FINAL TERM-FINAL EXAM

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SUBJECT:
“CALCULUS AND ANALYTICAL GEOMETRY”

Date:

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Q1 (A) Differentiate $\frac{3x^4 - 2x^3 + 5}{x^3 + 1}$ with respect to x

Solution:-

$$y = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

Diff w.r.t x.

$$\frac{dy}{dx} = \frac{(x^3+1)\, d/dx\,(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5)\, d/dx\,(x^3+1)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^6 - \cancel{6x^5} + 12x^3 - 6x^2 - 9x^6 + \cancel{6x^5} - 15x^2}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{3x^6 + 12x^3 - 21x^2}{(x^3+1)^2}$$

$$= \frac{3x^2(x^4 + 4x - 7)}{(x^3+1)^2} \quad \text{(Ans)}$$

Date:

1(b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to X?

Solution:-

$$y = \frac{(x^3+1)^2}{x^3-1}$$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{(x^3-1)\,d/dx\,(x^3+1)^2 - (x^3+1)^2\,d/dx\,(x^3-1)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3x^2\,(x^3+1)\,[2(x^3-1) - (x^3+1)]}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3x^2(x^3+1)\,[2x^3-2-x^3-1]}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3x\,(x^3+1)\,[x^3-3]}{(x^3-1)^2}$$ Ans.

Date:

Q2(A) Find the Integration of $\int \frac{1}{\sqrt{x^5}} dx$.

Solution:

$$\int \frac{1}{\sqrt{x^5}} dx$$

$$\int \frac{1}{(x^5)^{1/2}} dx$$

$$\int \frac{1}{x^{5/2}} dx$$

$$\int x^{-5/2} dx$$

$$\frac{x}{-5/2+1} + C$$

$$\frac{x^{\frac{-5+2}{2}}}{\frac{-5+2}{2}} + C$$

$$\frac{x^{-3/2}}{-3/2} + C$$

$$-\frac{2}{2} x^{-3/2} + C$$

$$-\frac{2}{3} \frac{1}{x\sqrt{x}} + C \rightarrow \text{Ans}$$

Date:

2b Find the Integration of $\int \frac{1}{(8x+7)^8} dx$

Solution:-

$$\int \frac{1}{(8x+7)^8} dx$$

$$\int (8x+7)^8 dx$$

Multiply and divide by 8

$$\frac{1}{8} \int (8x+7)^{-8} . dx$$

$$\frac{1}{8} \frac{(8x+7)^{-8+1}}{-8+1} + C$$

$$\frac{1}{8} \frac{(8x+7)^{-7}}{-7} + C$$

$$\frac{-1}{56} (8x+7)^{-7} + C$$

$$-\frac{1}{56} \frac{1}{(8x+7)^7} + C$$ (Ans)

Date:

Q3(A) Find the Integration of $\int \frac{-x+9}{2x^2-8x+6} dx$ by partial fractions?

Solution:

$$\int \frac{-x+9}{2x^2-8x+6} dx$$

$$\frac{-x+9}{2x^2-2x-6x+6} = \frac{x-9}{2x(x-1)-6(x-1)}$$

$$\frac{-x+9}{(x-1)(2x-6)} = \frac{A}{(x-1)} + \frac{B}{2x-6}$$

Multiply by both sides.
$(x-1)(2x-6)$

$$x+9 = A(2x-6) + B(x-1) - C$$

$$x-1 = 0$$

$$x = 1$$

$$-(1)+9 = A(2(1)-6) + B(1-1)$$

$$1+9 = A(2-6) + 0$$

$$10 = A(-4)$$

$$10 = -4A$$

$$-\frac{\cancel{10}\,5}{\cancel{4}\,2} = A$$

$$-\frac{5}{2} = A$$

Date:

$$\int \frac{-12}{x^2+2x+3}\,dx$$

$$-12\int \frac{1}{x^2+2x+3}\,dx$$

Solving for Integral

$$\int \frac{1}{x^2+2x+3}\,dx = \int \frac{1}{(x+1)^2+2}\,dx$$

$$y = \frac{x+1}{\sqrt{2}} \rightarrow$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}}(x+1)\frac{d}{dx} = \frac{1+0}{\sqrt{2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$$

Date:

So, $\frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$

$$\int \frac{4}{x^2+1}\,dx + \int \frac{-12}{x^2+2x+3}\,dx$$

$$\int \frac{4}{x^2+1}\,dx = 4\int \frac{1}{x^2+1}\,dx$$

Standard Integral for $\frac{1}{x^2+1} = \arctan(x)$

So

$$4\int \frac{1}{x^2+1}\,dx$$

$$= 4\arctan(x) + C$$

Substituing x

$$= \int \frac{\sqrt{2}}{2x^2+2}\,dx = \frac{1}{\sqrt{2}} \int \frac{1}{x^2} + 1 \; dx$$

$$\int \frac{1}{x^2+1}\,dx = \arctan(x)$$

So $\frac{1}{\sqrt{2}} (\arctan(x))$

$$= \arctan \frac{(x+1/\sqrt{2})}{\sqrt{2}}$$

$$-3 \cdot 2^{3/2} \arctan\left(\frac{2x+2}{2^{3/2}}\right) + c$$

Replacing x

$$\Rightarrow -3 \cdot 2^{3/2} \arctan(x+1/\sqrt{2}) + c$$

Final Answer

Date:

4(B) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

Solution:

$$X = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5-0 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$ Answer.

(c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Solution:

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$ Answer.

Date:

Q5 A) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Finds $A^2 + BC$

Solution: $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+8 & 4+2 \\ 2+2 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 2 \end{bmatrix}$$

$A^2 + BC$

$$\begin{bmatrix} 9 & 8 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 8 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 8 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$