Assignment No "1"

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Subject * Differential Equations
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Q (1) Use any method to solve the following questions and draw graphs.

① $x^2y'' - 4xy' + 6y = 0$ $\left[\begin{array}{l} y(1) = 0.4 \\ y'(1) = 0 \end{array}\right]$

* Solution:-

Lets substitute.

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

* into the given ODE this gives

$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$

$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} - 4\cancel{x} m x^m \cancel{x^{-1}} + 6x^m = 0$

* x^m as common factor

$m(m-1) - 4m + 6 = 0 \Rightarrow m^2 + 5m + 6 = 0$

* finding roots of equation.

$m^2 - 5m + 6 = 0 \Rightarrow m_{\frac{1}{2}} = \frac{5 \pm \sqrt{(-5)^2 + 4 \cdot 6}}{2}$

$\Rightarrow m_{\frac{1}{2}} = \frac{5 \pm 1}{2}$

* General Solution $\{m_1 = 3, m_2 = 2\}$

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^3 + c_2 x^2$

$\Rightarrow y' = 3c_1x^2 + 2c_2x$

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* Determine c_1 & c_2 from IVP

$$\begin{cases} 0.4 = y(1) = c_1 \cdot 1^3 + c_2 \cdot 1^2 \\ 0 = y'(1) = 3c_1 \cdot 1^2 + 2c_2 \cdot 1 \end{cases} \Rightarrow \begin{cases} 0.4 = c_1 + c_2 \\ 0 = 3c_1 + 2c_2 \end{cases}$$

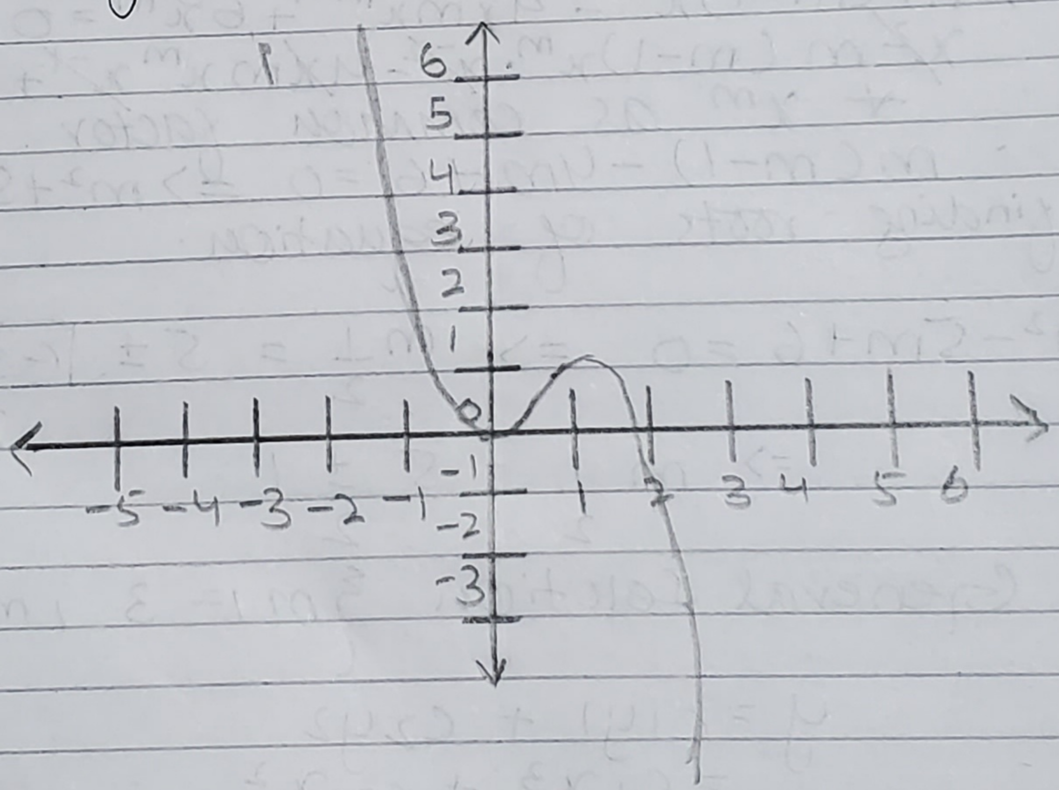
$$\begin{cases} 0.4 - c_2 = c_1 \\ 0 = 3(0.4 - c_2) + 2c_2 \end{cases}$$

$$\begin{cases} 0.4 - c_2 = c_1 \\ 0 = 1.2 - c_2 \end{cases}$$

$$\begin{cases} 0.4 - c_2 = c_1 \\ 1.2 = c_2 \end{cases} \Rightarrow \begin{cases} 0.4 - 1.2 = c_1 \\ 1.2 = c_2 \end{cases}$$

$$\begin{cases} -0.8 = c_1 \\ 1.2 = c_2 \end{cases}$$

$$y = -0.8x^3 + 1.2x^2$$



(2) $x^2y'' + 3xy' + 0.75y = 0$ $\left[\begin{array}{l} y(1) = 1 \\ y'(1) = -1.5 \end{array}\right]$

* Solution /-

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} + 3xmx^{m-1} + 0.75x^m = 0$

$x^2 m(m-1)x^m \cdot x^{-2} + 3xmx^m x^{-1} + 0.75x^m = 0$

* x^m as a common factor

$m(m-1) + 3m + 0.75 = 0$

$m^2 + 2m + 0.75 = 0$

* Finding roots of equation.

