NAME → Muhammad Usama

ID → # 16742

DEPT → (BSCS)

SUBJET → Linear Algebra

TEACHER → Mansoor Qadir

FINAL TERM EXAM

IQRA NATIONAL UNIVERSITY
PESHAWAR

QNO: 1-----

ANS:

VECTOR:

A (real) vector is an object that has both a magnitude And a direction. Geometrically, we can pecture a vector as a directed line segment, whose length is the Magnitude of the vector And with an arrow indicating the direction. The direction of the vector is ~~foomed~~ from its tails to its Head.

force, velocity, ac

Vector R^3:

A vector R^3 is a 3-tupple of Real numbers $(V1, V2, V_3)$.

Sol: Consider the following vectors in R_3: $V_1 = (1, 6, 7)$, $V_2 = (6, 7, 4)$ $V_3 = (7, 4, 2)$.

a) Verify that the General Vector $U = (x, y, z)$ can be Written as a linear Combination of V_1, V_2 and V_3.

Hint: the Coeffcients will be Expressed as functions of the entries x, y and z of u.

Note: This Shows That Span $\{V_1, V_2, V_3\} = R_3$.

(b) can R_3 can be Spanned By two vectors "W_1 and W_2" Be Sure to justify.

Linear Independence

Difinition:

$$\{V_1, V_2 --- V_p\}$$

$$a_1 V_1 + a_2 V_2 + ---- + a_p v_p = 0$$

is $a_1 = a_2 = --- = a_p = 0$

$$v_i = \begin{bmatrix} V_1 \\ \vdots \\ V_P \end{bmatrix}_{m \times 1}, \quad 0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{m \times 1}$$

$i = 1, ---, P$

SOLUTION :-

MY ID 16742

$$V_1 = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix} \cdot V_2 = \begin{bmatrix} 6 \\ 7 \\ 4 \end{bmatrix} \cdot V_3 = \begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix}$$

$$a_1 V_1 + a_2 V_2 + a_3 V_3 = 0$$

~~they are not~~

$$\begin{bmatrix} 1 & 6 & 7 \\ 6 & 7 & 4 \\ 7 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{Homogeneous System}$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 4 & 0 \\ 7 & 4 & 2 & 0 \end{array}\right]$$

$$\begin{cases} a_1 = 6a_3 \\ a_2 = a_3 \end{cases} \qquad \begin{cases} a_3 = 1 \\ a_1 = 6 \\ a_2 = 1 \end{cases}$$

linear independent

$$6V_1 + V_2 + V_3 = 0$$

$$\{V_1, V_2, V_3\}$$

QNO: 2 -----

ANS :- 3:-

Vector space:-

A Space in which:

Any two vectors in the Space
Can be added or scaled.
without leaving The Space

A vector space is
A Set of V. on which two
opration + and · are defined,
called vector addition and
Scaller ~~Addition~~ Multiplication

the opration + (vector addition) must Stisfy the following:

closure: if u and v are any ~~two~~ vectors in V then the sum $u+v$ belong to V:

1) - Commutive law: $u+v=v+u$.

2) - Associative law: $u+(v+w)=(u+v)+w$

3) - Additive Idintity: $0+v=v$ and $v+0=v$.

4- Additive inverses: $v+x=0$ and $x+v=0$

→ the opration (Scalar multiplication) is defined b/w real Number (or Scalar) and vectors, And must Satisfy the following Conditions.

↳ Distributive law: $c\cdot(u+v)=c\cdot u+c\cdot v$

↳ Distributive law: $(c+d)\cdot v=c\cdot v+d\cdot v$

↳ Associative law: $c\cdot(d\cdot v)=(cd)\cdot v$

↳ Unitary law: $1\cdot v=v$.

Vecter space over R:

A vector space over R is a nonempty set V of objects, called vectors, on which are defined two ~~vetors~~ oprations, called addition + and multiplication by scalars. satisfing following properties.

(closure of addition) for all $u, v \in V$, $u+v$ is defined and $u+v \in V$ ---

↳ Not vector space

Show that $S = \{ \underset{\sim}{x} \in \mathbb{R}^3 : 2x_1 + 3x_2^3 - 4x_3^2 = 0 \}$

is not vector space.

take $\underset{\sim}{v} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \in S$

Since $2 \times 2 + 3 \times 0^3 - 4 \times 1^2 = 4 + 0 - 9 = 0$

So $2\underset{\sim}{v} = 2 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \notin S$,

Since $2 \times 4 + 3 \times 0^3 - 4 \times 2^2 = 8 + 0 - 16$
$= -8 \neq 0$

therefore S is not closed under multiplacation.

Q NO: 3 PART (B):

- Pholynomials of degree n does not form a vector space b/c they don't form a set closed under addition.

For instance:

which is not of degree n.

So don't get Confused with The set of pholinomials of degree less or equal then n which form a vector space of dimension n+1

We often work with this space.

QNO: 2 PART (B)

linear tranformitionz.

1) $T(u+v) = T(u) + T(v)$

2) $T(cu) = cT(u)$

~~EXAM~~ Solution

Determine withe's $T: R^3 \to R^3$ defind by

$T([x, y, z]) = [x+y, x-y, z]$

is linear tranformation.

