Name :- Muhammad Hamza Rashid

ID # 7805

Section :- A

Semester :- 6th

Subject :- Plain & Reinforced Concrete Design - I

Instructor :- Engr. Fawad Khan

MID - TERM EXAMINATION

CIVIL ENGINEERING DEPARTMENT

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ID # 7805

QUESTION - 01

A rectangular beam that must carry a service live load of 2.47 kip/ft and a calculated dead load of 1.05 kips/ft (without self-weight) on an 18 ft. Simple span is limited to 10" width and 20" total depth for architectural reasons. If

$f_y = 60{,}000$ Psi

$f'_c = 4000$ Psi

What steel area must be provided? Draw sketch of your final design.

Given Data:-

Breadth (b) = 10"

Depth (h) = 20"

Service Live load = 2.47 kips/ft

Dead Load = 1.05 kips/ft

Span of Beam = 18'

f'_c = 4000 psi

f_y = 60,000 psi

Sol:-

First we have to find Effective Depth,

Effective depth (d) = h-3

= 20-3 = 17"

d = 17"

Now Assume,

Effective Cover (d') = 2.5"

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STEP # 01 :- (REINFORCEMENT RATIO)

By formula of Reinforcement Ratio,

Reinforcement Maximum :-

$$\rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \times \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$\beta \rightarrow 0.85$ (From Concrete compression strength)

$\varepsilon_u \rightarrow 0.003$ (In Compression Fibre)

$\varepsilon_u \rightarrow 0.005$ (In Tensile Fibre)

$$\Rightarrow \rho_{max} = 0.85 \times 0.85 \times \left(\frac{4}{60}\right) \times \left(\frac{0.003}{0.003 + 0.005}\right)$$

$$\boxed{\rho_{max} = 0.0181}$$

STEP # 02 :- (AREA OF STEEL)

By Formula,

$$\rho_{max} = \frac{\text{Area of steel}}{b \times d}$$

$$\Rightarrow \text{Area of Steel} = (b \times d) \times \rho_{max}$$

$$= (10 \times 17) \times 0.0181$$

$$\boxed{A_{st} = 3.07 \text{ in}^2}$$

STEP # 03 :- (DESIGN MOMENT)

⇒ First we have to find the Moment Produced due to External loads (Beam self wt + dead and live load).

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MOMENT DUE TO GIVEN LOADS:-

1- Beam Self Weight Per Feet :-

Wt = breadth * thickness × Unit wt. of concrete

$= \frac{10}{12} \times \frac{20}{12} \times 150 \text{ lb/ft}^3$ ($\gamma_c \to 150$ lb/ft³ for Rcc)

Wt = 208.33 lb/ft

2- Total Factored Load :-

By Formula,

Load = 1.2 DL + 1.6 (L.L)

$= 1.2((1.05 \times 1000) + 208.33) + 1.6(2.47 \times 1000)$

$= 5461.99$ lb/ft

= 5.46 kips/ft

3- Ultimate Factored Moment :-

By Formula,

$M_u = \frac{WL^2}{8} = \frac{5.46 \times (18)^2}{8} \times 12$

M_u = 2653.56 kip-inch

NOW DESIGN MOMENT :-

By Formula of (M_{u2}).

$M_{u2} = \phi \times A_{st} \times f_y \times (d - a/2)$

where $a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b}$

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$$a = \frac{3.07 \times 60}{0.85 \times 4 \times 10} = 5.41''$$

Now,

$$M_{u2} = 0.90 \times 3.07 \times 60 \times (17 - 5.41/2)$$

$$= 2369.82 \text{ kip-inch}$$

As, $M_u > M_{u2}$

i.e $2653.56 > 2369.82$

↓

So we have to design the beam section as Doubly Reinforced.

STEP #4 :- (DIFFERENCE B/W MOMENTS)

$$M_{u1} = M_u - M_{u2}$$

$$= 2653.56 - 2369.82$$

$$\boxed{M_{u1} = 283.74 \text{ kip - inch}}$$

STEP #5 :- (AREA OF STEEL IN COMPRESSION ZONE)

By Formula of Design Moment,

$$M_{u1} = \phi \times A'_{st} \times f_y \times (d - d')$$

$$\Rightarrow A'_{st} = \frac{M_{u1}}{\phi \times f_y \times (d - d')}$$

$$= \frac{283.74}{0.90 \times 60 \times (17 - 2.5)} = 0.36 \text{ in}^2$$

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STEP # 6 :- (TOTAL STEEL AREA)

$A_s = A_{s}' + A_{st}$

$= 3.07 + 0.36 = 3.43 \text{ in}^2$

STEP # 7 :- (SELECTION & NO. OF BARS IN TENSION ZONE)

A- In Tension Zone :-

Suppose we use # 8 bars;

Dia $= (8/8) = 1''$, Area $= \frac{\pi}{4}(8/8)^2 = 0.785 \text{ in}^2$

By Formula,

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

$$n = \frac{3.43}{0.785} = 4.36 \cong 5 \text{ bars}$$

So we use 5 # 8 bars in tension zone.