$m^2 + 2m + 0.75 = 0 \Rightarrow m\frac{1}{2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 0.75}}{2}$

$m\frac{1}{2} = \frac{-2 \pm 1}{2}$

General solution $\left\{ m_1 = \frac{1}{2}, m_2 = \frac{-3}{2} \right\}$

* Root gives us two solutions

$y_1 = x^{m_1} = x^{-\frac{1}{2}} = x^{-0.5}$

$y_2 = x^{m_2} = x^{-\frac{3}{2}} = x^{-1.5}$

so the General solution is

$y = c_1y_1 + c_2y_2$

$= c_1x^{-0.5} - c_2x^{-1.5}$

$y' = -0.5c_1x^{-1.5} - 1.5c_2x^{-2.5}$

* Determining c_1 and c_2 from IVP

$$\begin{cases} 1 = y(1) = c_1 \cdot 1^{-0.5} + c_2 \cdot 1^{-1.5} \\ -1.5 = y'(1) = -0.5 c_1 \cdot 1^{-1.5} - 1.5 c_2 \cdot 1^{-2.5} \end{cases}$$

$$\begin{cases} 1 = c_1 + c_2 \\ -1.5 = 0.5 c_1 - 1.5 c_2 \end{cases}$$

$$\begin{cases} 1 = c_1 + c_2 \\ 3 = c_1 + 3c_2 \end{cases}$$

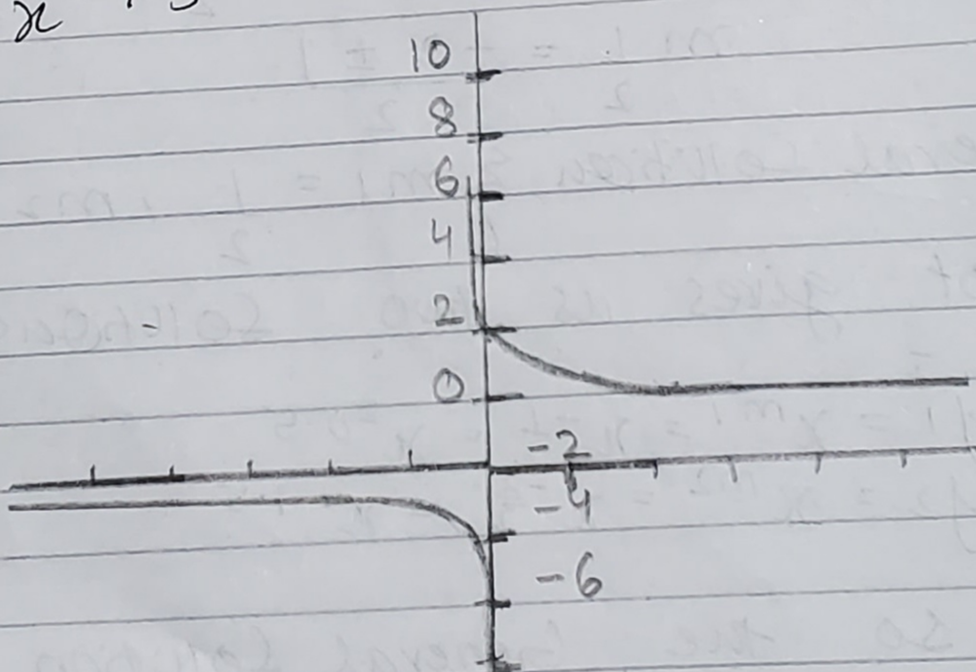
$$\begin{cases} 1 - c_2 = c_1 \\ 3 = 1 - c_2 + 3c_2 \end{cases}$$

$$\begin{cases} 1 - c_2 = c_1 \\ 2 = 2c_2 \end{cases}$$

$$\begin{cases} 1 - c_2 = c_1 \\ 1 = c_2 \end{cases}$$

$$\begin{cases} 0 = c_1 \\ 1 = c_2 \end{cases}$$

$$y = x^{-1.5}$$



3 $x^2y'' + xy' + 9y = 0$ $\left[\begin{array}{l} y(1) = 0 \\ y'(1) = 2.5 \end{array}\right]$

* Solution:-

$$y = x^m, \; y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$$
$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{2}} + mx^m + 9x^m = 0$$

* x^m as a common factor

$$m(m-1) + m + 9 = 0$$
$$m^2 - \cancel{m} + \cancel{m} + 9 = 0$$
$$m^2 + 9 = 0$$

* Finding roots of equation.

