(1)

NAME

JAWAD ALi

I.D

7351

Subject

Differential Equation

Submitted to, Engr. Shomaila Mazhar.

1

Q.

$$x^3 y''' + 2x^2 y'' + 2y = 100x + 10/x \longrightarrow (1)$$

Sol:

put $x = e^t$ then

$$\frac{dx}{dt} = et \Rightarrow \frac{dt}{dx} = e^{-t}$$

Now $= \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot e^{-t}$

or $y' = \frac{dy}{dx} = e^{-t} Dy \quad \because \frac{d}{t} \rightarrow D$

$$y''' = e^{-3t} \left[D(D-1)(D-2) \right] y$$

using These value in eq (1)

$$e^{3t} \, e^{-3t} \left[D(D-1)(D-2) \right] y + \cancel{2e^{2t}} e^{-2t} \left(D(D-1) y + 2 \right.$$

$$10^{et} + 10e$$

$$\Rightarrow (D^3 - 3D^2 + 2D) y + (2D^2 - 2D) y + 2y = 10e^t + 10^{et}$$

$$\Rightarrow D^3 y - D^2 y + 2y = 10^{et} + 10^{et}$$

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt} + 2y = 10^{et} + 10^{e-t} \longrightarrow (2)$$

The assosiate homogeneous eq of:

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 0.$$

Scanned with CamScanner

(2)

$d/dt^2 = k^3 = \frac{d^3}{dy^3} = k^3$

$\Rightarrow (k^3 y - k^2 y + 2y) = 0$

$= (k^3 y - k^2 y + 2y) = 0$

For Non travel sol $y \neq 0$

$k^3 - k^2 + 2 = 0.$

Q3

①

$x^2 y'' + 2xy' - 6y = 10x^2 \rightarrow ①$

Let

$x = e^t$ i.e $t = \log x$ $y'(1) = -6$

Now $xy' = Dy \Rightarrow x^2 y'' = D(D-1)y$

where $D = \frac{d}{dt}$

Then eq $= [D(D-1) + 2D - 6)y = 10e^{2t}$

$[D^2 - D + 2D - 6]y = 10e^{2t}$

$[D^3 + D - 6]y = 10e^{2t}$

$[D^2 + D - 6]y = 10e^{2t}$

Clear from eq. $D^2 + D - 6 = 0$

$D + 3D - 2D - 6 = 0$

$D = -3, D = 2$

complementary function

$Cf = ce^{-3t} + c_2 e^{2t}$

Also P Integral

$P.I = \frac{1 \times}{D_2 + D - 6}$

$= \frac{10 \cdot \frac{1}{2}}{2 + 2 - 6} e^{2t}$ replace by case of value e^t

$P.I = 10t \frac{1}{2} D + e^{2t} = 10t \cdot \frac{1}{2(2)+1} e^t$

Hence remesured $y = C.f + P.I$.

$y = c_1 e^{-3t} + c_2 e^{2t} + 2t e^{2t}$

$c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2$

Scanned with CamScanner

③

Apply initial cond

$y(1) = 1$ we get

$1 = C_1 + C_2 + 0$ — A

d. $y'(1) = -6$

$y' = -3C_1 x^{4} + 2C_2 x + 2x + 4x \log x$

$-6 = -3C_1 + 2C_2 = -8 \rightarrow$ B.

eq A × 3 and add with B

$5C_2 = -5$

$C_2 = 1$

eq A $= 1 = C_1 - 1$

$C_1 = 2$

Thus ★

$y = 3x^{-3} - x^{2} + 2x^{2} \log x$.

Q41

①

$x^2y'' + 7xy' + 5y = x^5$

$y(0) + 2$

$y'(1) = 2$

let $x = e^t \Rightarrow t = y_x$. $D = d/de$

Now

$xy = \Delta y \Rightarrow x^2y^2 = \Delta(\Delta-1)y$.

then

$(\Delta(\Delta-1) - 7\Delta + 5)y = e^{5t}$

$(\Delta^2 + 6\Delta + 5)y = e^{5t}$

$(\Delta^2 + 6\Delta + 5)y = e^{5t}$

Cheer eq is

$\Delta^2 + 6D + 5 = 0$

$\Delta^2 + 5D + \Delta + 5 = 0$.

