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ID# 16011

Programme : BSCS {2^{nd} Samester "2020"}

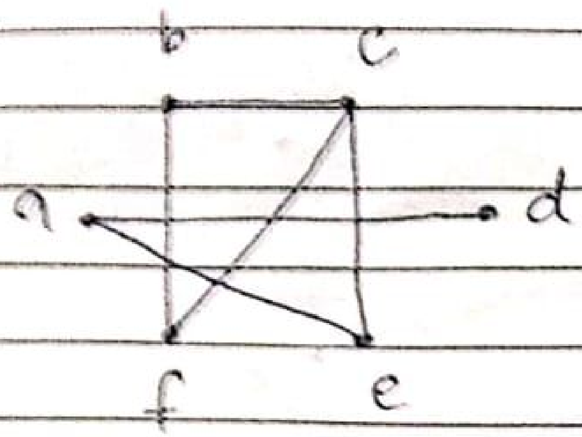
Subject : Discrete Structure

Teacher name : Sir Saifullah

Final Exams 2020

"Iqra National University"

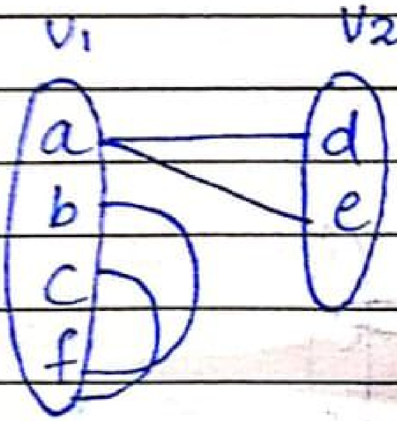
Question no :- "1"



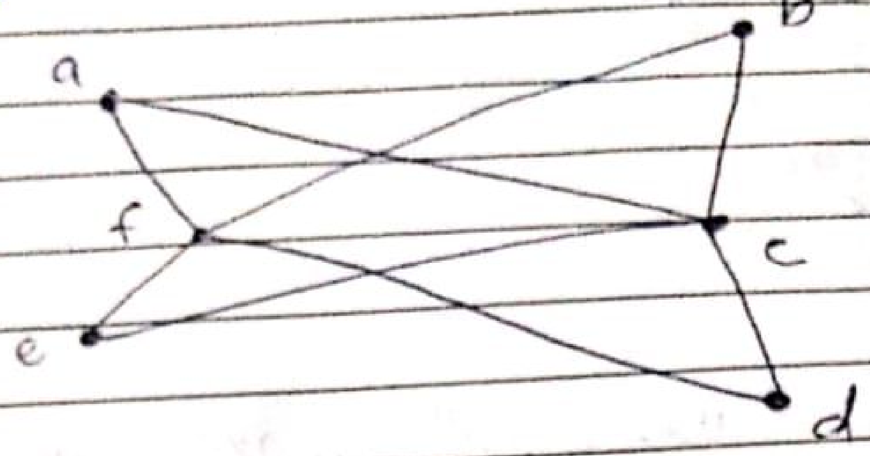
First we will make two different sets as S1 and S2

$V_1 = \{a, b, c, f\}$

$V_2 = \{d, e\}$

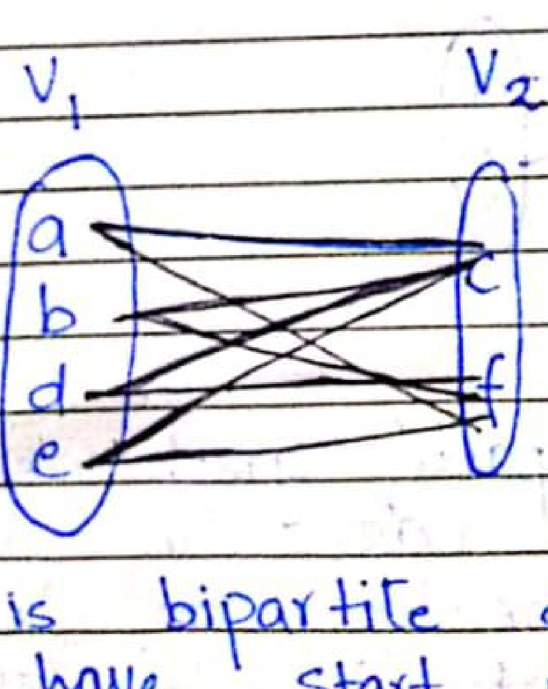


As in Bipartite graph definition that every edge in the graph connects with Two different sets (V_1 & V_2) but here a case where starting and ending edges are not different (same set) so as conclusion this graph is not bipartite graph