$$m^2 + 9 = 0, \Rightarrow m^2 - (3i)^2 = 0$$
$$(m - 3i)(m + 3i) = 0$$

* complex conjugate Roots

$$(m_1 = 3i \; \wedge \; m_2 = -3i)$$
$$x = e^{\ln x}$$
$$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i \ln x}$$
$$e^z = e^{a+ib} = e^a(\cos b + i \sin b) \; z \in C$$

So we have :-

$$e^{3i \ln x} = e^0(\cos(3\ln x) + i\sin(3\ln x)) = \cos(3\ln x) - i\sin(3\ln x)$$

This gives $\cancel{*}$

$$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x)$$
$$x^{m_2} = \cos(3\ln x) + i\sin(3\ln x)$$

* Now Adding two formulas

$x^{m_1} + x^{m_2} = \cos(3\ln x) + \cancel{i\sin(3\ln x)} + \cos(3\ln x)$
$\cancel{- i\sin(3\ln x)}$
$= 2\cos(3\ln x)$

* Dividing it by 2

$$\frac{x^{m_1} + x^{m_2}}{2} = \frac{2\cos(3\ln x)}{2} = \cos(3\ln x)$$

* Subtracting 2^{nd} formula from 1^{st}

$x^{m_1} - x^{m_2} = \cancel{\cos(3(\ln x))} + i\sin(3\ln x) +$
$\cancel{\cos(3\ln x)}\ i\ \sin(3\ln(x))$
$= 2\sin(3\ln x)$

* Dividing it by $2i$

$$\frac{x^{m_1} - x^{m_2}}{2i} = \frac{2\cancel{i}\sin(3\ln x)}{2\cancel{i}} = \sin(3\ln x)$$

* Quotient is not constant so the solution is

$y_1 = \cos(3\ln x) \wedge y_2 = \sin(3\ln x)$

* General solution is

$y = c_1y_1 + c_2y_2$
$= c_1\cos(3\ln x) + c_2\sin(3\ln x)$

$y' = -c_1\sin(3\ln x) \cdot (3\ln x)' + c_2\cos(3\ln x) \cdot (3\ln x)'$

$$= -\frac{3c_1}{x}\sin(3\ln x) + \frac{3c_2}{x}\cos(3\ln x)$$

Now all we need is to determine c_1 and c_2 from IVP

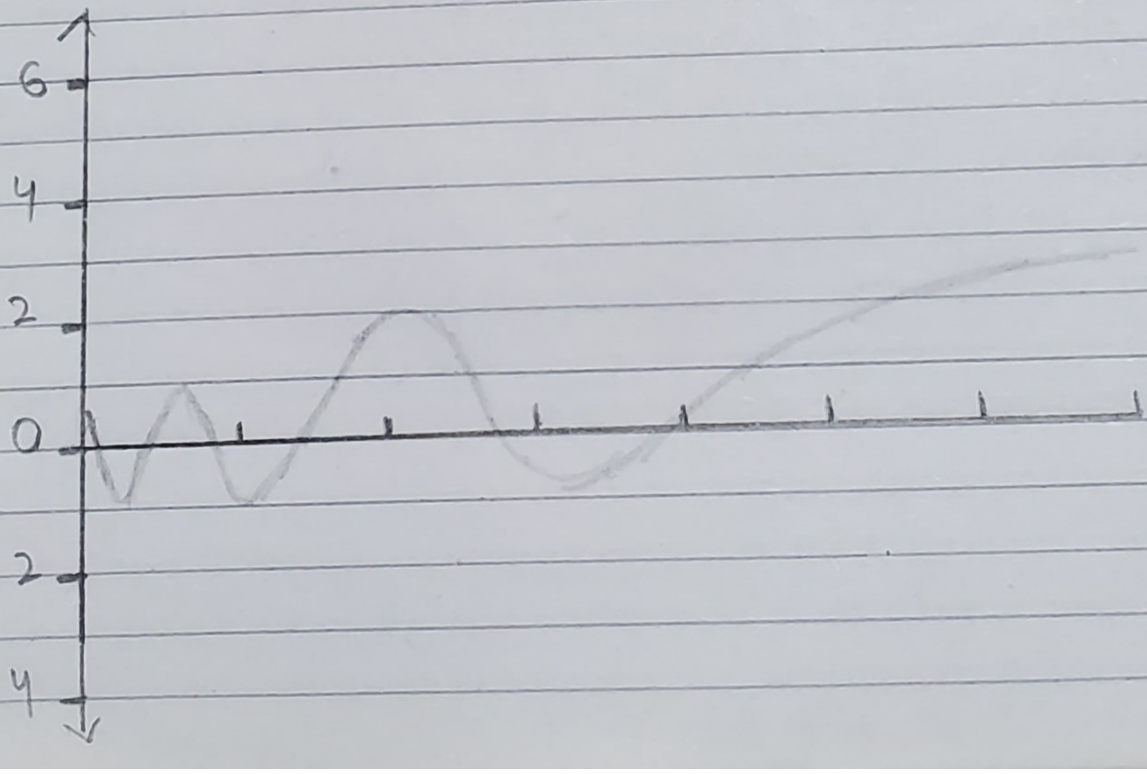
$$\begin{cases} 0 = y(1) = c_1 \cos(3\ln 1) + c_2 \sin(3\ln 1) \\ 2.5 = y'(1) = -3c_1 \sin(3\ln 1) + 3c_2 \cos(3\ln 1) \end{cases}$$

