Name	Asim Ali
ID	7763
Sec	C
Subject	Intro to S.D & Earth quick
Teacher	Engr. Yaseen Mahmood
Semistr	8th
Department	Civil (BS(c))
Date	29- June- 2020

* INU PESHAWAR *

Q: 1

Given DATA:

$E = 29,000$ ksi

$I = 150$ in^4

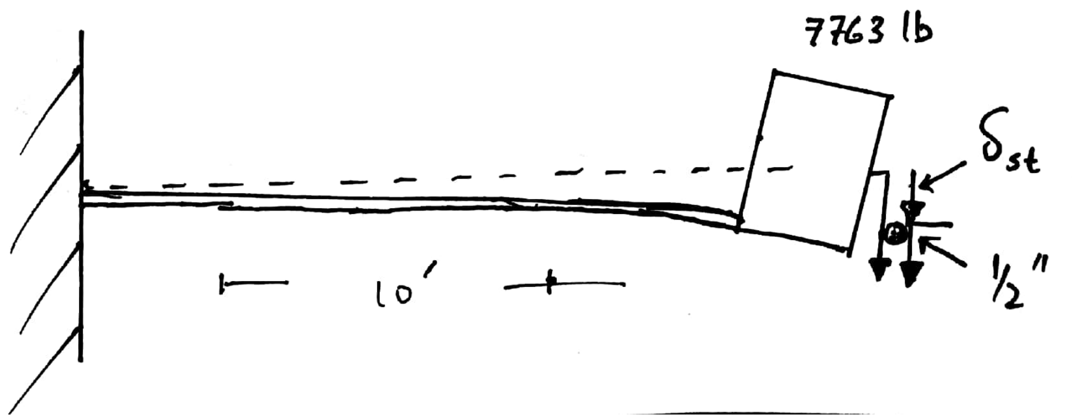
δ_{st} = Deflection due to 7763 lb.

Beam is pulled for 1/2 inch. in the downward

Required:

Determine natural time period of the system and develop and solve the equation of motion for vibrations resulting at free end. Also develop the equation showing variation in the equivalent static force with time. What will be the amplitude of equivalent static force? Also draw the graph?

Solution:



The General E.O.M for SDOF System is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t) = 0$)

Hence general EOM becomes

$$ku + m\ddot{u} = 0 \quad \ldots\ldots\ldots \quad (1)$$

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000 \times 150 \text{ in}^4}{(10 \times 12)^3}$$

$$K = 7.55 \text{ K/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sect or kg, m, sec.

$$K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}$$

$$m = \frac{7763 \text{ lb sec}^2}{32.2 \text{ ft}} = 241.09 \text{ slug}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{96025}{241.09}} = 19.96 \text{ rad/sec}$$

$$T_n = 2\pi/\omega_n = \frac{2\pi}{19.96} = 0.315 \text{ sec}$$

Substituting the corresponding values in eq - 1

$$90625\,u + 241.09\,\ddot{u} = 0$$

Where 'k' is in lb/ft & "m" lb sec/ft²

General Solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$$u(0) = 1/2'' = \frac{1}{24}\text{ ft} \quad \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.96t) + 0$$

