Name M. Yasir Shah

ID 13172

Subject EMF

Submitted to:

Dr. Rafiq Mansoor.

ID 13172

Problem 4.1: The value of E at $P(\rho = 2, \phi = 40°, z = 3)$ is given as $E = 100a_\rho - 200a_\phi + 300a_z$ V/m. Determine the incremental work required to move a 20 μC charge a distance of 6 μm.

(a) In the direction of a_ρ: The incremental work is given by $dW = -qE \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6 \times 10^{-6} a_\rho$. Thus.

$$dW = -(20 \times 10^{-6} C)(100 \text{ V/m})(6 \times 10^{-6} m)$$
$$= -12 \times 10^{-9} \text{ J} = \boxed{-12 \text{ nJ}}$$

(b) In the direction a_ϕ: In this case $dL = 2 d\phi\, a_\phi = 6 \times 10^{-6} a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6})$$
$$= 2.4 \times 10^{-8} \text{ J} = \boxed{24 \text{ nJ}}$$

(c) In this direction of a_z: Here $dL = dz\, a_z = 6 \times 10^{-6} a_z$ and so

$$dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6})$$
$$= -3.6 \times 10^{-8} \text{ J} = \boxed{-36 \text{ nJ}}$$

(d) In this direction of E: Here, $dL = 6 \times 10^{-6} a_E$

where $a_E = \dfrac{100a_\rho - 200a_\phi + 300a}{[100^2 + 200^2 + 300^2]^{1/2}} = 0.267a_\rho - 0.535a_\phi + 0.802a_z$.

Thus $dW = -(20 \times 10^{-6})[100a_\rho - 200a_\phi + 300a_z] \cdot [0.267a_\rho - 0.535a_\phi + 0.802a_z](6 \times 10^{-6})$

$$= \boxed{-44.9 \text{ nJ}}$$

ID 13172

(e) In this direction of $G = 2a_x - 3a_y + 4a_z$: In this case, $dL = 6\times10^{-6} a_G$ where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2+3^2+4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z.$$

So

$$dW = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z] \cdot [0.371a_x - 0.557a_y + 0.743a_z](6\times10^{-6})$$

$$\Rightarrow -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9](6\times10^{-6}).$$

where, at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos 40^\circ = 0.766$, $(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$ and $(a_\phi\cdot a_y) = -\sin(40^\circ) = -0.643$

Substituting these results in

$$dW = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = \boxed{-41.8\ nJ}$$

ID 13172

Problem 4.2: Let $E = 400a_x - 300a_y + 500a_z$ in the neighborhood of point $P(6, 2, -3)$. Find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by:

(a) $a_x + a_y + a_z$: write as

$$dW = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot \times \frac{(a_x + a_y + a_z)}{\sqrt{3}}(10^{-3}).$$

$$= -\frac{4 \times 10^{-3}}{\sqrt{3}}(400 - 300 + 500) = \boxed{1.39\ J}$$

(b) $-2a_x + 3a_y - a_z$: The computation is similar to that of part (a) but we change the direction.

$$dW = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \times \frac{(-2a_x + 3a_y - a_z)}{\sqrt{14}}(10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{14}}(-800 - 900 - 500)$$

$$= \boxed{2.35\ J}$$

ID 13172

Problem 4.3: If $E = 120 a_\rho$ V/m, find the incremental amount of work done in moving a 50 μm charge a distance of 2mm from.

(a) $P(1,2,3)$ toward $Q(2,1,4)$: The vecto along this direction will be $Q - P = (1,-1,1)$ from which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$. We writ

$$dW = -qE \cdot dL = -(50\times10^{-6})\left[120 a_\rho \frac{a_x - a_y + a_z}{\sqrt{3}}\right](2\times10^{-3})$$

$$= -(50\times10^{-6})(120)\left[(a_\rho \cdot a_x) - (a_\rho \cdot a_y)\right]\frac{1}{\sqrt{3}}(2\times10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho \cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho \cdot a_y) = \sin(63.4) = 0.894$. Substituting these we obtain

$$dW = \boxed{3.1\ \mu J}$$

(b) $Q(2,1,4)$ toward $P(1,2,3)$:

Note that the field has only a radial component and does not depend on ϕ or z. Note also that P and Q are at the same radius ($\sqrt{5}$) from the z-axis. but have different ϕ and z co-ordinates

ID 13172

We could just as well position the two points at the same z location and the problem would not change. If this were so, then moving along a straight line between P and Q would thus involve moving along a chord of a circle whose radius is d5. Halfway along this line is a point of symmetry in the field. This means that when starting from either point, the initial force will be same. Thus the answer is dW = 3.1 uJ as in part (a). This is also found by going through the same procedure as in part (a), but with the direction (roles of P and Q) reversed.

|ID 13172|

Problem 4.5: Compute the value of $\int_A^P G.dL$ for $G = 2y a_x$ with $A(1,-1,2)$ and $P(2,1,2)$ using the path.

(a) Straight-line segments $A(1,-1,2)$ to $B(1,1,2)$ to $P(2,1;2)$. In general we would have

$$\int_A^P G.dL = \int_A^P 2y\,dx$$

The change in x occurs when moving between B and P, during which $y = 1$. Thus

$$\int_A^P G.dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \boxed{2}$$

(b) Straight-line segment $A(1,-1,2)$ to $C(2,-1,2)$ to $P(2,1,2)$: In this case the change in x occurs when moving from A to C, during which $y = -1$

$$\int_A^P G.dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \boxed{-2}$$

ID 13172

Problem 4.7: Repeat problem 4.6 for $G = 3xy^3 a_x + 2z a_y$. Now things are different in that the path matter.

(a) Straight line $y = x - 1, z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2\, dx + \int_1^3 2z\, dy$$

$$= \int_2^4 3x(x-1)^2\, dx + \int 2(1)\, dy = \boxed{90}$$

(b) Parabola $6y = x^2 + 2, z = 1$ we obtain.

$$\int G \cdot dL = \int_2^4 3xy^2\, dx + \int 2z\, dy = \int_2^4 \frac{1}{12} x(x+2)^2\, dx$$

$$+ \int_1^3 2(1)\, dy = \boxed{82}$$