B- In Compression Zone:-

Suppose we use # 6 bars,

Dia $= (6/8) = 0.75''$, Area $= (\frac{\pi}{4})(6/8)^2 = 0.44 \text{ in}^2$

By Formula,

$$\text{No. of bars} = \frac{\text{Area of steel (compression zone)}}{\text{Area of single bar}}$$

$$= \frac{0.36}{0.44} = 0.81 \cong 1 \text{ bar}$$

So we use 1 # 6 bar in Compression Zone.

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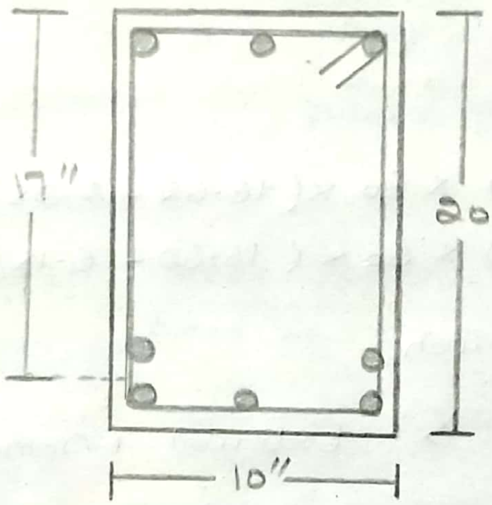
STEP #8 :- (MINIMUM WIDTH)

We have to check whether these bars accomodate in a single Layer or not.

$$b_{min} = 2(1.5) + 2(\tfrac{3}{8}) + 5(\tfrac{8}{8}) + 4(\tfrac{8}{8})$$

$$= 12.75'' > 10''$$

⇒ As value is greater so the bars will be in multiple Layers.



EFFECTIVE DEPTH :-

Effective depth for the designed beam will be,

$$d = 20 - 1.5 - 3/8 - 8/8 - 1/2(8/8)$$

$$\boxed{d = 16.62''}$$

EFFECTIVE COVER :-

Similarly Effective Cover will be,

$$d' = 1.5 + 3/8 + 1/2(6/8)$$

$$\boxed{d' = 2.25''}$$

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STEP # 9:- (DESIGN MOMENT)

By Formula,

$$M_d = \phi \times [A_{st} \times f_y \times (d - d') + (A_{st} - A'_{st}) \times f_y \times (d - a/2)]$$

Where

$$a = \frac{(A_{st} - A'_{st}) \times f_y}{0.85 \times f'_c \times b}$$

$$= \frac{(5 \times 0.785 - 1 \times 0.44) \times 60}{0.85 \times 4 \times 10} = 6.15''$$

By Formula,

$$M_d = 0.90 \times [(1 \times 0.44) \times 60 \times (16.62 - 2.25) + (5 \times 0.785 - 1 \times 0.44) \times 60 \times (16.62 - 6.15/2)]$$

$M_d =$ 2890.46 kip-inch

As Design Moment > External Moment

2890.46 > 2653.56

Design is **Ok!**

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STEP # 9:- (DESIGN MOMENT)

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$$M_d = \phi \times [A_{st} \times f_y \times (d - d') + (A_{st} - A'_{st}) \times f_y \times (d - a/2)]$$

Where $a = \frac{(A_{st} - A'_{st}) \times f_y}{0.85 \times f'_c \times b}$

$$= \frac{(5 \times 0.785 - 1 \times 0.44) \times 60}{0.85 \times 4 \times 10} = 6.15''$$

By Formula,

$$M_d = 0.90 \times [(1 \times 0.44) \times 60 \times (16.62 - 2.25) + (5 \times 0.785 - 1 \times 0.44) \times 60 \times (16.62 - 6.15/2)]$$

M_d = 2890.46 kip-inch

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Design is **OK!**

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(8)