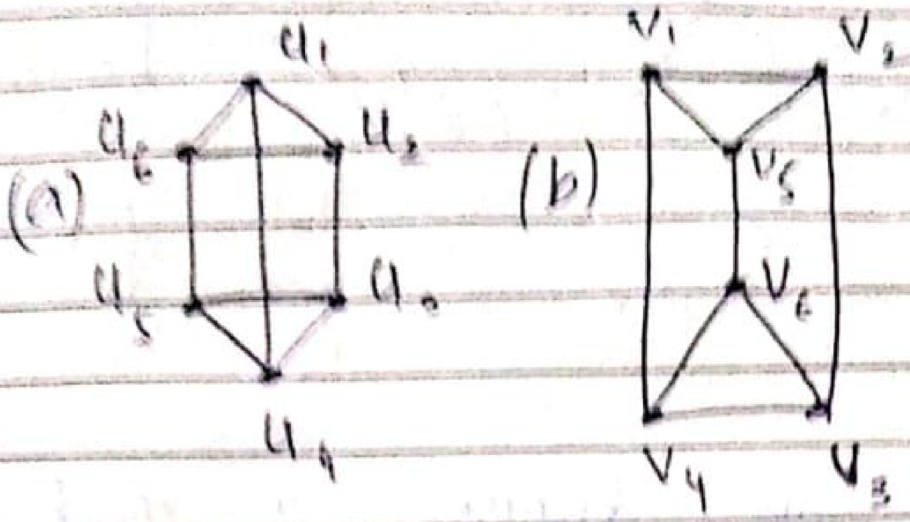
First we should make 2 sets as

V_1	V_2
(a, b, d, e)	(c, f)



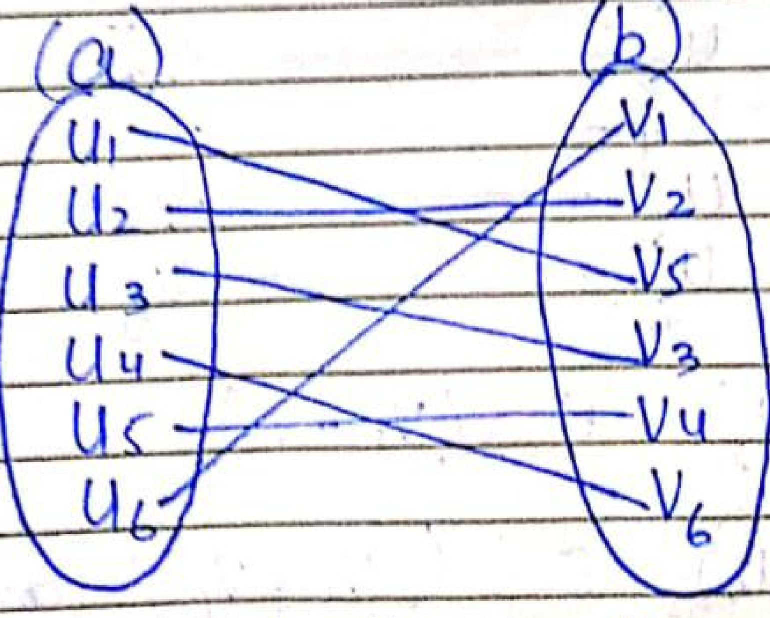
it is bipartite graph because V_1 have start point and V_2 have ended point and here no edge in same set