$$\begin{cases} 0 = c_1 \cos(0) + c_2 \sin(0) \\ 2.5 = -3c_1 \sin(0) + 3c_2 \cos(0) \end{cases}$$

$$\begin{cases} 0 = c_1 \\ 5/2 = 3c_2 \end{cases}$$

$$\begin{cases} 0 = c_1 \\ 5/6 = c_2 \end{cases}$$

$$y = \frac{5}{6} \sin(3 \ln x)$$



(4) $x^2y'' + 3xy' + y = 0$ $\left[\begin{array}{l} y(1) = 3.6 \\ y'(1) = 0.4 \end{array}\right]$

* Solution :-

$y = x^m, \; y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} + 3mx^m + x^m = 0$

$x^2 m(m-1)x^m \cdot x^{-2} + 3mx^m + x^m = 0$

$x^2 m(m-1)x^m$

$\therefore x^m$ is a common factor

$m(m-1) + 3m + 1 = 0 \Rightarrow m^2 + m + 3m + 1 = 0 \Rightarrow m^2 + 2m + 1 = 0$

* Finding Roots

$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$

$m = 1$

*Real solution is

$y_1 = x^m = x^{-1} = \frac{1}{x}$

* Using Reduction Order

$y'' + \frac{3}{x} \cdot y' + \frac{1}{x^2} \cdot y = 0$

$p(x) = 3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = 3\ln|x|$

*putting $y_2 = uy_1$

Where, $u = \int U\,dx \;\wedge\; U = \frac{1}{y_1^2} e^{-\int p\,dx}$

* finding U

$e^{-\int p\,dx} = e^{-3\ln|x|} = (e^{\ln|x|})^{-3} = x^{-3}$

$\Rightarrow U = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$

$u = \int \frac{dx}{x} = \ln|x|$

So, $y_2 = uy_1 = y_1 \ln x = \frac{1}{x} \cdot \ln x$

* The General solution is

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 \cdot \frac{1}{x} + c_2 \cdot \frac{1}{x} \cdot \ln x$$
$$= \frac{1}{x} \cdot (c_1 + c_2 \ln x)$$
$$y' = (x^{-1})'(c_1 + c_2 \ln x) + x^{-1}(c_1 + c_2 \ln x)'$$
$$= -x^{-2}(c_1 + c_2 \ln x) + 1/x\, c_2 \cdot 1/x$$
$$= 1/x^2 (-c_1 - c_2 \ln x + c_2)$$

* Determining c_1 and c_2 from IVP

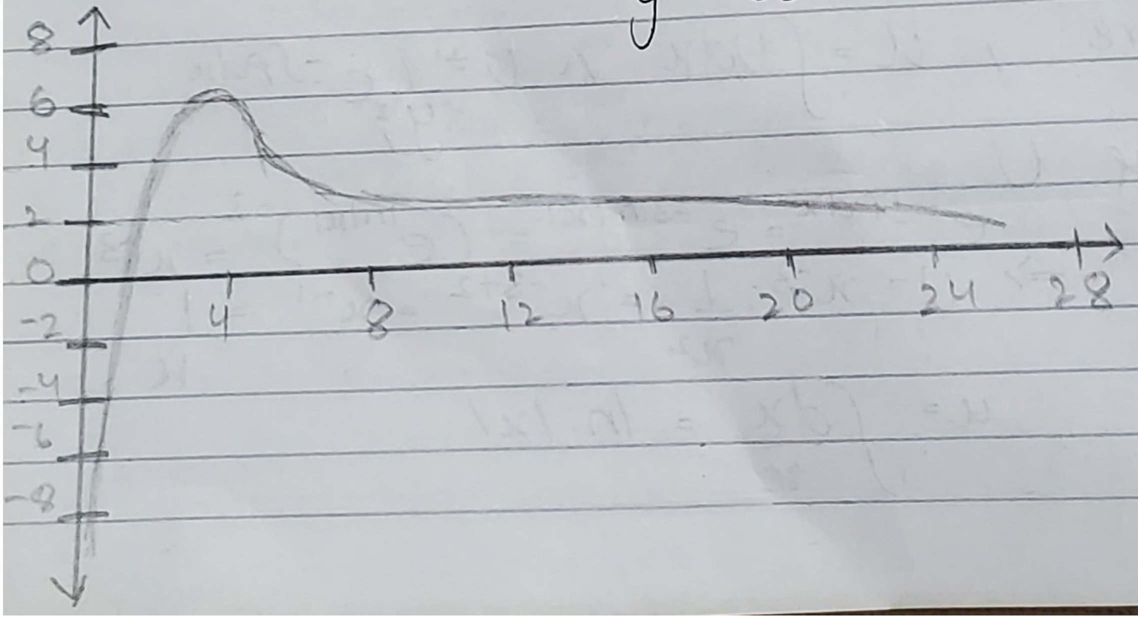
$$\begin{cases} 3.6 = y(1) = 1/1\,(c_1 + c_2 \ln 1) \\ 0.4 = y'(1) = 1/1^2(-c_1 - c_2 \ln 1 + c_2) \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = -c_1 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 0.4 = -3.6 + c_2 \end{cases}$$