QUESTION-2

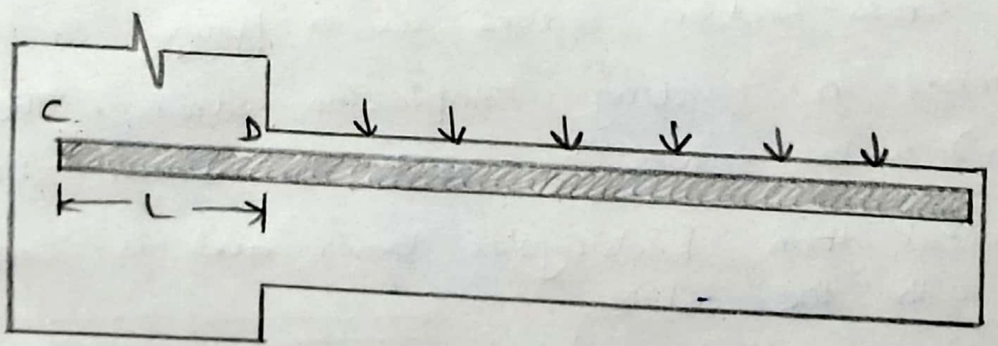
(PART-A)

BOND STRESS:-

Bond stress is the stress produced because of the bond between the concrete surface and the reinforcement steel. It varies depending upon the type of concrete and type of reinforcement used.
It is like a grip, if surface is rough and less grip and if surface is smooth and rounded

DEVELOPMENT LENGTH:-

Development length is certain minimum length of the bar required on either side of a point of maximum steel stress, in order to transfer the bar force to surrounding concrete through bond, without slip so to prevent bar from pulling out under tension.



In the above figure, CD represents the development length.

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(PART-B)

Doubly Reinforced Beam is used because of the following conditions / reasons :-

1- When the dimensions of the beam (breadth, depth) are limited for architectural or structural purposes.

2- When the sections such as, braces in water towers etc subjected to reversal of bending moment.

3- Due to high demand of loads (that causes moment) and due to restriction in dimensions, section is designed as doubly reinforced.

4- In continous T-Beams, the portion of the beam over middle support has to be designed as doubly reinforced section.

(PART-C)

T-BEAM ANALYSIS:-

A T-Beam consists of T-shaped structure of T-shaped cross-section. The top serves as a flange or compression member in resisting compression stresses. The lower (vertical section) web serves to resist the shear stresses.

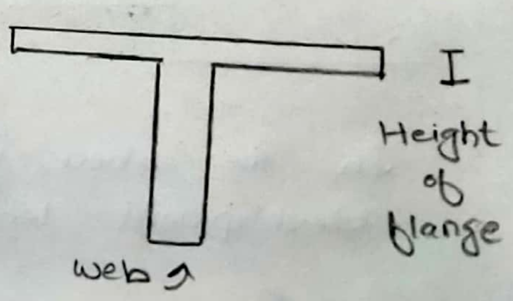
It is economical than Rectangular beam and is cast mololithically with the slab.

Its Analysis is required when

$a > h_f$

a = depth of commpression block

h_f = Slab thickness / flange height



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RECTANGULAR BEAM ANALYSIS:-

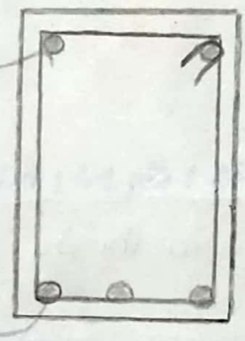
=> A Rectangular beam is one which is generally used as compression in top fibre and tension in bottom fibre.

=> Rectangular beams are most often used in office/commercial buildings. These can be cast in-situ using standard reinforcement or precast.

=> In rectangular beam, the stress block diagram shows that the top fibers are subjected to maximum compression that linearly reduces till the Neutral Axis.

=> Its analysis is required when

$$a \leq h_f$$



a = depth of compression block

h_f = height of flange

(PART- D)

STRENGTH REDUCTION FACTOR:-

=> It is defined as the Ratio of Elastic strength to Yield strength.

=> Denoted by ϕ.

EFFECT ON STRENGTH:-

Strength Reduction Factor shows the strength in terms of Percentage while designing a Section to resist the moment caused by loads.

Basically it respresents the uncertainty in determining the member's behaviour to the type of stresses to which it is subjected.

=> For example, during the design process of a beam, usually we take 0.75 for shear loads. It shows that 75 % of its strength is considered here and the rest of 25 % is for future accidental purposes.

