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Subject Calculus

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Q: No 2 (A)

Illustrate the center and radius of the sphere
$x^2 + y^2 + z^2 + 3x - 4z + 1$

Solution:-

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

$$(x^2 + 3y) + y^2 + z^2 - 4z + 1 = 0$$

$$(x^2 + 3x + (1/3)^2) + (y+0)^2 + (z^2 - 4z) + (-4/2)^2$$

$$= 1 + (3/2)^2 + (-4/2)^2$$

$$(x + 3/2)^2 + (y+0)^2 + (z-2)^2 = \frac{21}{4}$$

So $(x_0, y_0, z_0) \Rightarrow$ Center

$$(-3/2, 0, 2)$$

~~find~~ Radius $a = \sqrt{\frac{21}{4}}$

Q: No 2 (B)

The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$ and the x-axis is revoled about x-axis to generate a solid. Apply the integration find volume of solid.

Solution:-

$$y = \sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x \le b$$

as,

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx \Rightarrow \pi \frac{x^2}{2} \Big|_2^4$$

$$V = \frac{\pi}{2} \left((4)^2 - 0 \right)$$

$$V = 8\pi$$

Q: No 5 (A)

Estimate the angle between $A = i - 2j - 2k$, $B = 6i + 3j + 2k$

Solution:-

$$A = i - 2j - 2k$$

$$|A| = \sqrt{1 + 4 + 4} = \sqrt{9} \Rightarrow 3$$

$$|B| = 6i + 3j + 2k$$

$$|B| = \sqrt{36 + 9 + 4}$$

$$= \sqrt{49} \Rightarrow 7$$

$$Q = \cos^{-1} \frac{A \cdot B}{|A| \cdot |B|}$$

$$Q = \cos^{-1} \frac{((i - 2j - 2k) \cdot (6i + 3j + 2k))}{3 \times 7}$$

$$Q = \cos^{-1} \frac{((1)(6) + (-2)(3) + (-2)(2))}{21}$$

$$Q = \cos^{-1} \left(-4/21\right) \Rightarrow Q = 100.97$$

Q. No 5 (B)

Find a sperical coordinate equation for the sphere.

$$x^2 + y^2 + (z-1)^2 = 1$$

Solution:-

$$x^2 + y^2 + (z-1)^2 = 1$$

$$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2$$

$$+ (\rho \cos\phi - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos\theta^2 + \rho^2 \sin^2\phi \sin^2\theta$$

$$\rho^2 \cos\phi^2 + 1 - 2\rho \cos\phi = 1$$

$$\rho^2 \sin^2\phi \left(\cos\theta^2 + \sin^2\theta \right)$$

$$+\rho^2 \cos\phi^2 + 1 - 2\rho\cos\phi = 1$$

$$\rho^2(\sin^2\phi) + \rho^2\cos\phi^2 - 2\cdot\rho\cos\phi = 1-1$$

$$\rho^2(\sin\phi^2 + \cos^2\phi) - 2\rho\cos\phi = 0$$

$$\rho^2 = 2\rho\cos\phi$$

$$\rho = 2\cos\phi.$$

Q: No: 4

Find area of region between the graph and the x-axis where

$y = -x^2 + 5x - 4, \quad [0, 2]$

Solution:-

$$y = f x = x^2 + 5x - 4$$

and

$$[a, b] = [0, 2]$$

as

$$a = 0$$
$$b = 2$$

So area under graph will

$$A = \int_a^b f(x)\,dx \quad \text{(By Putting value)}$$

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$\frac{-x^3}{3} + \frac{5x^2}{2} - 4 \Big|_0^2$$

$$= \left| \frac{(-2)^2}{3} + \frac{5(2)^2}{2} - 4(2) - \frac{(0)^3}{3} + \frac{5(0)^2}{2} - 4(0) \right|$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8 \right) - 0 + 0 - 0$$

$$= -4/3 + 10 - 8$$

$$= -4/3 + 2 \Rightarrow \frac{-4+6}{3}$$

$$A = 2/3 \Rightarrow A = 0.666$$

Q:No 3

If $A = 2i - 4j + \sqrt{5}k$, and

$B = -2i + 4j - \sqrt{5}k$

then illustrate the vector proje AB

Solution:-

By Po? projet

$B \cdot A = (-2i + 4j - \sqrt{5}k)$

$(2i - 4j + \sqrt{5}k)$

$B.A = (-2j + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$B.A = (-\cancel{2j} + \cancel{2j} + \cancel{4j} - \cancel{4j} - \cancel{\sqrt{5}k} + \cancel{\sqrt{5}k})$