$$\begin{cases} 3.6 = c_1 \\ 4.0 = c_2 \end{cases}$$

$$y = (3.6 + 4.0 \ln x) \cdot \frac{1}{x}$$



⑤ $(x^2D^2 - 3xD + 4I)y = 0$ $\left[\begin{matrix} y(1) = -\pi \\ y'(1) = 2\pi \end{matrix}\right]$

Solution:

$x^2D^2y - 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y$
$= x^2y'' - 3xy' + 4y$

* substituting $\Rightarrow y = x^m, y' = mx^{m-1}, y'' = m(m-1)x^{m-2}$

$x^2m(m-1)x^{m-2} - 3xmx^{m-1} + 4xm = 0$

$x^2 m(m-1)x^m \cdot x^{-2} - 3xmx^m\cancel{x^{-1}} + 4x^m = 0$

$\cancel{x^2}\, m(m-1)x^m$ ... common factor

$\therefore x^m$ as a common factor

$m(m-1) - 3m + 4 = 0 \Rightarrow m^2 - 4m + 4 = 0$

* Finding Root

$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$

$y_1 = x^m = x^2$

* Using Reduction Order

$y'' - \frac{3}{x} \cdot y' + \frac{4}{x^2} \cdot y = 0$

$p(x) = -3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = -3 \ln|x|$

$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p\,dx}$

Lets find U:

$e^{-\int p\,dx} = e^{3\ln|x|} = (e^{\ln|x|})^3 = x^3$

$\Rightarrow U = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$

By integration we have

$u = \int \frac{dx}{x} = \ln|x|$

So, $y_2 = uy_1 = y_1 \ln x = x^2 \ln x$

* General Solution is

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 x^2 + x^2 \ln x$$
$$= x^2 (c_1 + c_2 \ln x)$$

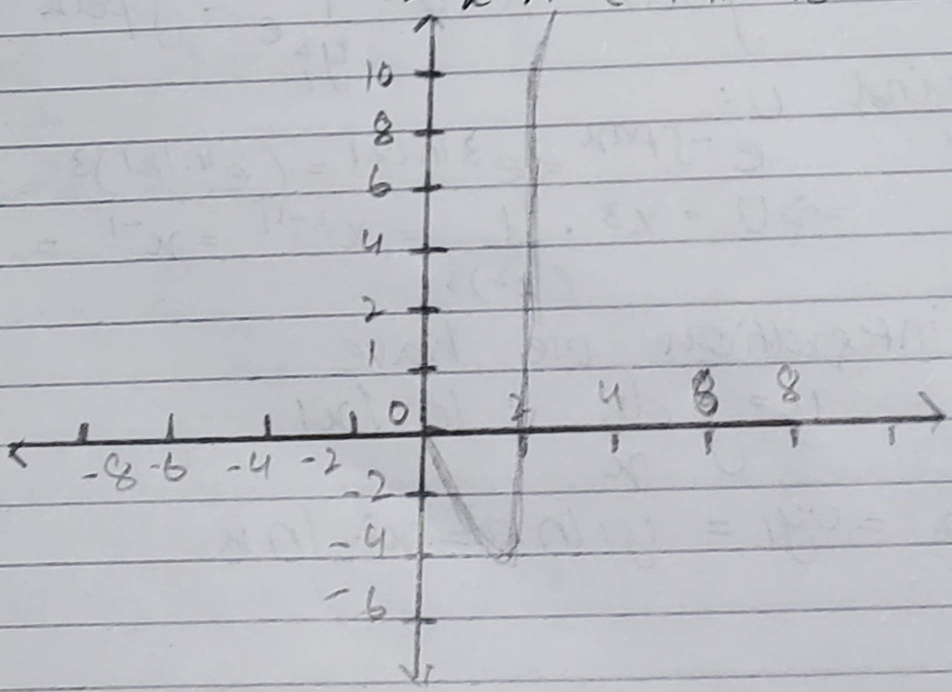
$$\Rightarrow y' = (x^2)'(c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$$
$$= 2x (c_1 + c_2 \ln x) + c_2 x^2 \cdot \frac{1}{x}$$
$$= 2c_1 x + 2c_2 x \ln x + c_2 x$$
$$= 2c_1 x + c_2 x (2 \ln x + 1)$$