(PART- E)

DESIGNING METHODS:-

Two methods are ~~[illegible]~~ worldly used for the designing of Concrete and different structural members.

1- ASD Method
(Allowable Stress Design) Method

2- USD Method
(Ultimate Strength Design) Method

1- Allowable Stress Design Method:-

ASD method is also known as working stress Design Method. It is based on the principle that stresses developed in the structural members should not exceed a certain limit / fraction of elastic limit.

=> In this method, all loads are taken as service loads and no factor is applied to increase these service loads.

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2 - Ultimate Strength Design Method :-

=> Ultimate strength Design method is also known as load factor method or Ultimate load method.

=> For the structure Subjected to large external loads, the Ultimate strength is determined by the inelastic (non-linear) analysis.

=> Ultimate strength Design method is best for designing different structural members because of the following reasons :

- As the Ultimate strength of the material is considered we will get much slender sections for columns and beams Compared to other methods.
- Ultimate design method results in more economical design for a building with fewer special needs for customized areas of reinforcement.
- ASD better determines design for higher safety factor needs where a building is more prone to environmental Pressures or must bear havier loads that stress a specific portion of a building.

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(QUESTION-3)

A concrete floor system consists of parallel T-beams spaced 10' on centers and spacing 32' b/w supports. The 6" thick slab is cast monolithically with T-beam webs having width bw = 14" and total depth measured from the top of the slab of h = 28". The effective depth will be taken 3" less than the total depth. In addition to its own weight each beam must carry a superimposed D.L of 50 psf and service L.L of 225 psf. Material strengths are fy = 60,000 psi and f'c = 4000 psi. Determine the required tensile steel area and select the reinforcement needed for a typical member. Draw sketch of your final design.

Given Data :-

Span of Beam = 32'

C/c distance b/w beams = 10'

Height of flange (h_f) = 6"

width of web = 14"

Total depth of beam = 28"

Dead Load = 50 psf

Service Live load = 225 psf

f_y = 60,000 psi

f'_c = 4000 psi

Sol:-

Effective depth (d) = h - 3
= 28 - 3

d = 25"

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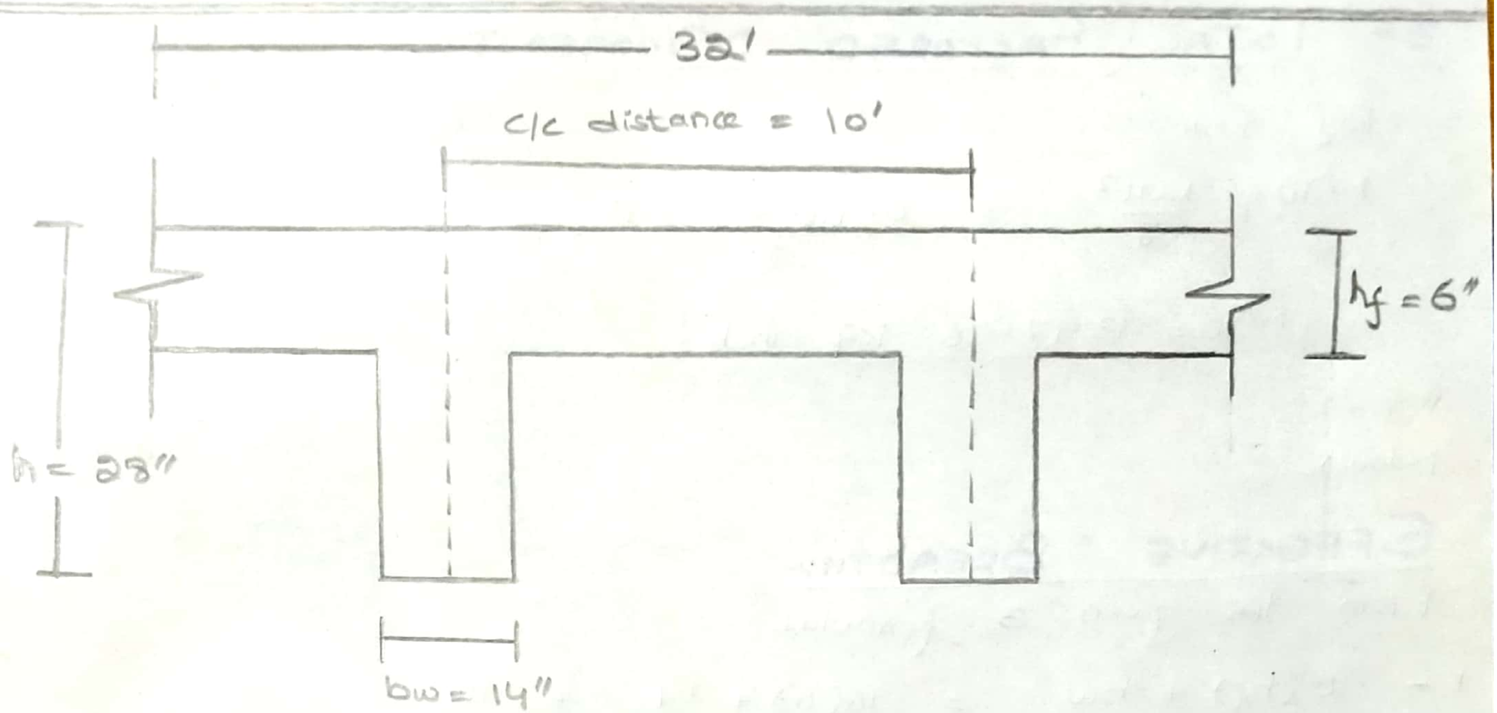

