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SUBJECT:

DIFFERENTIAL EQUATIONS:

Major Mid Assignment:

Qustion No 1:

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$$

$y(0)=0$ so $x=0$, $y=0$

$dy = e^{y} \cdot e^{-t} \sec(y)(1+t^2)\, dt$

$$\frac{1}{e^{y} \cdot \sec(y)} dy = (1+t^2) e^{-t} dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y} \cos y \, dy = \int (1+t^2) e^{-t} dt$$

Using integration by parts.

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) = (1+t^2) \int e^{-t} - \int \left(\int e^{-t} \cdot \frac{d}{dt} (1+t^2) \right)$$

L.H.S

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$e^{-y} \sin y - \int (\sin y \cdot e^{-y} (-1))$$

$$e^{-y} \sin y + \int (\sin y \cdot e^{-y})$$

$$e^{-y} \sin y + \int (e^{-y} \sin y)$$

Again Using Integration by parts.

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (\int \sin y \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (-\cos y \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int (\cos y e^{-y})$

since $\int (\cos y e^{-y}) = L.H.S$

Since it is again same to the first one so L.H.S will become

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2LHS = e^{-y}(\sin y - \cos y)$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$$

Now taking R.H.S

$\int (1+t^2) e^{-t} dt$

$(1+t^2)\int e^{-t} - \int (\int e^{-t} \cdot \frac{d}{dt}(1+t^2))$

$(1+t^2)e^{-t} - \int (-e^{-t}(2t))$

$-(1+t^2)e^{-t} + \int (2t)e^{-t})$

Again Using Integration By parts.

$$-(1+t^2)e^{-t}+\left(2t\int e^{-t}-\int\left(\int e^{-t}\frac{d}{dt}2t\right)\right)$$

$$-(1+t^2)e^{-t}+\left(-2t\,e^{-t}+\int(2e^{-t})\right)$$

$$-(1+t^2)e^{-t}+\left(-2t\,e^{-t}-2e^{-t}\right)+C$$

$$\Rightarrow -(1+t^2)e^{-t}-2te^{-t}-2e^{-t}+C$$

$$\Rightarrow -e^{-t}-e^{-t}t^2-2te^{-t}-2e^{-t}+C$$

$$\Rightarrow -(t^2+2t+3)e^{-t}+C = R.H.S$$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y-\cos y)}{2} = -(t^2+2t+3)e^{-t}+C$$

We know that

$t=0 \qquad y=0$

Put it above

$$\tfrac{1}{2}(0-1) = -3+C$$

$$C = 5/2$$

Put value of C

$$\frac{e^{-y}}{2}(\sin y-\cos y) = -(x^2+2t+3)e^{-t}+5/2$$

Ans

Question No 2:

$$(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad \text{——— (1)}$$

This is homogeneous differential eq^n in x and y to solve this

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (1) becomes

$$v+x\cdot\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v+x\frac{dv}{dx}=\frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$v+x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

Taking integrals on both sides

$$\int\frac{v\,dv}{(\sqrt{1-v^2})(1+\sqrt{1-v^2})} = \int\frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$= \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int\frac{-dt}{t} = \int\frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

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$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = 1/c$$

$$x+\sqrt{x^2-y^2} = c_1 \quad \because \frac{1}{c} = c_1$$

which is required solution.

Question No 3:

$$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$$

Sol:

$$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$$

$\rightarrow f(D)y = f(x)$

As it is non-homogeneous linear equation so, solution will be.

$$y = y_c + y_p \quad \text{——(i)}$$

Complementary solution y_c

$D^4-D^2=0 \Rightarrow D^2(D^2+1)=0$.

Either $D^2=0 \Rightarrow D=0$

$D^2+1=0 \Rightarrow D^2=-1$

$D=\sqrt{-1} \Rightarrow D=i$ or $D=0+i$

Roots are real and complex

$y_c = C_1e^{0x}+e^{0x}(C_2\cos x+C_3\sin x)$

$y_c = C_1 + C_2\cos x + C_3\sin x$

$y_p = \dfrac{1}{f(D)}F(x)$

$$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x-2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4+D^2$

at $D=0 \Rightarrow f(D)=0$

so $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(D)=0$

Again Differentiating

$f''(D) = 12D+2$

So for $D=0$

$f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2}\cdot 4\sin x - \frac{x^2}{12D+2}\cdot 2\cos x$$

Putting $D=0$ in all

So

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So

Putting in equation (i)

$$y = C_1 + C_2\cos x + C_3\cos x + \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4.$$

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