1) let $u = [x_1, y_1, z_1]$ and $v = [x_2, y_2, z_2]$. then we want to prove $T(u+v) = T(u) + T(v)$

$T(u+v) = T([x_1, y_1, z_1] + [x_2, y_2, z_2]$

$= T([x_1 + x_2, y_1 + y_2, z_1 + z_2])$

$$= (x_1 + x_2 + y_1 + y_2, x_1 + x_2 - (y_1 + y_2), z_1 + z_2.$$

• And

$T(u) + T(v) =$

$$= T([x_1, y_1, z_1]) + T[x_2, y_2, z_2])$$
$$= [x_1 + y_1, x_1 - y_1, z_1] + [x_2 + y_2, x_2 - y_2, z_2]$$
$$= [x_1 + y_1 + x_2 + y_2, x_1 - y_1 + x_2 - y_2, z_1 + z_2.$$
$$= [x_1 + x_2 + y_1 + y_2, x_1 + x_2 - (y_1 + y_2), z_1 + z_2.$$

therefore, $T(u+v) = (T(u) + T(v))$

2) We want to prove $T(cu) = cT(u)$.

$$T(cu) = T(c[x_1, y_1, z_1])$$
$$= T([cx_1, cy_1, cz_1])$$
$$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$$

And

$c\,T(u) =$

$= cT(x_1, y_1, z_1)$

$= c(x_1 + y_1, x_1 - y_1, z_1)$

$= [c(x_1 + y_1), c(x_1 - y_1), cz_1]$

$= [cx_1 + cy_1, cx_1 - cy_1, cz_1)$

So $T(cu) = cT(u)$.

Q4 PART A

$C = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$, $A = \begin{bmatrix} 4 & -2 \\ -4 & 2 \end{bmatrix}$

if matrize $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

then

$B = A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$,

if $ad - bc \neq 0$.

Notice $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

it follows that matrize

A inverse (nonsingular)

$\det(A) \neq 0$

$C = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$. $A = \begin{bmatrix} 4 & -2 \\ -4 & 2 \end{bmatrix}$

$\det(C) = \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix}$ d.

$\det(C) = (3(2) - (-1)(-2))$

$\Rightarrow \det(A) = \begin{vmatrix} 4 & -2 \\ -4 & 2 \end{vmatrix}$

$\det(C) = (3(2) - (-1)(-2))$.

$\det(A) = 8 - 8 = 0$

Matrize C is invertible

MAtrize A is not invertible

Scanned with CamScanner

17

Q4 PART (B)

det [ ]

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

(1) $\begin{vmatrix} 3 & 1 \\ -2 & 5 \end{vmatrix} = 15 - -2 = 17$

(2) $\begin{vmatrix} 1 & 0 & 3 \\ -2 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$

$= -0\begin{vmatrix} -2 & 3 \\ 5 & 4 \end{vmatrix} + 3\begin{vmatrix} -2 & 1 \\ 5 & 2 \end{vmatrix}$

$= (1)(-2) - (23) + 3(-9)$

$-2 \quad + -27$

-29

$\left|\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 3 \\ -2 & 1 & 3 & -2 & 1 & 3 \\ 5 & 2 & 4 & 5 & 2 & 4 \end{array}\right.$

$(4 + 0 + -12) - (15 + 6 + 0)$

$-8 - 21$

-29

QNO: 4 PART C

Solutions.

Determinent with 2x2 bit matrices

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ det(A) $|A| = ad - bc$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow |A| = 1 \cdot 4 - 2 \cdot 3 = 4 - 6 = -2$

$B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \rightarrow |B| = 3 \cdot 2 - 4 \cdot 1 = 6 - 4 = 2$

$C = \begin{bmatrix} 4 & 5 \\ 6 & 10 \end{bmatrix} \rightarrow |C| = 4 \cdot 10 - 5 \cdot 6 = 40 - 30 = 10$

$D = \begin{bmatrix} 1 & 4 \\ 4 & 8 \end{bmatrix} \rightarrow |D| = 2 \cdot 8 - 4 \cdot 4 = 16 - 16 = 0$

20

Q3: PART B

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3 + \lambda_4 E_4 = \begin{pmatrix} \lambda & \lambda_2 \\ \lambda_3 & \lambda_4 \end{pmatrix}$$

$\lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3$ S is spanning

$+ \lambda_4 E = 0$

$\lambda_1 = 0 \quad \lambda = 0 \quad \lambda_3 = 0 \quad \lambda_4 = 0$

S is lin. inde...

$\dim M_{22}(\mathbb{C}) = 4$

by Analogy:

$\dim M_{mk}(\mathbb{C}) = m \times k$

19

Q4 PAR D

$$A = \begin{bmatrix} 2 & -9 & 7 \\ 8 & -\tfrac{1}{2} & 3 \\ 6 & -9 & 5 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & -9 & 7 \\ 8 & \tfrac{1}{2} & 3 \\ 6 & -9 & 5 \end{vmatrix} \begin{matrix} 2 & -9 \\ 8 & \tfrac{1}{2} \\ 6 & -9 \end{matrix}$$

$21 \quad -59 \quad -160 \quad 5 \quad -72 \quad -509$

$5 + (-72) + (-509) - 21 - (-59) - (-160)$

2)

the End