Question 2



Answer

* here in figure a and b number of edges are same
* Mapping



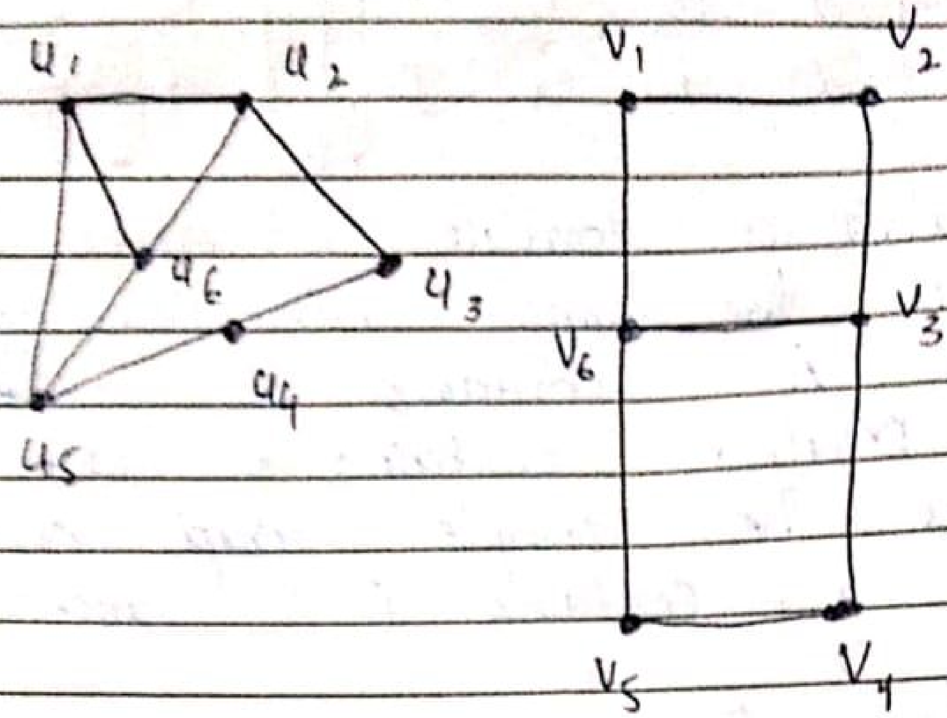
Now checking the edges

$(u_6, u_1) \cong (v_1, v_5)$

$(u_6, u_2) \cong (v_1, v_2)$

Hence it is Isomorphi

Question 2
part (b)



Answer

let us determine the degree of every vertex in left graph

$deg(u_1) = 3$
$deg(u_2) = 3$
$deg(u_3) = 2$
$deg(u_4) = 2$
$deg(u_5) = 3$
$deg(u_6) = 3$

degree sequence = 3, 3, 3, 3, 2, 2.

let us next determine the degree Vertex in the right path

$$deg(V_1) = 2$$
$$deg(V_2) = 3$$
$$deg(V_3) = 3$$
$$deg(V_4) = 2$$
$$deg(V_5) = 2$$
$$deg(V_6) = 4$$

degree sequence = 4, 3, 3, 2, 2, 2

Isomorphic graphs need to have the same number of vertices the same number of edges and the same degree sequences we then note that the two graphs do not have the same degree sequences and thus the two graphs are not

Isomorphic

Question 3.

(a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Answer

let us add all elements in the two matrices, which will represent the number of connection of a vertex to another vertex

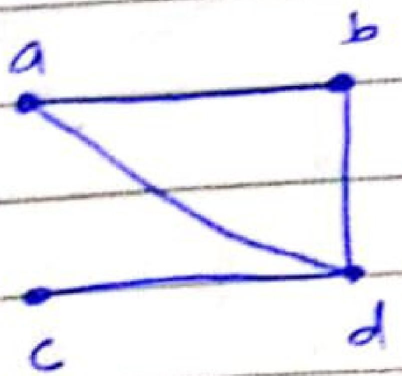
A (Matrix) contain 8 ones and thus the simple graph corresponding to A contains 8 connections
B (Matrix) contains 10 ones and thus the simple graph corresponding to A contain 10 connections

Since number of connection of the two graphs are not the same the number of edges in the graph are not the same and then graphs are not isomorphic

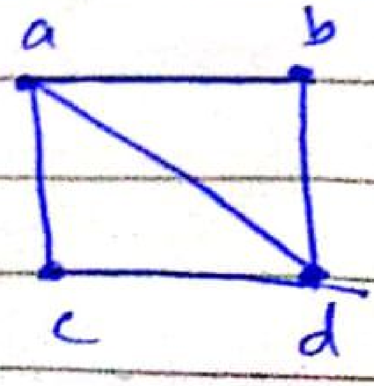
Question 3
Part (a)

graph.

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



$$B = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



(b)

$$a = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Matrix a contain 8 ones and thus the simple corresponding to A contains 8 connection b Matrix contains 6 ones and thus the simple graph corresponding to a contains 6 connection.

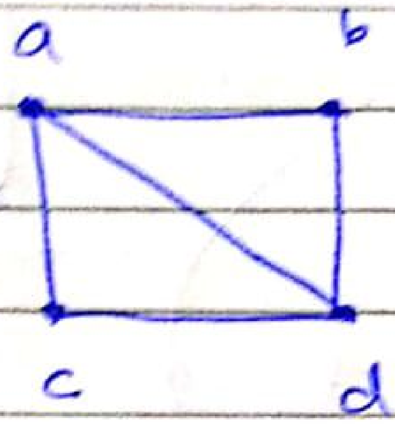
Since the number of connections of the two graphs are not the same, the number of edges in the graphs are not the same and then the graps are not isomorphic.