$\Delta = -5 - 1$

$Cf = c_1 e^{-5t} + c_2 e^t$

P integral 5t

$P.I = \dfrac{1}{\Delta^2 + 6\Delta + 5}$

$= \frac{1}{2} e^{5t}$ replacing Δ by

$5^2 + 6(5) + 5$

thus $y_p = 5rt$

$y = c_1 e^{-5t} + c_2 e^t + \frac{1}{60} e^{5t}$

Scanned with CamScanner

$y' = -5c_1 x^{-6} = c_2 x^{-2} + \frac{1}{12} x^4$ ②

$y(0) = 2 \quad x = 1 \quad y = 2$

$2 = -5c_1 - c_2 + \frac{1}{12}$

$-5c_1 - c_2 = \frac{23}{12}$

$A + B - 4c_1 = \frac{234}{60} = c_1 = \frac{-117}{120}$

Now

$y = \frac{-117}{120} x^{-5} + c_2 x^{-1} + \frac{1}{60} e^5$

$c_1 = \frac{-117}{120}$ put in eq A.

$\frac{-117}{120} + c_2 = \frac{19}{60}$

Let $x = e^t \Rightarrow t = \log x,\ D\ d/d$

Now $xy = Dy \Rightarrow 2x^2y^2 = \Delta(\Delta - \Delta y)$

then $xy = \Delta y \Rightarrow x^2 y\ e^{5t}$

$(\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$

cheer eq is $D + 6\Delta + 5 = 0$

$\Delta^2 + 5\Delta + \Delta + 5 = 0.$

complementary eq is

$C.f = c_1 e^{5t} + c_2 e^{-t}$

P integral

$P.I \cdot \frac{1}{D^2 + 6\Delta + 5}$ replacing Δ by 5 thus

$y = c e^{-5t} + C_2 e^{-t} + \frac{1}{e} e^{5t}$

$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$

$y' = -6 C_1 x^{-6} \; C_2 x^{-2} + \frac{1}{2} x^4$

$y(0) = 2 \quad x = 0, y = 2$

$2 = C_1 + C_2 + \frac{1}{60}$

$C_1 + C_2 = \frac{119}{60} \; - A$

$y'(1) = 2 \; (x = 1) \; y = 2$

$2 = -5C_1 - C_2 + \frac{1}{12}$

$-5C_1 + C_2 = \frac{23}{12} \Rightarrow B$

$A + B - 4C_1 = \frac{234}{60} = 4 = \frac{117}{120}$

Now $y = \frac{117}{120} x^{-5} + C_2 x^{-1} + \frac{1}{60}$

$C_1 = \frac{-117}{120} x^{-5} \cancel{+} C_2 x^{-1} + \frac{1}{60}$

$C_1 = \frac{-117}{120}$ put in eq A.

$\frac{117}{12} + C_2 = \frac{119}{60}$

$C_2 = \frac{119}{60} + \frac{117}{12}$

$= \frac{238}{120} + 117 \; \frac{-335}{120}$.

Scanned with CamScanner

9) $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2 \rightarrow ①$

$x+1 = e^t \Rightarrow x = e^t - 1$

Diff $\log(x+1) = t$

Also $(x+1)y' = \Delta y \; d/dt = \Delta$.

$\Delta (c+1)^2 y'' = \Delta(\Delta-1) - 3\Delta + 4)y = (e^t-1)^2$

$(\Delta^2 - 4\Delta + 4)y = e^{2t} - 2e^t + 1)$

Chear eq is $\Delta^2 - 4\Delta + 4 = 0$

$(\Delta-2)^2 = 0$

$\Delta = 2, 2$

C. Fuction is

$C.f = (c_1 + c_2 t)e$

Also particule integral

$P.I = \frac{1}{(\Delta-2)^2}(e^{2t} - 2e^t + 1)$

$= \frac{1}{(\Delta-2)^2} e^{2t} - 2\frac{1}{(D-2)^2} e^t + \frac{1}{(\Delta-2)^2} \rightarrow ②$.

Now

$\frac{1}{(\Delta-2)^2} e^{2t} = \frac{1}{\Delta+2-2} 2e^{2t} = \frac{1}{0} e^{2t}$

case of failure.

Scanned with CamScanner