* Determining c_1 and c_2 from IVP

$$\begin{cases} -\pi = y(1) = 1^2 (c_1 + c_2 \ln 1) \\ 2\pi = y'(1) = 2c_1 + c_2 (2 \ln 1 + 1) \end{cases} \Rightarrow \begin{cases} -\pi = c_1 \\ 2\pi = 2c_1 + c_2 \end{cases}$$

$$\begin{cases} -\pi = c_1 \\ 4\pi = c_2 \end{cases}$$

$$y = x^2 (-\pi + 4\pi \ln x)$$
$$= x^2 \pi (4 \ln x - 1)$$



⑥ $(x^2D^2 + xD + I)y$ $\left[\begin{array}{l} y(1) = 1 \\ y'(1) = 1 \end{array}\right]$

* Solution:-

$x^2D^2y + xDy + Iy = x^2D(Dy) + xDy + y$

$= x^2y'' + xy' + y$

$x^2y'' + xy' + y = 0$

$y = x^m, \; y' = mx^{m-1}, \; y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} + \cancel{x} m x^{m-1} + x^m = 0$

$\cancel{x^2} m(m-1)x^m \cdot x^2 + xmx^m \cancel{x^{-1}} + x^m = 0$

* x^m as a common factor

$m(m-1) + m + 1 = 0 \Rightarrow m^2 - m + m + 1 = 0$

$m^2 + 1 = 0$

* Finding Roots

$m^2 + 1 = 0 \Rightarrow m^2 - i^2 = 0 \Rightarrow (m-i)(m+i) = 0$

* Complex Conjugate Roots

$m_1 = i \wedge m_2 = i$

* Using $x = e^{\ln x}$:

$x^{m_1} = x^i = (e^{\ln x})^i = e^{i\ln x}$

$x^{m_2} = x^{-i} = (e^{\ln x})^{-i} = e^{-i\ln x}$

$e^z = e^{a+ib} = e^a(\cos b + i\sin b)$

So we have,

$e^{i\ln x} = e^0(\cos(\ln x) + i\sin(\ln x))$

$= \cos(\ln x) + i\sin(\ln x)$

$e^{-3i\ln x} = e^0(\cos(\ln x) + i\sin(\ln x))$

$= \cos(\ln x) - i\sin(\ln x)$

This gives $x^{m_1} = \cos(\ln x) + i\sin(\ln x)$

$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$

$x^{m_1} + x^{m_2} = \cos(\ln x) + \cancel{i\sin(\ln x)} + \cos(\ln x)$

$- \cancel{i\sin(\ln x)} \; \cancel{=} c$

$= 2\cos(\ln x)$

* Dividing it by 2

$$\frac{x^{m_1} + x^{m_2}}{2} = \frac{\cancel{2}\cos(\ln x)}{\cancel{2}} = \cos(\ln x)$$

$$x^{m_1} - x^{m_2} = \cos(\cancel{\ln x}) + i\sin(\ln x) - \cancel{\cos(\ln x)}$$
$$+ i\sin(\ln x)$$
$$= 2i\sin(\ln x)$$

* Dividing by $2i$

$$\frac{x^{m_1} - x^{m_2}}{2i} = \frac{\cancel{2i}\sin(\ln x)}{\cancel{2i}} = \sin(\ln x)$$