STEP #1 :- ULTIMATE FACTORED MOMENT

By formula

$$M_u = \frac{wL^2}{8}$$

=> First we have to find "w"

1- BEAM SELF WEIGHT PER FEET

$Wt = b \times t \times \gamma_c$

$= \frac{14}{12} \times \frac{28}{12} \times 150$ ($\gamma_c = 150\ lb/ft^3$ For Rcc)

$\boxed{Wt = 408.34\ lb/ft}$

2- TOTAL FACTORED LOAD:-

$= 1.2\,DL + 1.6\,LL$

$= 1.2(50 + 408.34) + 1.6(225)$

$= 910.1\ lb/ft$

$= 0.9101\ kip/ft$

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3- TOTAL FACTORED MOMENT:-

By Formula,

$$M_u = \frac{WL^2}{8} = \frac{0.910 \times (32)^2}{8} \times 12$$

$$\boxed{M_u = 1397.76 \text{ kip-inch}}$$

Now,

EFFECTIVE BREADTH:-

From the given 3 formulas

1 - $16(h_f) + b_w = 16(6) + 14 = 110''$

2 - c/c distance $= 10(12)'' = 120''$

3 - Span/4 $= \frac{32 \times 12}{4} = 96''$

We have to choose least value

So $\boxed{b_e = 96''}$

STEP #3 :- (CHECKING FOR RECTANGULAR OR T-BEAM ANALYSIS)

Trial #1 :-

Let $a = h_f = 6''$

By formula of Area of steel

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)} \qquad (\phi = 0.90)$$

$$= \frac{1397.76}{0.90 \times 60 \times (25 - 6/2)} = 1.17 \text{ in}^2$$

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Trial # 2 :-

By formula of "a"

$$a = \frac{Ast \times fy}{0.85 \times f'c \times b} = \frac{1.17 \times 60}{0.85 \times 4 \times 96} = 0.2 < 6''$$

As, depth of Compression block is less than height of flange, so we have to design a Rectangular Beam.

By Formula,

$$Ast = \frac{Mu}{\phi \times fy \times (d - a/2)} = \frac{1397.76}{0.90 \times 60 \times (25 - \frac{0.2}{2})}$$

$$\boxed{Ast = 1.03 \ in^2}$$

Trial # 3 :-

By formula,

$$a = \frac{Ast \times fy}{0.85 \times f'c \times b} = \frac{1.03 \times 60}{0.85 \times 4 \times 96} = 0.19''$$

Also,

$$Ast = \frac{Mu}{\phi \times fy \times (d - a/2)} = \frac{1397.76}{0.90 \times 60 \times (25 - \frac{0.19}{2})}$$

$$\boxed{Ast = 1.03 \ in^2}$$

So Area of steel obtained is same in previous 2 trials.