$$= \left(\frac{1}{24}\right) \times \cos(19.96t)$$

Equivalent Static force at any time "t" is

$$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(19.96t)}{24}$$

$$f_s(t) = 3776 \cos(19.96t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

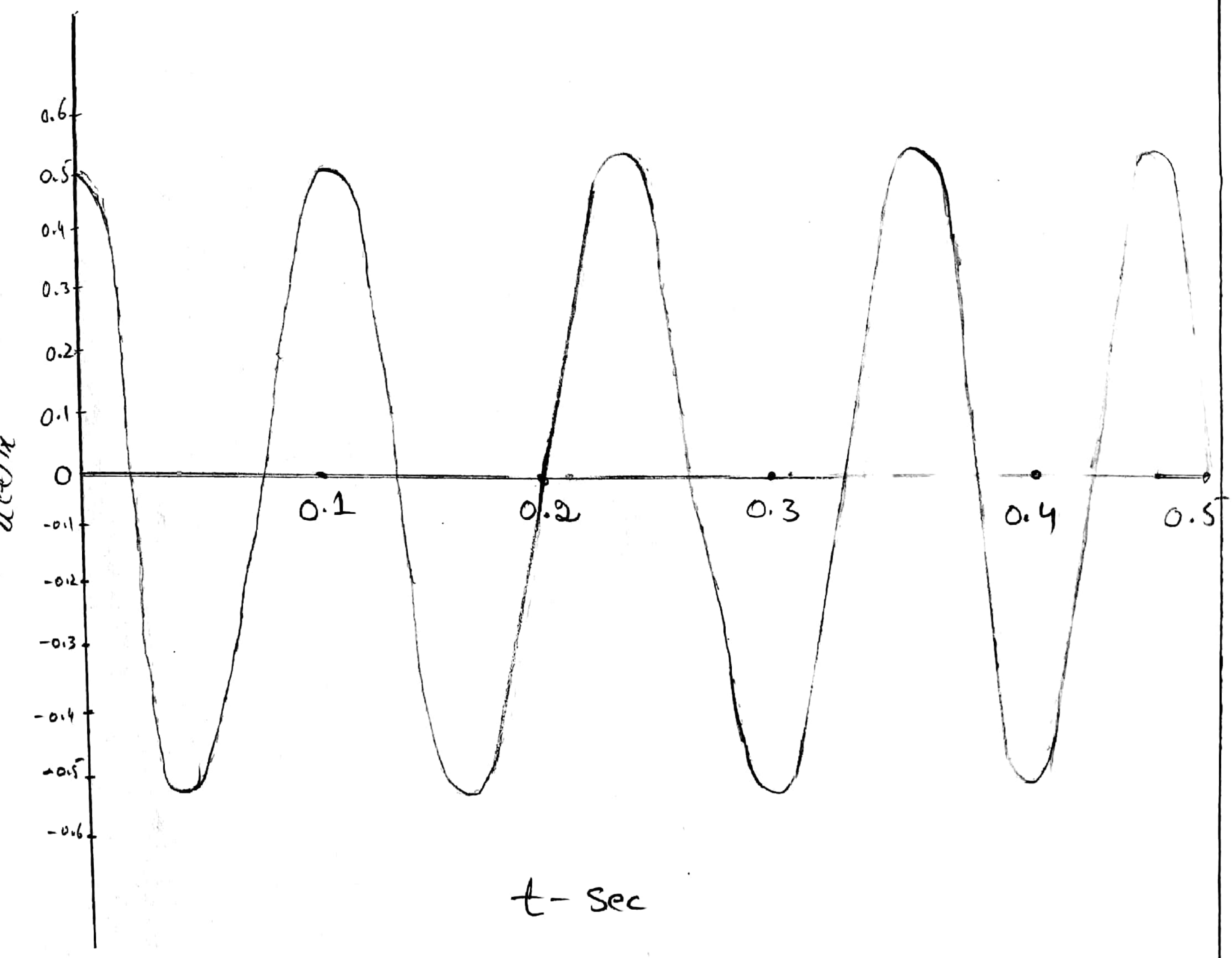
$$u_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