Question 3

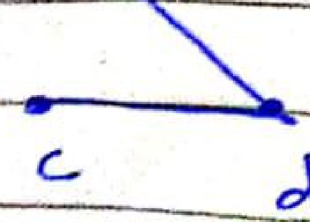
part (b)

graph

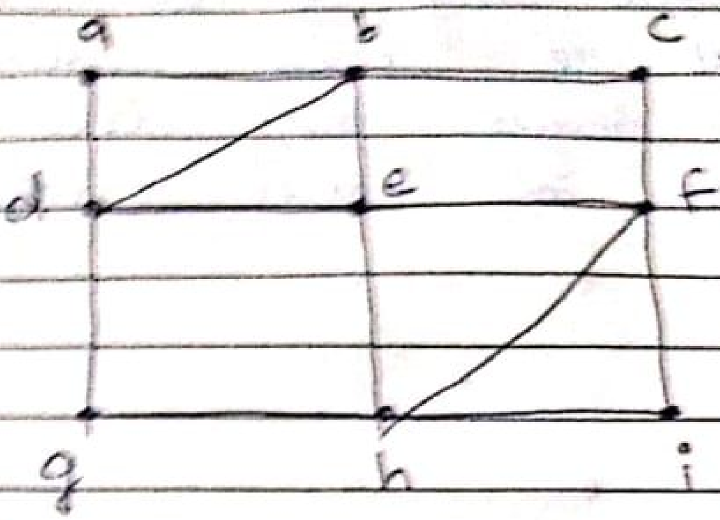
$$A = \begin{array}{c c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



$$B = \begin{array}{c c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



Question 4



Answer

Euler Circuit

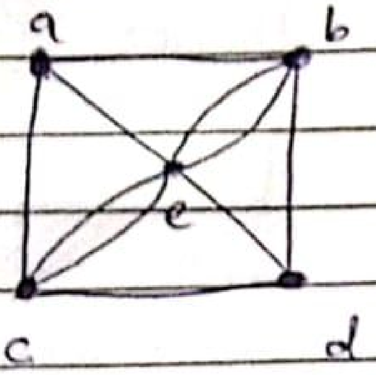
Definition ;

Euler circuit is a simple circuit that contains every edge of the graph.

let us first determine the degree of every matrix (vertex) in given graph

degree a = 2
degree b = 4
degree c = 2
degree d = 4
degree e = 4
degree f = 4
degree g = 2
degree h = 4
degree i = 2

A graph has a Euler circuit if and only if the vertices has an even degree. Since all degrees are even there exist an Euler circuit.



Euler Path;

Euler path is a directed graph or simple path that contain every edge of the graph

let us first determine the degree of every vertex in given path

deg a = 3
deg b = 4
deg c = 4
deg d = 3
deg e = 6

A graph has an Euler circuit if and only if each of the vertices has an even degree. Since a degrees are odd, there is no Euler circuit

A graph has a Euler path if and only if there are exactly two vertices who have an odd degree. We note that vertices a and d odd degree and thus an Euler path exists.

Here in figure 1

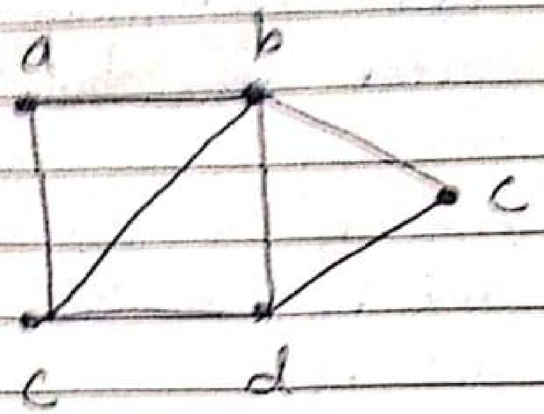
Euler circuit exist

and
in figure 2

No Euler circuit exist
but euler path exist.

Q5;

(a)



Answer.

Hamilton Circuit;

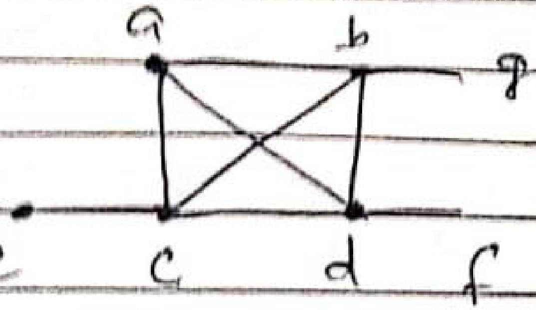
is a circuit that passes through every vertex.

let us first determine the degree of every vertex in the given graph.

$\deg(a) = 2$
$\deg(b) = 4$
$\deg(c) = 2$
$\deg(d) = 3$
$\deg(e) = 3$

we note that the graph contain the cycle C_5 and the cycle C_5 within the given graph form a Hamilton circuit (as the circuit will pass through all vertices exactly once).

(b)



Vertixes.

deg a = 3
deg b = 4
deg c = 4
deg d = 4
deg e = 1
deg f = 1
deg g = 1

we note here that there is only one edge (d,f) connecting to less than any circuit that contains f need to pass through d twice, which means that no circuit can be Hamilton Circuit

" Hamilton Circuit does not exist because f has only 1 edge connecting to it "