Department of Electrical Engineering
Final – Term Assignment Spring 2020
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Course Details

Course Title: Numerical Analysis **Module:**
Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Find the root of the equation given below by Bisection method, accuracy must be up to three decimal places $$x^3 - x^2 + x - 7 = 0$$	Marks 10 CLO 1
Q2.	(a)	Use Regula-Falsi method to compute the root of the following equation in the interval [0, 1] after third iteration. $$f(x) = \cos x - xe^x$$	Marks 07 CLO 1
	(b)	Use Regula-Falsi (method of false position) to solve the following equation, accuracy must be up to four decimal places. $$x^3 - 4x - 9 = 0$$	Marks 07 CLO 2
Q3.	(a)	Find the real root of the following equation using Newton-Raphson method in the interval [2,3] after third iteration. $$x^3 - 3x - 5 = 0$$	Marks 08 CLO 2
	(b)	Solve the following equation by using Muller's method, only perform three iterations. ($x_0 = 0.5$, $x_1 = 1$, $x_2 = 0$) $$x^3 - 7x^2 + 14x - 6$$	Marks 08 CLO 2
Q4.	(a)	Using Gaussian Elimination method, solve the following system of equations $$2x - y + 2z = 2$$ $$x + 10y - 3z = 5$$ $$x - y - z = 3$$	Marks 10 CLO 1

Muhammad Maaz Akhunzada ID 11448 Pg No: 1

Q No 1:-

Find the root of the equation given below by bisection Method, accuracy must be upto three decimals places.

$$x^3 + x^2 + x - 7 = 0$$

Sol:-

$$f(x) = x^3 - x^2 + x - 7 = 0$$

Step 1:

Assume Limit

$$f(1) = (1)^3 - (1)^2 + (1) - 7 = 0$$

$$= 1 - \cancel{1} + \cancel{1} = 7$$

$$\boxed{= -6}$$

$$f(2) = (2)^3 - (2)^2 + (2) - 7$$

$$= 8 - 4 + 2 - 7$$

$$\boxed{= -1}$$

$$f(3) = (3)^2 - (3)^2 + 3 - 7$$

$$= 27 - 9 + 3 - 7$$

$$\boxed{= 14}$$

$$[2, 3) = f(2) \times f(3)$$

$= (-1)(14)$

$\boxed{= -14 < 0}$

$C = \frac{2+3}{2} = \frac{5}{2}$

$\boxed{= 2.5}$

Step 2:

Mid Points:

$C = \frac{2+3}{2} = \frac{5}{2}$

$\boxed{= 2.5}$

$f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7$

$\boxed{= 4.875}$

$f(2) \times f(2.5) = (-1) \times (4.875)$

$\boxed{= -4.875 < 0}$

Step 3:

Mid Point

$C = \frac{2+2.5}{2}$

$= \frac{4.5}{2}$ $\boxed{= 2.25}$

Mohammad Maaz Akhunzada 11448 Pg No 3

$f(2.25) = (2.25)^3 - (2.25)^2 + (2.25) - 7$

$= \boxed{1.5181}$

$f(2) \times f(2.25) = (-1 \times 1.5781)$

$= (-1.5781) < 0$

Step 4:

Midpoints:

$c = \frac{2.25}{2}$

$\boxed{= 2.125}$

$f(2.125) = (2.125)^3 - (2.125)^2 + (2.0625) - 7$

$\boxed{= -0.4177}$

$f(2) \times f(2.0625) = (1) \times (0.4177)$

$\boxed{= +0.4177 > 0}$

Root of the equation lies in limit $(2, 2.125)$ ie

$\boxed{0.20507}$

Muhammad Maaz Akhunzada ID # 11448 Pg # 05

Q2a.

$f(x) = \cos x - xe^x$

Sol

$f(x) = \cos x - xe^x$

$[0, 1] =$

$f(0) = \cos(0) - (0)e^0$

$= 1 - 0$

$\boxed{= 1}$

$f(1) = \cos(1) - (1)e^1$

$= 0.999 - 2.7182$

$\boxed{= -1.7192}$

Here $a = 0$, $b = 1$

$f(a) = 1$, $f(b) = -1.7192$

Formula =

$$\frac{af(a) - bf(b)}{f(b) - f(a)}$$

$$= \frac{0(1) - 1(-1.7192)}{-1.7192 - 1}$$

$$= \frac{+1.7192}{-2.7192}$$

$$\boxed{= -0.6323}$$

$f(-0.6322) = \cos(0.6322) - (0.6322)e^{-(0.6322)}$

$= 0.9999 - (0.6322)(0.5314)$

$= 0.9999 + 0.3359$

$\boxed{= 1.3358}$

Step#2

$a = 0.6322$ $b = 1$

$f(a) = 1.3358$ $f(b) = -1.7192$

Formula:

$$= \frac{af(a) - bf(b)}{f(b) - f(a)}$$

$$= \frac{0.6322(1.3358) - 1(-1.7192)}{-1.7192 - 1.3358}$$

$$= \frac{0.8444 + 1.7192}{-3.055}$$

$$= \frac{2.5636}{-3.055} \quad \boxed{= -0.839}$$

$f(-0.8391) = \cos(0.8391) - (-0.8391)e^{-0.8391}$

$= 0.9998 - (0.8391)(0.4320)$

$0.9998 + 0.3625 \quad \boxed{= 1.3623}$

Step:

$a = -0.83912$ $b = 1$

$f(a) = 1.3623$ $f(b) = -1.7192$

$$= \frac{-0.8391(1.3623) - 1(-1.7192)}{-1.7192 - 1.3623}$$

$$= \frac{1.1431 + 1.7192}{3.0815} \quad \boxed{= 0.9287}$$

Muhammad Maaz Akhunzada ID #11448 Pg # 06

Q2 b.

b. Use Regula Falsi (method of false position) to solve the following equation, accuracy must be up to four decimal places.

$$x^2 - 4x - 9 = 0$$

Solution

$$f(x) = x^3 - 4x - 9$$

$$f(0) = (0)^3 - 4(0) - 9$$

$$\boxed{= -9}$$

$$f(1) = (1)^3 - 4(1) - 9$$

$$\boxed{= -12}$$

$$f(2) = (2)^3 - 4(2) - 9$$

$$= 8 - 8 - 9$$

$$\boxed{= -9 \text{ (negative)}}$$

$$f(3) = (3)^3 - 4(3) - 9$$

$$= 27 - 12 - 9$$

$$\boxed{= 6 \text{ (positive)}}$$

Root lies between [2, 3]

First Approx:

$a = 2 \quad , \quad b = 3$

Using Formula:

$$x = \frac{af(a) - b\,f(b)}{f(b) - f(a)}$$

$$= \frac{2f(2) - 3f(3)}{f(3) - f(2)}$$

$$= \frac{2(-9) - 3(6)}{6 - (-9)}$$

$$= \frac{-36}{-15}$$

$$\boxed{= -2.4}$$

$$f(-2.4) = (-2.4)^3 - 4(-2.4) - 9$$

$$= -5.76 - (-9.6) - 9$$

$$\boxed{= -5.16}$$

Root lies between $f(-2.4)$ & $f(3)$

$a = (-2.4) \quad , \quad b = 3$

$f(-2.4) - 5.16 \quad , \quad f(3) = 6$

$$\frac{af(a) - bf(b)}{f(b) - f(a)}$$

$$= \frac{(2.4)(-5.16) - 3(6)}{6-(-2.4)}$$

$$= \frac{12.384 - 18}{6 + 2.4}$$

$$= \frac{-5.016}{8.4}$$

$$\boxed{= -0.5971}$$

$$f(-0.5971) = (0.5971)^3 - 4(-0.5971) - 9$$

$$= -0.2128 - (-2.284) - 9$$

$$\boxed{= -6.9288}$$

Roots lies b/w $(0.5971, 3)$

$a = 0.5971$, $b = 3$

$f(a) = -6.9288$ $\quad f(b) = 6$

$$= \frac{0.5771(-6.9288) - 3(6)}{6-(-6.9288)}$$

$$= \frac{4.1371 - 18}{12.1288} \quad \boxed{= 2.7448}$$

Muhammad Maaz Akhunzada ID#11448 Pg#009

Q3 a)

$$x^3 - 3x - 5 = 0$$

Sol:-

$$f(x) = x^3 - 3x - 5$$
$$f'(x) = 3x^2 - 3$$

Since Roots lies between [2, 3]

Initial Point:-

$$x_0 = \frac{2+3}{2}$$
$$= \frac{5}{2}$$
$$\boxed{= 2.5}$$

NRM Formula:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{x_n(3x_n^2 - 3) - (x_n^3 + 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{3x_n^3 - 3x_n - x_n^3 - 3x_n - 5}{3x^2 - 3}$$