$y_1 = \cos(\ln x) \wedge y_2 = \sin(\ln x)$

$y = c_1y_1 + c_2y_2 = c_1\cos(\ln x) + c_2\sin(\ln x)$

$\Rightarrow y' = -c_1\sin(\ln x)\cdot(\ln x)' + c_2\cos(\ln x)\cdot(\ln x)'$

$= -\frac{c_1}{x}\sin(\ln x) + \frac{c_2}{x}\cos(\ln x)$

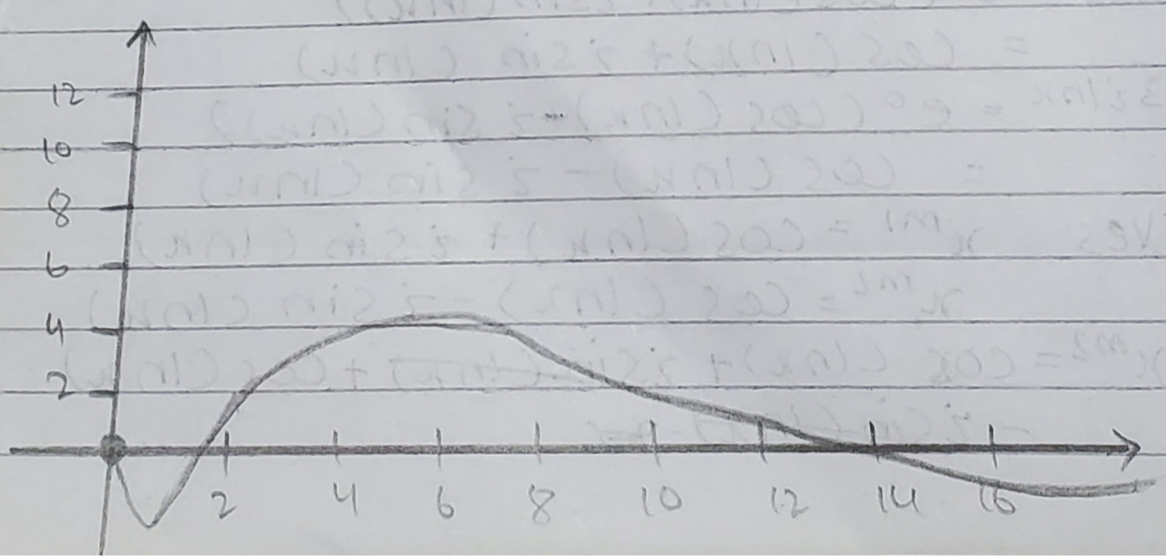
* Determine c_1 & c_2 from IVP

$$\begin{cases} 1 = y(1) = c_1\cos(\ln 1) + c_2\sin(\ln 1) \\ 1 = y'(1) = -c_1\sin(\ln 1) + 3c_2\cos(\ln 1) \end{cases}$$

$$\begin{cases} 1 = c_1\cos(0) + c_2\sin(0)) \\ 1 = -c_1\sin(0) + c_2\cos(0)) \end{cases} \Rightarrow \begin{cases} 1 = c_1 \\ 1 = c_2 \end{cases}$$

$$y = \sin(\ln x) + \cos(\ln x)$$

$$\{y = \sin(\ln x) + \cos(\ln x)\}$$



⑦ $(9x^2D^2 + 3xD + I)y = 0$ $\left[\begin{matrix} y(1) = 1 \\ y'(1) = 0 \end{matrix}\right]$

* Solution.

$9x^2D^2y + 3xDy + Iy = 9x^2D(Dy) + 3xDy + y$

$= 9x^2y'' + 3xy' + y$

$9x^2y'' + 3xy' + y = 0$

$y = x^m, y' = mx^{m-1}, y'' = m(m-1)x^{m-2}$

$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$

$9x^2m(m-1)x^m \cdot x^2 + 3xmx^m x^{-1} + x^m = 0$

* x^m as a common factor

$9x^2m(m-1)x^{m-2} + 3x\cancel{m}\cancel{x}x^{m-1} + x^m = 0$

$9x^2m(m-1)x^m \cdot x^{\cancel{-2}} + 3xmx^m \cancel{x^{-1}} + x^m = 0$

$9\cancel{x^2}m(m-1)x^m \ldots$ $9m^2 - 9m + 3m + 1 = 0$

$9m(m-1) + 3m + 1 = 0 \Rightarrow 9m^2 - 9m + 3m + 1 = 0$

$9m^2 - 6m + 1 = 0$

* Finding Roots

$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0$

$9m^2 - 6m + 1 = 0 \Rightarrow m\frac{1}{2} = \frac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18}$

$m\frac{1}{2} = \frac{6}{18} = \frac{1}{3}$

$y_1 = x^m = x^{\frac{1}{3}}$

$y'' + \frac{1}{3x} \cdot y' + \frac{1}{9x^2} \cdot y = 0$

$p(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int p\,dx = \frac{1}{3} \ln|x|$

$y_2 = uy_1$

where,

$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p\,dx}$

*finding U

$e^{-\int p\,dx} = e^{-\frac{1}{3}\cdot \ln|x|} = \left(e^{\ln|x|}\right)^{-\frac{1}{3}} = x^{-\frac{1}{3}}$

$U = x^{-\frac{1}{3}} \cdot \frac{1}{(x^{\frac{1}{3}})^2} = x^{-\frac{1}{3}-\frac{2}{3}} = x^{-1} = \frac{1}{x}$

$u = \int \frac{dx}{x} = \ln|x|$

so, $y_2 = u y_1 = y_1 \ln x = x^{\frac{1}{3}} \ln x$

*general solution:

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^{\frac{1}{3}} + x^{\frac{1}{3}} \ln x$

$= x^{\frac{1}{3}} (c_1 + c_2 \ln x)$

$\Rightarrow y' = (x^{\frac{1}{3}})'(c_1 + c_2 \ln x|) + x^{\frac{1}{3}} (c_1 + c_2 \ln x)'$