STEP # 4 :- (CHECKING OF REINFORCEMENT RATIO)

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By formula,

=> Max. Reinforcement ratio is,

$$\rho_{max} = 0.85 \times \beta \times \frac{f'c}{fy} \times \left(\frac{\varepsilon u}{\varepsilon u + \varepsilon y}\right)$$

$$= 0.85 \times 0.85 \times \frac{4}{60} \times \left(\frac{0.003}{0.003 + 0.005}\right)$$

$$\boxed{\rho_{max} = 0.018}$$

=> Mininum Reinforcement Ratio is,

$$\rho_{min} = 200/fy \qquad = 200/60{,}000$$

$$\boxed{\rho_{min} = 0.003}$$

$$\Rightarrow \rho = \frac{Ast}{b \times d} = \frac{1.03}{14 \times 25} = 0.002.$$

$$\boxed{\rho = 0.002}$$

=> **Checking the Order :-**

$$\rho_{min} < \rho < \rho_{max}$$

$$\boxed{0.003 < 0.002} < 0.018$$

↓

As ρ is less than ρ_{min}, So we have to find Ast on ρ_{min},

Then,

$$\Rightarrow \rho_{min} = \frac{Ast}{b \times d} \Rightarrow Ast = \rho_{min} \times b \times d$$

$$Ast = 0.003 \times 14 \times 25$$

$$\boxed{Ast = 1.05 \text{ in}^2}$$

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STEP # 5 :- (SELECTION AND NO. OF BARS REQUIRED)

Let we use #8 bar,

So

Dia = (8/8) = 1" , Area = $\frac{\pi}{4}(8/8)^2$ = 0.785 in²

=> By Formula,

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

$$= \frac{1.05}{0.785} = 1.3 \simeq 2$$

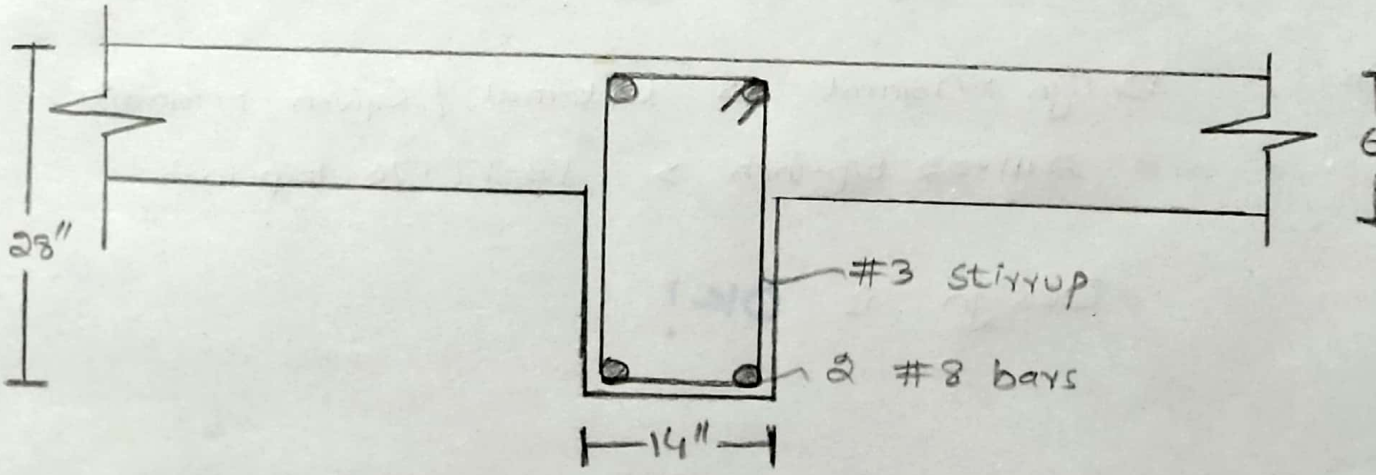
So we have to use 2 #8 bars.

STEP #6 :- (MINIMUM WIDTH FOR ACCOMODATION OF BARS)

$b_{min} = 2(1.5) + 2(3/8) + 2(8/8) + 1(8/8)$

$= 6.75" < 14"$

So bars will be good in a single Layer.



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STEP # 7 :- (DESIGN MOMENT)

By formula of Design Moment :-

$$M_d = \phi \times f_y \times A_{st} \times (d - a/2)$$

As from formula,

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

$$\Rightarrow A_{st} = \text{No. of bars} \times \text{Area of single bar}$$

$$= 2 \times 0.785$$

$$\boxed{A_{st} = 1.57 \text{ in}^2}$$

Also,

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \frac{1.57 \times 60}{0.85 \times 4 \times 96} = 0.2''$$

Putting in the formula,

$$M_d = 0.90 \times 60 \times 1.57 \times (25 - 0.2/2)$$

$$= 2111.02 \text{ kip-inch}$$

As, Design Moment > External / Given Moment

2111.02 kip-inch > 1397.76 kip-inch

Design is **OK!**