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Date: _____

Q3:- Solve the following systems of linear equations by Gauss-Jordan Method.

$$2x + 2y + 4z = 18$$

$$x + 3y + 2z = 13$$

$$3x + 2y - 3z = 14$$

Solution:-

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & 3 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & 3 & 14 \end{bmatrix} R_{2,3} =$$

$$R_1 = \frac{1}{2} R_1$$

$$= \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & 3 & 14 \end{bmatrix}$$

$$\begin{aligned} R_2 &= R_2 - R_1 \\ R_3 &= R_3 - 3R_1 \end{aligned} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -3 & -13 \end{bmatrix}$$

$$R_2 = \frac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -3 & -13 \end{bmatrix}$$

$$R_3 = R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & -3 & -9 \end{bmatrix}$$

$$\begin{aligned} R_1 &= R_1 - R_2 \\ R_3 &= -\frac{1}{3} R_3 \end{aligned} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$R_1 = R_1 - 2R_3$$

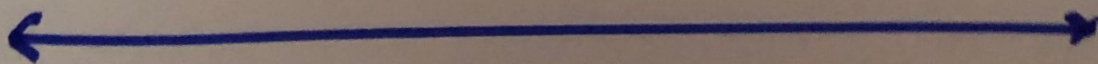
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}$$

solution

so

$$z = 3 \quad 3$$



Q6:- Reduce the matrix to normal form and find its rank

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

solution

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Reduce matrix to reduced row-echelon form

Swap matrix rows $R_1 \rightarrow R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in rows R_2 by performing

$$R_2 \leftarrow R_2 - \frac{1}{3} R_1$$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in R_3 row
 R_3 by performing

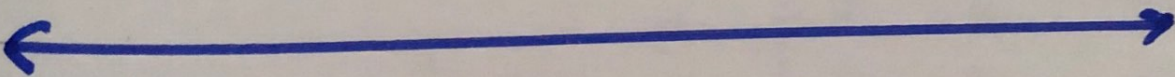
$$R_3 \leftarrow R_3 - \frac{1}{3} R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number
of all zero rows

Rank of

$$\begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} = 2$$



Q5:- Determine if the following homogeneous...
 solution set

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 2x_2 + 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \end{aligned}$$

Solution:-

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array} \right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array} \right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array} \right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

So, we have a solution of

$$x = \begin{bmatrix} 4/3 \cdot 8 \\ 0 \\ 8 \end{bmatrix} = 8 \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$

Q1:- Determine if the following or not.

$$\begin{aligned} x_1 - (3^{\text{rd}} \text{ID})x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

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$$\begin{aligned} x_1 - 7x_2 + 8x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -40 & -10 & 10 \end{array} \right] \quad R_3 - 5R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array} \right] \begin{array}{l} R_2/4 \\ R_3/10 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & -15 \end{array} \right] ; R_3 - 4R_2$$

↓ consistent because of triangle

$$-15x_3 = -15$$

$$x_3 = 1$$

$$x_2 - 4x_3 = 4$$

$$x_2 = 4 + 4x_3$$

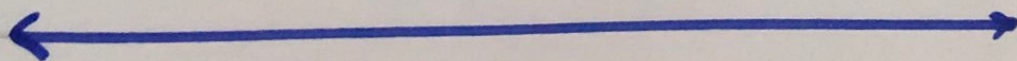
$$x_2 = 8$$

$$x_1 - 7x_2 + x_3 = 0$$

$$x_1 = 7x_2 - x_3$$

$$x_1 = 7 - 1$$

$$\boxed{x = 6}$$



Q2:- Find the inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$

by adjoint method.

Sol:- $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$

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$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$$

To find the inverse using the formula, we first determine the cofactors of A.

We have

$$C_{11} = \begin{vmatrix} -1 & 4 \\ -2 & 7 \end{vmatrix} = -11, \quad C_{12} = - \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} = 6$$

$$C_{13} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -1$$

$$C_{23} = \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} = 6$$

$$C_{21} = - \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 38$$

$$C_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 21$$

$$C_{23} = - \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -26$$

$$C_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 4 \end{vmatrix} = -21, \quad C_{32} = - \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} = -2$$

$$C_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11$$

therefore the adjoint of A
matrix A is

$$A_{\text{adj}}(A) = A^t = \begin{bmatrix} 1 & 38 & 21 \\ 6 & 21 & -2 \\ 6 & 26 & -11 \end{bmatrix}$$

using the formula, we obtain
the inverse formula

formula

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$$A^{-1} = \frac{1}{\det(A)} \text{Adj}(A) = \begin{bmatrix} 1 & 38 & 21 \\ 6 & 21 & -2 \\ 6 & 26 & -11 \end{bmatrix}$$

Ans