Iteration 1

$$x_0 = 2.5$$

$$x_{0+1} = \frac{2(2.5)^3 - 6(2.5) - 5}{3(2.5)^2 - 3}$$

$$= \frac{31.25 - 15 - 5}{18.75 - 3}$$

$$= \frac{11.25}{15.75} \quad \boxed{= 0.7142}$$

Iteration 2:

$$x_{1+1} = \frac{2(x_1)^3 - 6x_1 - 5}{3x_1^2 - 3}$$

$$x_2 = \frac{2(0.7142)^3 - 6(0.7142) - 5}{3(0.7142)^2 - 3}$$

$$x_2 = \frac{0.7286 - 4.2852 - 5}{1.5302 - 3}$$

$$x_2 = \frac{-8.5566}{-1.4697} \quad \boxed{= 5.8220}$$

Iteration: 3

$$x_{2+1} = \frac{2(x_2)^3 - 6(x_2) - 5}{3x_2^2 - 3}$$

$$= \frac{2(5.8220)^3 - 6(5.8220) - 5}{3(5.8220)^2 - 3}$$

$$= \frac{394.68 - 34.93 - 5}{101.68 - 3}$$

$$= \frac{354.67}{98.687} \quad \boxed{= 3.594}$$

Q3 b

$$f(x) = x^3 - 7x^2 + 14x - 6$$

$$x_0 = 0.5, \quad x_1 = 1, \quad x_2 = 0$$

$$f(x_0) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6$$

$$f(0.5) = 0.25 - 1.75 + 7 - 6$$

$$\boxed{f(0.5) = -0.625}$$

$$f(x_1) = x_1^3 - 7(1)^2 + 14(1) - 6$$

$$f(1) = 1 - 7 + 14 - 6$$

$$\boxed{f(1) = 2}$$

$$f(x_2) = x_2^3 - 7x_2^2 + 14x_2 - 6$$

$$= (0)^3 - 7(0)^2 + 14(0) - 6$$

$$f(0) = -6$$

$$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$$

$$h_2 = x_2 - x_1 = 0 - 1 = -1$$

$$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{2 - (-0.625)}{0.5} = \frac{2.625}{0.5} = \boxed{5.25}$$

$$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{-6 - (-0.625)}{-1} = \boxed{5.375}$$

$$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{5.375 - 5.25}{-1 + 0.5} = \frac{0.125}{-1.5} \boxed{= -0.0833}$$

$b = a \times h_2 + \delta_2 = -0.0833(-1) + 5.375$
$= 5.4583$

$c = f(x_2) = -6$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 0 + \frac{-2(-6)}{5.4583 \pm \sqrt{(5.4583)^2 - 4(-0.0833)(-6)}}$$

$$= \frac{12}{5.4583 \pm \sqrt{29.79 - (1.9992)}}$$

$$= \frac{12}{5.4583 \pm 5.2717}$$

$$= \frac{12}{0.1866} = 64.30$$

or

$$= \frac{12}{10.73} \quad \boxed{= 1.11}$$

<u>Relative Error</u>:

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\% \quad \left| \frac{1.11 - 0}{1.11} \right| \times 100\%$$

$= 100\%$ error.

Now $x_0 = x_1 = 1$

$x_1 = x_2 = 0$

$x_2 = x_3 = 1.11$

2nd Iteration

$f(x_0) = (1)^3 - 7(1)^2 + 14(1) - 6$

$= 1 - 7 + 14 - 6 = 2$

$f(x_1) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$

$f(x_2) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6$

$= 1.367 - 8.624 + 15.54 - 6$

$= 2.283$

$h_1 = x_1 - x_0 = -6 - 2 = -8$

$h_2 = x_2 - x_1 = 2.283 - 0 = 2.283$

$\delta_1 = \dfrac{f(x_1) - f(x_0)}{h_1} = \dfrac{-6-2}{-8} = 1$

$\delta_2 = \dfrac{f(x_2) - f(x_1)}{h_2} = \dfrac{2.283 - (-6)}{2.283} = \dfrac{8.283}{2.283} \boxed{= 3.628}$

$a = \dfrac{\delta_2 - \delta_1}{h_2 + h_1} = \dfrac{3.6281 - 1}{2.283 + (-8)} = \dfrac{-2.628}{-5.716} \boxed{= -0.459}$

