$$

are eigen values.

For eigen vector

$\lambda_1 = 3$ put in eq ① & ②

So

$x_1 + x_2 = 3x_1$

$-2 + 4x_2 = 3x_1$

$-2x_1 + x_2 = 0 \Rightarrow \boxed{x_1 + \frac{1}{2}x_2}$

$-2x_1 + x_2 = 0 \Rightarrow \boxed{x_1 = \frac{1}{2}x_2}$

for

$\lambda_2 = 2$ put in eq ① & ②

$x_1 + x_2 = 2x_1 \Rightarrow x - 2x + x_2 = 0$

$-x_1 + x_2 = 0 \rightarrow \boxed{x_1 = x_2}$

eq ②

$-2x_1 + 4x_2 = 2x_2 \Rightarrow -2x_1 + 4x_1 - 2x_1 = 1$

$2x_1 + 2x_2 = 0$

$\boxed{x_1 = x_2}$

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