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**SUBJECT NAME:
MULTIVARIATED CALCULUS**

QUESTION#1:

If $(x + yi) / i = (7 + 9i)$ where x and y are real, what is the value of $(x + yi)(x - yi)$?

ANSWER:

$$(x + yi) / i = (7 + 9i)$$

$$(x + yi) = i(7 + 9i) = -9 + 7i$$

$$(x + yi)(x - yi) = (-9 + 7i)(-9 - 7i)$$

$$= 81 + 49$$

$$= 130$$

QUESTION#2:

Find the values of x and y in the following equation, given further that $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$(x+iy)(2+i)=3-i$$

ANSWER:

$$(x+iy)(2+i)=3-i$$

$$2x-(x)i+(2y)i-i^2y=3-i$$

$$\underline{2x+y+(2y-x)i=3-i}$$

Real imaginary

Comparing the real & imaginary parts,

$$2x+y=3\text{-----}(1)$$

$$2y-x=-1\text{-----}(2)$$

Solving eq(1) & eq(2),

$$4x+2y=6$$

$$-3x+2y=-3$$

$$x=9$$

$$y=-15$$

$$\therefore(x,y)=(9,-15)$$

QUESTION#3:

Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in \mathbb{C}$.

ANSWER:

$$2z^2 - 2iz - 5 = 0$$

$$x = \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2i \pm \sqrt{4i^2 + 20}}{2}$$

$$x = \frac{2i \pm \sqrt{-4 + 20}}{2}$$

$$x = \frac{2i \pm \sqrt{16}}{2}$$

$$x = \frac{2i \pm 4}{2}$$

$$x = \frac{i \pm 2}{1}$$

QUESTION#4:

Express $4 - \sqrt{5}i$ in polar form.

ANSWER:

Polar form: (r, θ)

$a+bi$

$a=4$

$b=-\sqrt{5}i$

$$r = \sqrt{a^2 + b^2}$$

$$= \sqrt{16+5}$$

$$= \sqrt{21}$$

$$\tan \theta = \frac{-\sqrt{5}i}{4} = \tan(\theta) = -\frac{\sqrt{5}}{4}$$

$$\left(\sqrt{21}, -\frac{\pi}{4} \right)$$

QUESTION#5:

Find the limit $\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$

ANSWER:

$$2z^2 - 17z + 8 / 8 - z$$

$$\lim_{z \rightarrow 8}$$

$$2(8)^2 - 17(8) + 8 / 8 - 8$$

$$128 - 136 + 8 / 0$$

$$0 / 0$$

0 answer

QUESTION#6:

Differentiate

(i). $f(x)=(\ln x)^4$

(ii). $g(x)=x^2 \cdot \ln|x|$

ANSWER:

(i) $\frac{dy}{dx}=4(\ln x)^3x$

Explanation:

differentiate using the chain rule

given $y=f(g(x))$ then

$$\frac{dy}{dx}=f'(g(x)) \times g'(x) \leftarrow \text{chain rule}$$

$$y=(\ln x)^4$$

$$\Rightarrow \frac{dy}{dx}=4(\ln x)^3 \times \frac{d}{dx}(\ln x)$$

$$=4(\ln x)^3x$$

- (ii) If you are studying math's, then you should learn the Product Rule for Differentiation, and practice how to use it:

$$\frac{d}{dx}(uv)=u\frac{d}{dx}v+v\frac{d}{dx}u, \text{ or, } (uv)'=(u)'v+u(v)'$$

I was taught to remember the rule in words; "*The first times the derivative of the second plus the derivative of the first times the second*".

This can be extended to three products:

$$d(uvw) = uvdw + udvdx + dudxvw$$

So, with $f(x) = x^2 \ln x$ we have;

$$\{ \text{Let } u = x^2 \implies du/dx = 2x \text{ And } v = \ln x \implies dv/dx = 1/x \}$$

$$d(uv) = udv + vdu$$

$$d(x^2 \ln x) = (x^2)(1/x) + (2x)(\ln x)$$

$$d(x^2 \ln x) = x + 2x \ln x$$