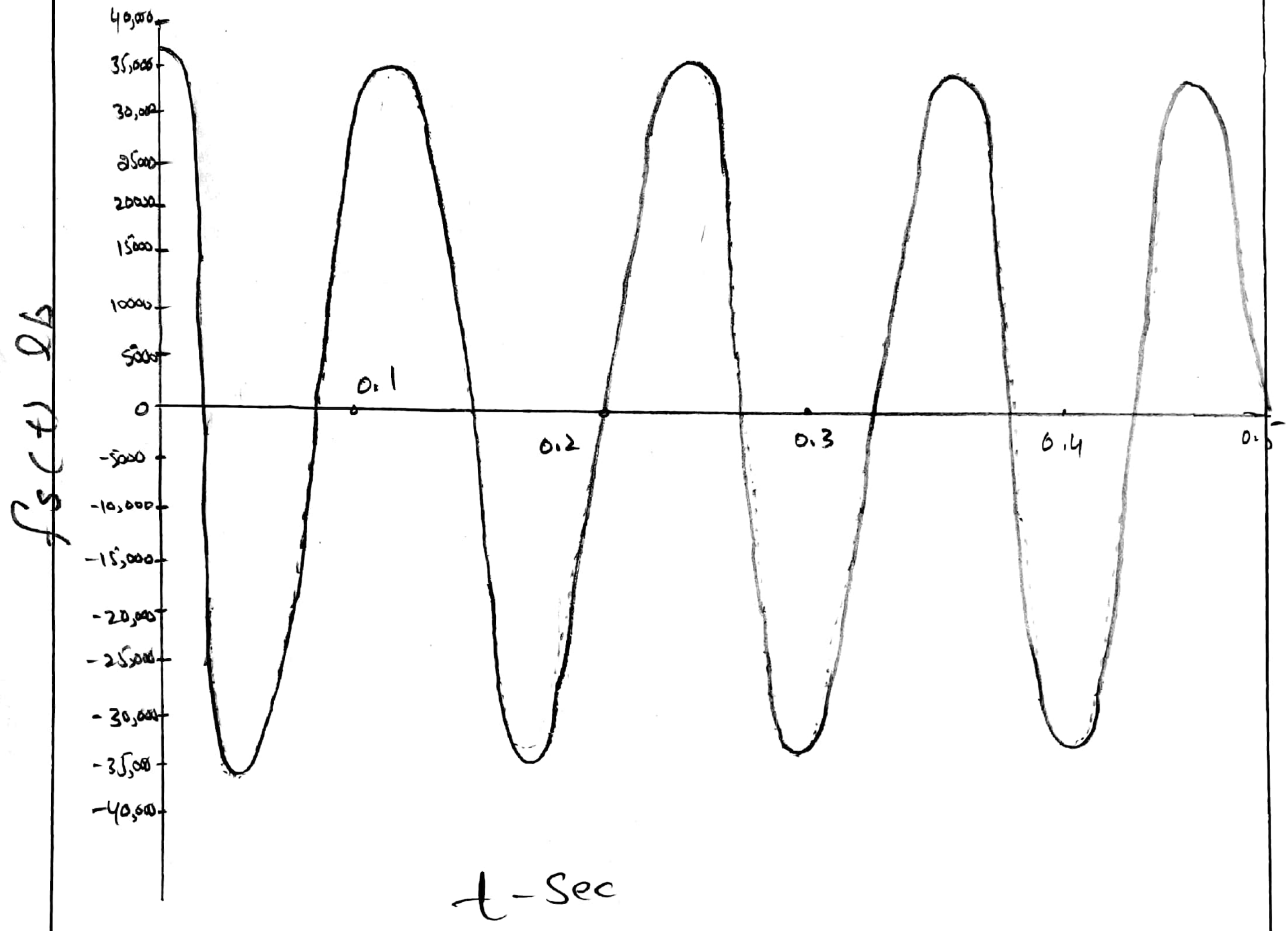
$$= \frac{1}{24} \text{ ft}$$

Amplitude of equivalent static force, f_{s0}

$$Ku_0 = 90625 \times \frac{1}{24} = 3776.04 \text{ lb}$$



Variation of Displacement with time



Variation of Equivalent Static forces with time.

Question # 02

Given DATA:

Using DATA of the beam of Q:1

$E = 29,000$ ksi

$I = 150$ in^4

δ_{st} = Deflection due to 7763 lb

Beam is pulled for ½ inch in the downward direction

Damping ratio of RCC with considerable cracking $= 3-5\%$

$= 3\%$

Required:

Develop & Solve the equation of motion for vibrations resulting at free end. Also develop an equation showing variation in the Equivalent Static Forces with time. Also Draw the Graph?