$B.A = -4 - 16 - 5$

$B.A = -25$

Now/

$A.A = (2j - 4j + \sqrt{5}k) \cdot (2j - 4j + \sqrt{5}k)$

$A.A = (2ij + 2i) + (-4j - 4j) + (\sqrt{5}k + \sqrt{5}k)$

$A.A = 4 + 16 + 5$

$A.A = 25$

So $proj_A B = \left(\frac{B.A}{A.A}\right) A.$

As $Proj_A B = \left(\frac{B.A}{A.A}\right) A$

By putting values

$= \frac{\cancel{-25}}{\cancel{25}} (2i - 4j + \sqrt{5}k)$

$= (-1)(2i + 4j + \sqrt{5}k)$

$= -2i + 4j - \sqrt{5}k$ Ans

Q: No 1 (B)

Estimate by using substitute method:

$$\int_0^1 x^3 (1+x^4)^3 \, dx$$

Solution:-

$$\int_0^1 x^3 (1+x^4)^3 \, dx$$

let, $t = 1+x^4$

$$\frac{dt}{dx} = \frac{d}{dx}(1+x^4)^{\cancel{3}}$$

$$dt/dx = 4x^3$$

or $dt = 4x^3 \, dx$

$$dt/4 = x^3 \, dx$$

So $\frac{1}{4} \int_0^1 t^3 \, dt$

$$\frac{1}{4} \left(\frac{t^4}{4}\right)\Big|_0^1$$

$$\frac{1}{16}(1^4 - 0^1)$$

$\frac{1}{16}$ (1)

$= \frac{1}{16}$ Ans

Q.No 1 (A)

Estimate $\int \sqrt[4]{1-\phi^2}\, d\phi$

Solution:-

$$\int \sqrt{1-\phi^2}\, d\phi$$

let, $1 - \phi^2 = u$

then $1 - \phi^2 = u$

$d/d\phi \,(1-\phi^2 = du/d\phi$

$0 - 2\phi = du/d\phi$

$\phi\, d\phi = -\frac{1}{2} du$

Put $= \frac{1}{2} du = \phi du$

$\phi \; (1-\phi^2) = u$

$$= \int -4 \sqrt{4/2} \; du$$

$$= -1/2 \int 4\sqrt{4} \; du$$

$$= -1/2 \int u^{1/4} \; du$$

$$= -1/2 \; \frac{u^{1/4+1}}{1/4+1} + C$$

Put $u = 1 - \theta^2$

$$= -\frac{1}{2} (1-\theta^2)^{5/4} + C$$

$$= \cancel{-\tfrac{2}{5}} \; -\frac{1}{\cancel{2}} \; 4^2 \left(\frac{1 \; \theta^2}{5}\right)^{5/4} + C$$

$$= -2/5 \, (1 - \theta^2)^{5/4} + C$$

$$= -2/5 \left(\frac{-\theta^2 + 1}{5}\right)^{5/4} + C$$

Ans

Q: No 1 (A)

Given

$$\int \phi \sqrt[4]{1-\phi^2}\, d\phi$$

Solution:-

Let $1-\phi^2 = u$

$$\frac{d}{d\phi}(1-\phi^2 = \frac{d}{d\phi}u$$

$$-2\phi = \frac{du}{d\phi}$$

$$= \phi\, d\phi = -\frac{1}{2}\, du$$

Now, $= \int (u)^{1/4} \cdot (-1/2)\, du$

$$= -\frac{1}{2}\int u^{1/4}\, du$$

$$= -\frac{1}{2} \cdot \frac{4}{5} u^{5/4} + c$$

$$= -\frac{2}{5} u^{5/4} + c$$

By back substitution

$$= -\frac{2}{5}(1-\phi^2)^{5/4} + c$$

Q. No 1 (B)

$\int_0^1 x^3 (1+x^4)^3 dx$

Paking $(1+x^4) = u$

$u = (1+x^4)$

applying d/dx b.s

$\frac{du}{dx} = d/dx (1+x^4)$

$\frac{du}{dx} = d/dx + d/dx\, x^4$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\int_0^1 x^3 (1+x^4)^3 dx$

xing & $\div$ing 4

$\therefore 4x^3 dx = du$

So $1/4 \int_0^1 u^3 du$

So take

$u = (1+x^4)$

Put $x = 0$

$u(1+0)$

Now

$x = 1$

$u = (1 + 1^4)$

$u = 2$

So limit are

$\frac{1}{4} \int_1^2 u^3 dx$

Apply Integration

$\frac{1}{4} \int_1^2 u^3 dx$

$= \frac{1}{4} \left(3x^2 \Big|_1^2 \right)$

Putting limits

$= \frac{1}{4} \left(3(2)^2 - 3(1)^2 \right)$

$= \frac{1}{4} \left(3(4) - 3 \right)$

$= \frac{1}{4} (12 - 3)$

$= \frac{1}{4} (9)$

$= \boxed{\frac{9}{4}}$