$b = a \times h_2 + \delta_2 = (-0.459)(2.283) + 3.628 \boxed{= 2.580}$

$c = f(x_2) = 2.283$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 0 + \frac{-2(2.283)}{2.580 \pm \sqrt{(2.580)^2 - 4(-0.459)(2.283)}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{6.656 + 4.1915}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{10.84}}$$

$$= \frac{4.566}{2.580 \pm 3.290}$$

$$= \frac{4.566}{5.873} \quad \boxed{= 0.7774}$$

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%$$

$$= \left| \frac{0.7744 - 0}{0.7744} \right| \times 100\% = 100\%$$

Now

$x_0 = x_1 = 0$

$x_1 = x_2 = 1.11$

$x_2 = x_3 = 0.7744$

Muhammad Maaz Akhunzada ID # 11448 Pg # 15

$f(x_0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$

$f(x_1) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6 = 2.283$

$f(x_2) = (0.7744)^3 - 7(0.7744)^2 + 14(0.7744) - 6$

$= 0.418 - 3.874 + 10.416 - 6$

$\boxed{= 0.953}$

$h_1 = x_1 - x_0 = 1.11 - 0 = \boxed{1.11}$

$h_2 = x_2 - x_1 = 0.7744 - 1.11 = \boxed{0.335}$

$$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{2.283 - (-6)}{1.11} = \frac{8.28}{1.11} = \boxed{7.462}$$

$$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{0.953 - 2.283}{0.335} \boxed{= 3.970}$$

$$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{3.970 - 7.462}{-0.335 + 1.11} = -4.505$$

$b = a \times h_2 + \delta_2 = (-4.505) \times (-0.335) + 3.970 = \boxed{5.479}$

$c = f(x_2) = 0.953$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 1.11 + \frac{-2(0.953)}{5.479 \pm \sqrt{(5.479)^2 - 4(-4.505)(0.953)}}$$

$$= 1.11 + \frac{(-1.906)}{5.479 \pm 3.584}$$

$$= 1.11 + \frac{(-1.906)}{9.333}$$

$$= \frac{9.333(1.11) - (1.906)}{9.333}$$

$$= \frac{10.3596 - 1.906}{9.333}$$

$$\boxed{x_3 = 8.45363}$$

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%$$

$$= \left| \frac{8.4536 - 1.11}{8.4536} \right| \times 100\%$$

$$= 0.868 \times 100\%$$

$$\boxed{= 86.8\%}$$

Muhammad Maaz Akhunzada. ID# 11448 Pg# 17

Q4 A#

in matrix form

$$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

In Augmented Form

$$\left[\begin{array}{ccc:c} 2 & -1 & 2 & 2 \\ 1 & 10 & -3 & 5 \\ 1 & -1 & -1 & 3 \end{array}\right]$$

applying operations

$R_2 \to 2R_2 - 1R_1$

$R_3 \to 2R_3 - R_1$

$$\left[\begin{array}{ccc:c} 2 & -1 & 2 & 2 \\ 0 & 19 & 4 & 8 \\ 0 & -1 & -4 & 4 \end{array}\right]$$

$R_3 \to 19R_3 + R_2$

$$\left[\begin{array}{ccc:c} 2 & -1 & 2 & 2 \\ 0 & 19 & 4 & 8 \\ 0 & 0 & -70 & 84 \end{array}\right]$$

$$\begin{bmatrix} 2 & -1 & 2 \\ 0 & 19 & 4 \\ 0 & 0 & -70 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 84 \end{bmatrix}$$

writing in equation form

$2x - y + 2z = 2$ — (i)

$19y + 4z = 8$ — (ii)

$-70z = 84$

$$z = \frac{84}{-70} = -1.2$$

putting in eq (ii)

$$19y + 4(-1.2) = 8$$

$$19y + (-4.8) = 8$$

$$19y = 8 + 4.8 = 12.8$$

$$y = \frac{12.8}{19} = 0.673$$

putting values of y, z in eq (i)

$$2x - (0.673) + 2(-1.2) = 2$$

$$2x - 0.673 + (-2.4) = 2$$

$$2x - 3.073 = 2$$

$$2x = 5.073$$

$$x = \frac{5.073}{2} = 2.5365$$

$$\boxed{x = 2.536 \quad y = 0.673 \quad z = -1.2}$$

Answer.