Solution:

E.O.M for damped free Vibration is

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \text{———} \quad (1)$$

From Question one

$K = 90625$ lb/ft

$m = 241.09$ lb sec^2/ft

$W_n = 19.96$ rad/sec

$$C = \xi \times 2mW_n$$

$$C = (0.03) \times 2(241.09)(19.96)$$

$$C = 288.72 \text{ lb.sec/ft}$$

Put values in eq (1)

$$90625u + 288.72\dot{u} + 241.09\ddot{u} = 0$$

Solution to the EOM for damped Free vibration is

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\ddot{u}(0) + u(0)\xi\omega_n\right] \times \sin(\omega_D t)\right]$$

$W_D = 19.96$ rad/sec

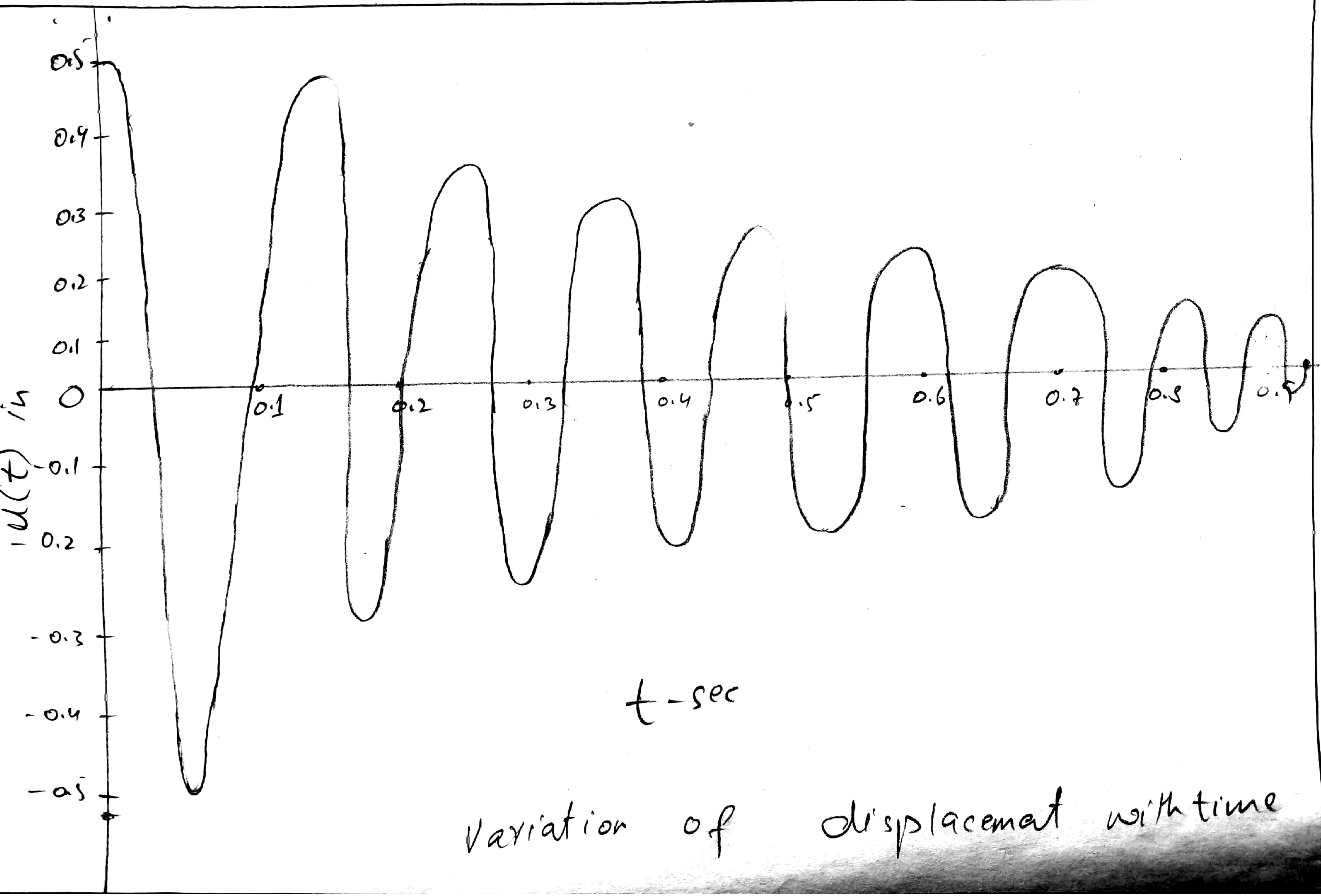
$$u(t) = e^{-0.5988t}\left[0.041 \times \cos(19.96t) + 0.00125 \times \sin(19.96t)\right]$$