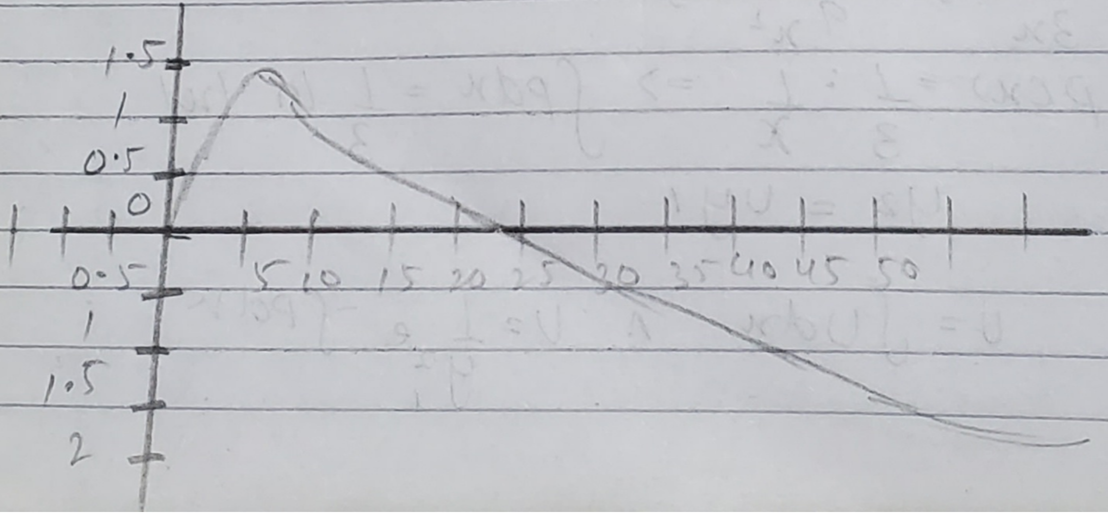
$= \frac{1}{3} \cdot x^{-\frac{2}{3}} (c_1 + c_2 \ln x) + x^{\frac{1}{3}} c_2 x \cdot \frac{1}{x}$

$= \frac{1}{3} \cdot x^{-\frac{2}{3}} (c_1 + c_2 \ln x) + x^{\frac{1}{3}} c_2$

$\begin{cases} 1 = y(1) = 1^{\frac{1}{3}} (c_1 + c_2 \ln 1) \\ 0 = y'(1) = 1/3 \cdot 1^{-\frac{2}{3}} (c_1 + c_2 \ln 1) + 1^{1/3} c_2 \end{cases}$

$\begin{cases} 1 = c_1 \\ 0 = c_1/3 + c_2 \end{cases}$ $\quad \begin{cases} 1 = c_1 \\ -1/3 = c_2 \end{cases}$

$y = x^{\frac{1}{3}} \left(1 - \frac{1}{3} \ln x\right)$



8 $(x^2D^2 - xD - 15I)y = 0$ $\left[\begin{matrix} y(1) = 0 \\ y'(1) = 4.5 \end{matrix}\right]$

Solution:-

$$x^2D^2y - xDy - 15Iy = x^2D(Dy) - xDy - 15y$$
$$= x^2y'' - xy' - 15y$$
$$x^2y'' - xy' - 15y = 0$$
$$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$$
$$x^2m(m-1)x^{m-2} - xmx^{m-1} - 15x^m = 0$$
$$\cancel{x^2}m(m-1)x^m \cdot x^{-2} - xmx\,\cancel{x^{-1}} - 15x^m = 0$$

* x^m As a Common Factor

$$m(m-1) - m - 15 = 0 \Rightarrow m^2 - 2m - 15 = 0$$

* Finding Roots

$$m^2 - 2m - 15 = 0 \Rightarrow m\tfrac{1}{2} = \frac{2 \pm \sqrt{(-2)^2 + 4\cdot 15}}{2}$$
$$= m\tfrac{1}{2} = \frac{2 \pm 8}{2}$$

$$m1 = 5 \wedge m2 = -3$$
$$y1 = x^{m1} = x^5 \wedge y2 = x^{m2} = x^{-3}$$

* General Solution.

$$y = c_1y_1 + c_2y_2 \Rightarrow c_1x^5 + c_2x^{-3}$$
$$y' = 5c_1x^4 - 3c_2x^{-4}$$

* Determining C_1 & C_2 from IVP

$$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 \cdot 1^{-3} \\ -4.5 = y'(1) = 5c_1 \cdot 1^4 - 3c_2 \cdot 1^{-4} \end{cases}$$

$$\begin{cases} 0.1 = c_1 + c_2 \\ -4.5 = 5c_1 - 3c_2 \end{cases}$$

$$\begin{cases} 0.1 - C_2 = C_1 \\ -4.5 = 5(0.1 - C_2) - 3C_2 \end{cases}$$

$$\begin{cases} 0.1 - c_2 = c_1 \\ 5 = 8c_2 \end{cases}$$

$$\begin{cases} 0.1 - c_2 = c_1 \\ 0.625 = c_2 \end{cases}$$

$$\begin{cases} 0.1 - 0.625 = c_1 \\ 0.625 = c_2 \end{cases}$$

$$\begin{cases} -0.525 = c_1 \\ 0.625 = c_2 \end{cases}$$

$$y = -0.525x^5 + 0.625x^{-3}$$