Ans (1)

$$dy/dt = e^{y-t} \cdot Sec(y)(1+t^2)$$

$$y(0) = 0$$

Solution

$$dy/dx = e^{y} \cdot e^{-t} \cdot 1/\cos y \, (1+t^2)$$

$$e^{-y} \cdot \cos y \cdot dy = e^{-t}(1+t^2)\,dt$$

"Integrating"

$$\int e^{-y} \cos y \cdot dy = \int (1+t^2) e^{-t} \cdot dt$$

$\Rightarrow$ (Using Integrating by Parts)

$$e^{-y}\int \cos y \cdot dx - \int \cos y \cdot d/dy \, e^{-y} = (1+t^2)\int e^{-t} \cancel{\frac{d}{dt}(1+t^2)}$$

$$= (1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \frac{d}{dt}(1+t^2)\right) \text{---} (i)$$

Taking L.H.S

$e^{-y} \int \cos y \cdot dx - \int \left(\int \cos y \cdot \frac{d}{dy} \cdot e^{-y} \right.$

$e^{-y} \sin y - \int (\sin y \cdot e^{-y} (-1))$

$e^{-y} \cdot \sin y + \int (\sin y \cdot e^{-y})$

$e^{-y} \cdot \sin y + \int (e^{-y} \cdot \sin y)$ — (ii)

Again Using Integration by parts

eq(ii) $\Rightarrow$ $e^{-y} \sin y + e^{-y}(-\cos y) - \int \left(\sin y \cdot \frac{d}{dy} e^{-y} \right)$

$e^{-y} \cdot \sin y - e^{-y} \cos y - \int \left(-\cos y \cdot \frac{e^{-y}}{-1} \right)$

$e^{-y} \cdot \sin y - e^{-y} \cdot \cos y - \int (\cos y \cdot e^{-y})$

Since $\int (\cos y . e^{-y}) = L \cdot H \cdot S$

Now again same to the first to the first one So L.H.S

$$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$$

$$L.H.S = e^{-y}(\sin y - \cos y)$$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2} \quad \text{—— (iii)}$$

Now taking R.H.S

$$\int (1+t^2) e^{-t} . dt$$

$$(1+t^2)\int e^{-t} - \int \left(\int e^{-t} . \frac{d}{dt}(1+t^2)\right)$$

$$-(1+t^2)e^{-t} - \int (-e^{-t}(2t))$$

$$-(1+t^2)e^{-t} + \int (2t)e^{-t} \quad \text{—— (iv)}$$

⇒ Integration again "by parts"

eq (iii)

$$-(1+t^2)e^{-t} + \left(2t \int e - \int \left(\int e^{-t} . \frac{d}{dt} . 2t\right)\right)$$

$-(1+t^2)e^{-t} + \left(-2te^{-t} + \int (2e^{-t}\right)$

$-(1+t^2)e^{-t} + \left(-2te^{-t} - 2e^{-t}\right) + c$

$-(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$

$-e^{-t} - e^{-t} \cdot t^2 - 2te^{-t} - 2e^{-t} + c$

$-(t^2 + 2t + 3)e^{-t} + c = R.H.S$

Now putting value in eq "i"

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2 + 2t + 3)e^{-t} + c \rightarrow (v)$$

$e^{-0}(\sin(0) - \cos(0)) = ((0)^2 + 2(0) + 3)e^{-0} + c$ | As $t = 0$, $y = 0$

$\cancel{1/2\,(0-1)\,\cos(0)}$

$1/2\,(0-1) = -3 + c$

$-1/2 + 3 = c$

$\Rightarrow \boxed{c = 5/2}$

By putting in eq "V"

$$\frac{e^{-y}(\sin y - \cos y)}{2} = (t^2+2t+3)e^{-t} + 5/2$$

(Required Solution)

Ans (02);

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

Solution

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx=\left(\sqrt{x+y}-\sqrt{x-y}\right)dy$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad \text{———— (i)}$$

$\Rightarrow$ eq "i" is a homogenous equation.

