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Assignment probability

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Submitted to Daud sir.

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$S = \{(1,1), (1,2), (1,3), (1,4),$
$(1,5), (1,6), (2,1), (2,2),$
$(2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4),$
$(3,5), (3,6), (4,1), (4,2),$
$(4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4),$
$(5,5), (5,6), (6,1), (6,2),$
$(6,3), (6,4), (6,5), (6,6)\}$

A = {The Sum is 7}

B = {The Sum is even}

C = {Sum is greater than 8}

D = {The two dise had the same outcomes}

Now

$A = \{(1,6), (2,5), (3,4), (4,3),$
$(5,2), (6,1)\}$

$B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$A \cap B = \{\}$ OR ϕ

$A \cap C = \{\}$ OR ϕ

$A \cap D = \{\}$ OR ϕ

$P(A) = 6/36$, $P(B) = 18/36$

$P(C) = 10/36$ $P(D) = 6/36$

$$P(A|B) = \frac{P(A \cap B)}{PB} = 0 \times 18/36$$

$P(A|B) = 0$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = 0 \times 10/36$$

$P(A|C) = 0$

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{\cancel{6}^{1}}{\cancel{36}\,6} \quad 0 \times 1/6$$

$$P(A|D) = 0$$

2/4

Question # 2 :

Sol:-

We simply have to divide the number of desired outcomes by total number of possible outcomes.

Explanation:-

When we rolling two dice, there are 36 different combinations (6 possible to the power of 2 dice), counting these up, there are 15 possibilities less than 7: (1·1), (1·2), (1·3), (1·4), (1·5) (2·1), (2·2), (2·3), (2·4), (3·1), (3·2) (3·3), (3·4), (4·1), (4·2), (5·1) the probability of getting less than a 7 is

$\frac{15}{36} = \frac{5}{12}$

There are 6 possible combinations of getting a 7 (which gives a probability of $\frac{1}{6}$, this means 21 possibilities for getting less than or equal to 7. So there are 15 remaining possibilities of getting more than 7. This is the same as the probability of getting less than 7. So the probability must be $\frac{5}{12}$ as well. In calculating this we must assume that each combination is equally likely to roll as any other and therefore the dice are fair.

OR 2nd method

Sum of 2 has 1 way 1, 1
" " 3 " 2 1, 2 and 2, 1
" " 4 " 3 1, 3, 2, 2, 3, 1
5 has 4 ways
6 " 5 "
8 " 5 "
9 " 4 "
10 " 3 "
4 " 2 "
12 " 1 "
Those are 15/36 for each side
with a sum of 30/36
that leaves a 6/36 = 1/6 prob. for

a sum of 7.

6

(4) A and B play a game in which A's probability of wining is 2/3 In a series of 8 games. What is the probability that "A" will win?

(1) Exactly 4 games
(2) At least 4 games
(3) From 3 to 6 games

Sol:-

Given that $p = 2/3$ $n = 8$

$q = 1 - p$

$= 1 - 2/3$

$q = 1/3$

Let "z" denotes the number of games win by A, so:

case 1:- $P(z = 4)$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= 1120/6561$$

$$= 0.1707$$

case 2: $P(z \geq 4)$

$$= 1 - P(x < 4)$$

$$= 1 - \sum_{z=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-z}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$$

$$1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

case 3:- $P(3 \le z \le 6)$

$$\sum_{x=3}^{6} \binom{8}{z} \left(\frac{2}{3}\right)^{z} \left(\frac{1}{3}\right)^{8-z}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}$$
$$\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^{8}}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

(7) Since the C_i's from the partition of the sample space we can apply the law of total probability for $A \cap B$.

$$p(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i)\, p(C_i)$$

$$p(A \cap B) = \sum_{i=1}^{M} p(A/C_i)\, p(B/C_i)\, P(C_i)$$

(A and B are condontionally independant.

$$p(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\, p(B)\, p(C_i)$$

B is independent of all C_i's

$$p(A \cap B) = P(B) \sum_{i=1}^{M} P(A/C_i)\, P(C_i)$$

$$p(A \cap B) = P(B)\, P(A)$$

total probability

A & B are independent.

(5) (9)

The probability function for a binomial random variable is:

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

this is the probability of having x successes in a series of n independent trails when the probability of success in any one of the trials is p. If x is the random variable with this probability distribution

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since the $x=0$ term vanishes Let $y = x-1$ and $m = n-1$ Subbing $x = y+1$ and $n = m+1$

Since the $x=0$ term vanishes. Let $y = x-1$ and $m = n-1$ Subbing $x = y+1$ and $n = m+1$ into the last Sum (and using the fact that the limit $x=1$ and $x=n$ correspond to $y=0$ and $y = n-1 = m$

$$E(X) = \sum_{y=0}^{m} \frac{(m+1)}{y!\,(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}$$

the binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

Setting $a = p$ and $b = 1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y} = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

$$= (a+b)^m = (p+1-p)^m = 1$$

So that $E(X) = np$

Similarly, but this time using $y = x-2$ and $m = n-2$

$$E(X(X-1)) = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(X(X-1)) = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^x (1-p)^{n-x}$$

$$n(u-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

$$= n(u-1)\,p^2\,(p+(1-p))^m$$

$$= n(n-1)\,p^2$$

So the variance of x is

$$E(x^2) - E(x)^2 = E(x(x-1)) + EX - E(x^2) = n(u-1)\,p^2 + np - (np)^2$$

$$= np(1-p)$$

6

Binomial Frequency distribution:

The relative frequency of a discrete random variable which has only two possible outcomes. As with all random variable the mean or expected value and the variance can be calculated from the probability distribution.

Example:-

A Servey with only yes/No response.

Formula:-

$$p(x) = \begin{bmatrix} n \\ x \end{bmatrix} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where

n = number of trials

x = " " successes desired

p = probability of getting a success in one trial.

q = 1-p = the probability of getting a failure in one trial.

2 Binomial distribution:

Many experiments consists of repeated independent trials each trials having two possibilities e.g it may be head or tail success or failure right and wrong etc.

Formula:-

$$p(X=x)\ F(x) = {}^nC_x\, p^x q^{n-x}$$

n = The no of Sample size
x = The no of success
p = The probability of success
q = The probability of failure

Difference:-
The main difference b/w them is what binomial distribution is discrete. This means that in bionomial distribution there are no data points b/w any two data points. This is very different from from a normal distribution which has coutinuous data points.

Or
2nd defination of Binomial frequency distribution:
If the binomial probability distribution is multiplied by N, the no of experiments or sets the resulting distribution is known as binomial freq. distribution

(7)

Coefficient of variations:-

Data in case A:-

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = 3/45 \times 100$$

$$CV = 6.7$$

Data in case B:-

$$CV = 11/60 \times 100$$

$$CV = 18.3$$

Data in case C:-

$$CV = 5/50 \times 100$$

$$CV = 10$$

Data in case D:-

$$CV = 15/25 \times 100$$

$$CV = 60$$