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Degree : BS (CS) II

Subject : Linear Algebra.

Assignment : final.

QNo1: Solve the system and find if these vectors are linearly independent?

$$V_1 = \begin{bmatrix} ID_1 \\ ID_2 \\ ID_3 \end{bmatrix}, V_2 = \begin{bmatrix} ID_2 \\ ID_3 \\ ID_4 \end{bmatrix}, V_3 = \begin{bmatrix} ID_3 \\ ID_4 \\ ID_5 \end{bmatrix}$$

Ans: Since ID = 16547

$$V_1 = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}, V_2 = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}, V_3 = \begin{bmatrix} 5 \\ 4 \\ 7 \end{bmatrix}$$

for linearly independent

$$a_1 \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix} + a_2 \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} + a_3 \begin{bmatrix} 5 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So,

$$\left[\begin{array}{ccc|c} 1 & 6 & 5 & 0 \\ 6 & 5 & 4 & 0 \\ 5 & 4 & 7 & 0 \end{array}\right]$$

$R_2 - 6R_1 \to R_2$; $R_3 - 5R_1 \to R_3$

$$\begin{bmatrix} 1 & 6 & 5 & 0 \\ 0 & -31 & -24 & 0 \\ 0 & -24 & -18 & 0 \end{bmatrix}$$

$4R_1 + R_3 \to R_1$

$$= \begin{bmatrix} 6 & 0 & 2 & 0 \\ 0 & -31 & -24 & 0 \\ 0 & -24 & -18 & 0 \end{bmatrix}$$

$\frac{R_2}{-31} \to R_2$

$$= \begin{bmatrix} 6 & 0 & 2 & 0 \\ 0 & 1 & \frac{24}{31} & 0 \\ 0 & -24 & -18 & 0 \end{bmatrix}$$

$\frac{R_3}{-24} \to R_3$

$$= \begin{bmatrix} 6 & 0 & 2 & 0 \\ 0 & 1 & \frac{24}{31} & 0 \\ 0 & 1 & \frac{18}{24} & 0 \end{bmatrix}$$

$R_3 - R_2 \to R_3$

$$= \begin{bmatrix} 6 & 0 & 2 & 0 \\ 0 & 1 & \frac{24}{31} & 0 \\ 0 & 0 & \frac{-18}{744} & 0 \end{bmatrix}$$

$\frac{-744}{18} R_3 \to R_3$

$$= \begin{bmatrix} 6 & 0 & 2 & 0 \\ 0 & 1 & \frac{24}{31} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1/6 \rightarrow R_1$

$$= \begin{bmatrix} 1 & 0 & 2/6 & 0 \\ 0 & 1 & 24/31 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$31/24\, R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 2/6 & 0 \\ 0 & \frac{31}{24} & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 - R_3 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 2/6 & 0 \\ 0 & \frac{31}{24} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$24/31\, R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 2/6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$6/2\, R_1 \rightarrow R_1$

$$\begin{bmatrix} 6/2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\frac{2}{6} R_1 \to R_1$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 - R_3 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Since~~

~~It~~ is

The vectors are linearly independent.

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Q No: 3

(a) find the total cost.

total cost = {number of units} {Cost per unit}

total cost for product X = ?

Cost per unit for $X = \begin{bmatrix} 4 & 5 & 0 \\ 2 & 5 & 0 \\ 1 & 5 & 0 \end{bmatrix}$

In question number of unit = A = 1000

total cost = {number of unit} {Cost Per Unit}

total cost of product X = A × X

$= 1000 \begin{bmatrix} 4 & 5 & 0 \\ 2 & 5 & 0 \\ 1 & 5 & 0 \end{bmatrix}$

total cost of X = $\begin{bmatrix} 450000 \\ 250000 \\ 150000 \end{bmatrix}$

Now we will find total cost for Y Product.

Cost per Unit for $Y = \begin{bmatrix} 400 \\ 350 \\ 150 \end{bmatrix}$

Total cost of Y = {number of unit} {cost per unit}

~~cost~~ number of unit = 500 = B

Total cost for Product Y = B . Y

$$= 500 \begin{bmatrix} 400 \\ 350 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 200,000 \\ 175000 \\ 75000 \end{bmatrix} = \text{total cost of } Y$$

~~(b) Explain the linear transformation properties~~

Now adding X and Y Products

total cost X + Y

$$\text{Total cost} = \begin{bmatrix} 450000 \\ 250000 \\ 150000 \end{bmatrix} + \begin{bmatrix} 200000 \\ 175000 \\ 75000 \end{bmatrix}$$

$$\text{total cost} = \begin{bmatrix} 650000 \\ 425000 \\ 225000 \end{bmatrix}$$

Ans

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(b) Explain the linear transformation

Properties:

- $T(u+v) = T(u) + T(v)$
- $T(cu) = cT(u)$

Ans:

$$T(u+v) = T(u) + T(v)$$

It means for all u, v in the domain of T.

$$T(cu) = c\,T(u)$$

It means for all u and all scalars c.

Property (i) says that the result $T(u+v)$ of the first adding u and v in R^n and the applying T is the same as applying T first to u and v and then adding $T(u)$ and $T(v)$ in R^n.

These two Properties lead easily useful facts.

Q No: 3

~~four things for vector space~~:

~~Ans~~:

(a) The Set V of all matrices of the form $\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}$ where $a, b \in \mathbb{R}$

with standard addition and scallar multiplication.

Note V is not closed under the addition.

$$\begin{bmatrix} 1 & c \\ d & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} + \begin{bmatrix} 1 & c \\ d & 1 \end{bmatrix} = \begin{bmatrix} 2 & a+c \\ b+d & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & c \\ d & 1 \end{bmatrix}$$

we have

$$\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & c \\ d & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a+c \end{bmatrix}$$

$$K \cdot \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & Ka \\ ab & 1 \end{bmatrix}$$

A

we calculated that v is not a vector space with the given operation

$$= \begin{bmatrix} 1 & Ka \\ Kb. & 1 \end{bmatrix}.$$

we see from that V is indeed a vector space with the given operation field that is closed under the condition and scalar.

Q No: 4

Determinate Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 matrix.

(a) for which values of det M does M have an inverse?

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$M^{-1} = \frac{Adj M}{|M|}$$

for inverse $M > 0$ or $M \neq 0$

$$|M| = ad - bc$$

$$Adj M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$M^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

Since $ad - bc \neq 0$

$$ad \neq bc$$

(b) Write down all 2×2 bit matrices with determinate 1.

Let A is a matric.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a matrix whose determinate is equal to 1.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|A| = 1 - 0 = 1$

(c) write down 2×2 bit matrices with determinate 0.

Let B is a ~~b~~ matrix

$B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is a bit 2×2 matrix Whose determinate is equal to 0.

(d) Compute determinate A for below 3×3 matrix.

$$A = \begin{bmatrix} ID_1 & ID_1 & ID_1 \\ ID_2 & ID_3 & ID_2 \\ ID_4 & ID_1 & ID_5 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 5 & 6 \\ 4 & 1 & 7 \end{bmatrix}$$

$|A| = 1(35-6) - 6(7-1) + 4(6-5)$

$|A| = 29 - 6(6) + 4(1)$

$|A| = 29 - 36 + 4$

$|A| = -3$ Ans