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subject :- Linear Algebra.

Q1

(i) $\begin{bmatrix} 1 & 2 & \text{2nd - ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

ii) :- 15889

2nd number is 5

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$A^{adj} = ?$

Replace all values its cofactors then take transpose also equal to adjoint of A.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = (1) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1$$

$$= 5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = (-1)(4-3) = -1$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(2-9) = -7$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = (-1)(4-2) = -2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (1)(2-6) = -4$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (-1)(1-6) = 5$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = (1)(2-6) = -4$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = (-1)(1-4) = 3$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (1)(3-4) = -1$$

$$A\ \text{cofactor} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -2 & -4 & 5 \\ -4 & 3 & -1 \end{bmatrix}$$

transpose

$$A_{adj} = A^T = \begin{bmatrix} 5 & -2 & -4 \\ -1 & -4 & 3 \\ -7 & 5 & -1 \end{bmatrix}$$

~~A13~~

Q1 (ii)

$$B \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Sol $A_{adj} = ?$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (1)(-8+16) = 8$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = (1)(16-40) = 24$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1)(-4+5) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (-1)(32+10) = -42$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = (1)(24-25) = -1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (-1)(-6-20) = 26$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = (1)(32+5) = 37$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (-1)(24-10) = -14$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (1)(-3-8) = -11$$

Now replace all value factor.

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Now transpose

$$B_{adj} = B^t = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

Ans.

Q2 Find the cofactor of A_{21}, A_{31} A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution:

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)(-4+9)$$

$$\boxed{A_{21} = -5}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(-2-9)$$

$$\boxed{A_{31} = -11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (1)(3-4)$$

$$\boxed{A_{33} = -1}$$

Q3 $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol

$\boxed{|A - \lambda I| = 0}$ → formula

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

Now taken Determinent.

$2-\lambda((3-\lambda)(2-\lambda)-2)-1(2-(2-\lambda)+1(1+(3-\lambda)=0$

$2-\lambda(6-3\lambda-2\lambda+\lambda^2-2)-1(2-2+\lambda)+1(1+3-\lambda)=0$

$2-\lambda(\lambda^2-5\lambda+4)-1(\lambda)+1(4-\lambda)=0$

$2r^2 - 10r + 8 - r^3 + 5r^2 - 4r - r + 4 - r = 0$

by ordering

$-r^3 + 7r^2 - 16r + 12 = 0$

xing by (-1)

$r^3 - 7r^2 + 16r - 12 = 0 \rightarrow (A)$

Now put $r - 2 = 0$

then $r = 2$ in above eq (A)

$r^3 - 7r^2 + 16r - 12 = 0$

Put $r = 2$

$(2)^3 - 7(2)^2 + 16(2) - 12 = 0$

$8 - 7(4) + 32 - 12 = 0$

$8 - 28 + 32 - 12 = 0$

$-20 + 20 = 0$

$0 = 0$

Equ value $= r = 2$

or $r - 2 = 0$

Ans.