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SUBJECT : DIFFERENTIAL EQUATION

DEPARTMENT : BEE

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TEACHER NAME

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QUESTION NO 1

PART A

⇒ Estimate the general solution of $y' = (x+2)y^2$

SOLUTION:-

$$y' = (x+2)y^2$$

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

$$\Rightarrow \int \frac{1}{y^2}\, dy = \int (x+2)\, dx$$

$$\Rightarrow \int y^{-2}\, dy = \int (x+2)\, dx$$

$$\Rightarrow \int \frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\Rightarrow \frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

→ Multiplying both sides by -1

$$\Rightarrow y^{-1} = -\left(\frac{x^2}{2} + 2x + C\right)$$

$$\Rightarrow \boxed{y = -\left(\frac{1}{x^2/2 + 2x + C}\right)}$$

Ans

QUESTION NO 1

Part B

⇒ Estimate the general solution of $y' = (y+9x)^2$ —(i)?

Solution :-

Let $y + 9x = u$

$$\Rightarrow \frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 9$$

So (i) becomes

$$\Rightarrow \frac{du}{dx} - 9 = u^2$$

$$\Rightarrow \frac{du}{dx} = u^2 + 9$$

$$\Rightarrow \int \frac{1}{u^2+9}\, du = \int dx$$

$$\Rightarrow \int \frac{1}{(3)^2 + u^2}\, du = \int dx$$

$$\Rightarrow \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) = x + C_1$$

$$\Rightarrow \tan^{-1}\left(\frac{u}{3}\right) = 3x + 3C_1$$

$$\Rightarrow \frac{u}{3} = \tan(3x + C)$$

$$\Rightarrow u = 3\tan(3x + C)$$

$$\Rightarrow y + 9x = 3\tan(3x + C)$$

$$\Rightarrow \boxed{y = -9x + 3\tan(3x + C)}$$

Ans

QUESTION NO 2

PART A

⇒ Estimate the general Solution $x^3 dx + y^3 dy = 0$

SOLUTION:-

$$x^3 dx + y^3 dy = 0$$

$$\Rightarrow M du + N dy = 0$$

$$\Rightarrow M = x^3 \quad , N = y^3$$

$$\Rightarrow \frac{\partial M}{\partial y} = 0 \quad \frac{\partial N}{\partial x} = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ so exact.

$$\Rightarrow u = \int M du + k(y)$$

$$\Rightarrow u = \int x^3 du + k(y)$$

$$\Rightarrow u = \frac{x^4}{4} + K(y) \quad -(i)$$

$$\Rightarrow \frac{\partial u}{\partial y} = 0 + \frac{d}{dy} K(y)$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{d}{dy} K(y)$$

we know that

$$\Rightarrow \frac{\partial u}{\partial y} = N = y^3$$

$$\Rightarrow y^3 = \frac{d}{dy} K(y) \Rightarrow \int y^3 = \int \cancel{d} K(y)$$

$$\Rightarrow K(y) = \frac{y^4}{4} + C_1 \quad \text{putt in (i)}$$

$$\Rightarrow C_1 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\Rightarrow C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\Rightarrow C_2 - C_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\Rightarrow \boxed{C = \frac{x^4}{4} + \frac{y^4}{4}}$$

Ans

QUESTION NO 3

PART A

➡ Find the general solution

$$4y'' - 20y' + 25 = 0$$

SOLUTION:-

This is second order homogeneous differential equation with constant coefficient.

$$ay'' + by' + cy = 0$$

and the solution for this is $y = e^{\lambda}$ —(i)

General solution:

$$y = c_1 e^{\lambda} + c_2 + e^{\lambda}$$

Now

$$4 \frac{d^2}{dx^2}(y) - 20 \frac{d}{dx}(y) + 25(y) = 0 \text{ eq } (A)$$

Put eq (i) in eq (A)

$\Rightarrow 4\frac{d^2}{dx^2}(e^{\lambda x}) - 20\frac{d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0$

$\Rightarrow \frac{d^2}{dx^2} e^{\lambda x} = \lambda^2 e^{\lambda x}$ — (B)

$\Rightarrow$ put eq (B) and eq (1) in eq (A)

$\Rightarrow 4\lambda^2 e^{\lambda x} - 20 e^{\lambda x} + 25 e^{\lambda x} = 0$

$\Rightarrow e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$

$\Rightarrow e^{\lambda x} \neq 0$

$\Rightarrow 4x^2 - 20\lambda + 25 = 0$

$\Rightarrow (2\lambda - 5)^2 = 0$

$\Rightarrow \lambda = 5/2$ or $\lambda = 5/2$

$\Rightarrow y(x) = y_1(x) + y_2(x)$

$\Rightarrow \boxed{y(x) = C_1 e^{5/2 x} + C_2 x e^{5/2 x}}$

Ans

QUESTION NO 3

PART B

⇒ Estimate general solution of

$$4y'' - 6y - 7y = 0$$

SOLUTION:-

Assume $y(x) = e^{\lambda x}$

Put in eq

$$\Rightarrow 4 \cdot d^2 \cdot y(x) - 6 \frac{d}{dx} y(x) - 7y(x) = 0$$

$$\Rightarrow 4 \cdot \frac{d^2}{dx^2}(e^{\lambda x}) - 6 \frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0$$

$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \longrightarrow A$$

$$\Rightarrow \frac{d^2}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \longrightarrow B$$

put Ⓐ and Ⓑ in eq (i)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow (4\lambda^2 - 6\lambda - 7)e^{\lambda x} = 0$$

$$\Rightarrow \lambda = \frac{3}{4} - \sqrt{37}/4$$

$$\Rightarrow \lambda = \frac{3}{4} + \sqrt{37}/4$$

$$\Rightarrow y(x) = y_1(x) + y_2(x)$$

$$\Rightarrow \boxed{y(x) = C_1 e^{(3/4 - \sqrt{37}/4)x} + C_2 e^{(3/4 + \sqrt{37}/4)x}}$$

Ans//

THE END