$$f_s(t) = K \cdot u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.5988t}\left[(90625 \times 0.041)\cos(19.96t) + (90625 \times 0.00125)\sin(19.96t)\right]$$

$$f_s(t) = e^{-0.5988t}\left[3715.625\cos(19.96t) + 113.28\sin(19.96t)\right]$$

Scanned with CamScanner



Variation of displacemat with time

Q: 3

Given data:

Force = 60 kips

Displacement of tank = $\left(\frac{ID}{1000}\right)''$

$= \left(\frac{7763}{1000}\right) = 7.763$

Time taken to complete 7 cycles = 3.57 sec

Amplitude of displacement

= 2.286 cm

= 0.9"

Required DATA:

1) Damping ratios ?
2) Natural period of un-damped vibration
3) Stiffness of structure
4) Weight of tank
5) Damping coefficient

b) Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

→ Displacement of tank, $u = 7.763$"

After 7 cycles, i-e After $j = 7$

$$u_{j+1} = u_8 = 0.9"$$

Damping Ratio:

Damping ratio is find as

$$j = \frac{1}{2\pi \zeta} \ln \left[\frac{u_1}{u_{1+1}} \right]$$

$$7 = \frac{1}{2\pi\zeta} \ln \left[\frac{7.763}{0.9} \right]$$

$$\zeta = 0.0489 = 4.89\%$$

2. Natural Period of undamped vibration, $T_n = ?$

As the 7 cycles of Vibrations are ~~compdretd~~ completed in 3.57 Sec

=> Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

Now

$$W_D = W_n \sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{W_D} = \frac{2\pi}{(W_n \sqrt{1-\xi^2})}$$

$$T_D = \frac{T_n}{(1-\xi)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0489)^2}$$

$$\Rightarrow T_n = 0.5094 = 0.51 \text{ sec}$$

$$\Rightarrow T_n = 0.51 \text{ sec}$$

So the natural period of undamped vibration, $T_n = 0.51$ sec

3) Stiffness of Structure, $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.763} = 3.86 \text{ K/in}$$

$$K = 3.86 \text{ K/in} = 46320 \text{ lb/ft}$$

4) Weight of Tank :-

Weight of tank, w is find as

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{W/g}} = \sqrt{\frac{K.g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{K \times g}{W n^2}$$

Also ~~W_n~~ $\omega_n = \frac{2\pi}{T_n}$

$$W = \frac{Kg}{\left[\frac{4\pi^2}{T_n^2}\right]}$$

$$W = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46320 \text{ lb}}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2}\right] \times \frac{(0.5 \text{ sec})^2}{4\pi^2}$$

$W = 9445.06\ lb = 9.45\ k$

(5) Damping Coefficient, $c = ?$

It is know that

$$\zeta = \frac{c}{2 m \omega_n}$$

$$\Rightarrow c = \zeta \times 2m \times \omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{(0.0489) \times 4 \times \pi \times \frac{9445.06}{32.2}}{0.51}$$

$$c = 353.42\ lb.sec/ft$$

6) Number of cycles to reduce the displacement amplitude to 0.5", j

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0489} \ln\left[\frac{7.763}{0.5}\right]$$

$$j = 8.92 \text{ or } 9 \text{ cycles}$$