By putting $y=vx$, $v=y/x$

$$\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}$$

$$v+x\cdot\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$= 1 + \cancel{v} + \frac{(-\cancel{v} + 2\sqrt{1-v^2})}{2v}$$

$$v + x.dv/dx = \frac{\cancel{x}(1+\sqrt{1-v^2})}{\cancel{x}v}$$

$$x.\ dv/dx = \frac{1+\sqrt{1-v^2}}{v} - v$$

taking L.C.M

$$x.\ dv/dx = \frac{1+\sqrt{1-v^2} - v}{v}$$

$$x.\ dv/dx = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{v}$$

$$\frac{v.dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{1}{x}.dx$$

" Integrating "

$$\int \frac{v.dv}{\sqrt{1-v^2}(1+\sqrt{1+v^2})} = \int 1/x.dx$$

By putting $1+\sqrt{1-v^2} = t$

$$= 1/2(1-v^2)^{-1/2}(-2v)dv = dt$$

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$$\frac{v \cdot dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int 1/x \, dx$$

$$-\ln(t) = \ln(x) + \ln(c)$$

$$-\ln(1+\sqrt{1-v^2}) = \ln x \cdot c$$

$$\ln(1+\sqrt{1-v^2}) = \ln x \cdot c$$

$$\ln(1+\sqrt{1-v^2}) = \ln(xc)^{-1}$$

$$\ln(1+\sqrt{1-v^2}) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = (x \cdot c)^{-1}$$

$$1+\sqrt{1-v^2} = 1/x \cdot c$$ Xing b. sides By "x"

$$x(1+\sqrt{1-v^2}) = 1/c$$

$$x+\sqrt{x^2-v^2 x^2} = 1/c$$

put = $(v = y/x)$

$$x+\sqrt{x^2-y^2/x^2 \cdot x^2} = 1/c$$

ANS 03

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x.$$

Sol'

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$f(D)y = f(x)$$

non homogenous linear equation

G. Soloution will be $y = y_c + y_p$

Complementry Solouticn $y_c = ?$

$$D^4 - D^2 = 0 \Rightarrow D^2(D^2 + 1) = 0$$

$$D^2 = 0, \quad D^2 + 1 = 0$$

$$D = 0, \quad D^2 = -1$$

$$D = \sqrt{-1}$$

$$\boxed{D = i}$$

Roots are reall and Complex

$$y_c = C_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$$

Now $yp = 1/f(D) \; F(x)$

$$yp = \frac{1}{D^4 + D^2}\left(3x^2 - 4\sin x - 2\cos x\right)$$

$$yp = \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 - D^2}$$

$$f(0) = D^4 + D^2$$

at $\quad D = 0 \rightarrow f(D) = 0$

So, $f(0) = 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f'(0) = 0$

"Again differentiating,"

$$f''(0) = 12D + 2$$

So, for $D = 0$

$$f''(0) = 12(0) + 2$$
$$= 2$$

Now replacing $\frac{1}{f(0)}$ with $\frac{x^2}{f''(D)}$

$$y_p = \frac{x^2 \, 3x^2}{12D + 2} + \frac{x^2}{12D + 2} \cdot 4\sin x \cdot \frac{x^2}{12D + 2} \cdot 2\cos x$$

By putting $D=0$ in all

$$yP = \frac{x^2\, 3x^2}{12(0)+2} + \frac{x^2}{12(0)+2} \cdot 4\sin x - \frac{x^2}{12(0)2} \cdot 2\cos x$$

$$yP = \frac{3}{2}x^4 + \frac{\cancel{2}4x^2 \cdot \sin x}{2} - \frac{2x^2 \cdot \cos x}{2} \quad \text{—— (ii)}$$

eq (ii)

$$\frac{3}{2}x^4 + 2x^2 \cdot \sin x - x^2 \cdot \cos x$$

By putting in eq ①

$$y = C_1 + C_2 \cos x + C_3 \cdot \cos x + \frac{3}{2}x^2 + 2x^2 \sin x - x^2 \cdot \cos x$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2) \sin x + \frac{3}{2}x^4$$

—— x —— x —— X