IQRA NATIONAL UNIVERSITY

Digital Signal Processing

Sessional Assignment 2020

Name: Kaleem Ullah

ID: 13170

Submitted to: Pir Meher Ali Shah Sir

Question No (1)

Determine the response $y(n)$, $n \geq 0$ of the System desribed by the 2nd order difference equation:

$$y[n] - 3y(n-1) - 4y(n-2) = x(x) + 2x(n-1)$$

to the input $x[n] = 4^n u(n)$

Solution :- $y(n) - 3y(n-1) - 4y(n-2) = x[n] - 2x(n-1) \rightarrow ①$

The homogenous equation of the System

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The charastic equation of the System

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^{1} - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 4, \lambda = -1$$

So

$$y_h(n) = c_1[-1]^n u[n] + c_2[4]^n u[n]$$

Scanned with CamScanner

$\Rightarrow$ Since 4 is a charcteristic roo and the eviction is

$$x[n] = 4^n u[n]$$

Assume a particular Solution.

$$y_p[n] = Kn4^n u[n]$$

Than

$$Kn\,4^n u[n] - 3K[n-1]\,4^{n-1} u[n-1] - 4K[n-2]\,4^{n-2}$$

$$u[n-2] = 4^n u[n] + 2\,(4)^{n-1} u(n-1)$$

For $n=2$

$$K(32-12) = 4^2 + 8 = 24$$

$$K = \frac{6}{5}$$

The total Solution is

$$y[n] = yp(n + yh(n)$$

$$= \left[\frac{6}{5} n4^n + C1\,4^n + C2(-1)^n\right] u(n)$$

The Solve For C1 C4 C2 we assume that $y(-1) = y(-2) = 0$

$y(0) = 1$ and $y(1) = 3y(0) + 4 + 2 = 9$

$\Rightarrow$ Hence

$c_1 + c_2 = 1$

$\frac{24}{5} + 4c_1 - c_2 = 9$

$4c_1 - c_2 = \frac{21}{5}$

$c_1 = \frac{26}{25}$

$c_2 = \frac{-1}{25}$

The total solution is

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4n - \frac{1}{25} (-1)^n\right] u(n).$$

$\longleftrightarrow$

Question NO (1)

=> Part (B)

Determine the Impulse response and unit step response of the System described by different equation.

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n).$$

Solution :- $y[n] = 0.6y[n-1] - 0.08y[n-2] + x[n]$

$$y[n] - 0.6y[n-1] + 0.08y[n-2] - x[n]$$

=> homogenous eq

$$x[n] = 0$$

$$y[n] = 0.6y(n-1) + 0.08y(n-2) = 0$$

$$y_n(n) = \lambda^n$$

So

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^1 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$(\lambda - 0.2)(\lambda - 0.8) = 0$$

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$$

Thus the genral form of the Solution to the homogenious equation.

$$y_n[n] = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$y(n) = C_1(0.2)^n + C_2(0.4)^n \longrightarrow (1)$$

$$y_n(n) = C_1 \frac{1}{5}^n + C_2\left(\frac{2}{5}\right)^n$$

$$y(0) = 1 \quad , \quad y(1) - 0.6y(0) = 0$$

$$y(1) = 0.6$$

$$\rightarrow \quad C_1 = -1 \qquad C_2 = 3$$

there for

$$h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(2/5\right)^n\right]u(n)$$

The Step response

$$y(n) = \sum_{K=0}^{n} h(n-K), \quad n > 0$$

$$= \sum_{K=0}^{n} \left[2\left(2/5\right)^{n-K} - \left(1/5\right)^{n-K}\right]$$

$$= \left(\frac{1}{0.12}\left(\frac{2}{5}^{n+1} - 1\right) - \frac{1}{0.16}\left(\frac{1}{5}^{n+1} - 1\right]u(n)\right.$$

$$\longleftrightarrow$$

Question NO (2)

=> Part (A)

Determine the casual Signal (x(n)) having Z - transform.

$$x(Z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution :- $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$

By Partial Fraction method

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^1} = \frac{A}{(1-2z^{-1})} \quad \frac{B}{(1-z^{-1})} \quad \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})$$ eq (1)

put $z = 1$

$$1 = A(1-0)^2 - B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$\boxed{C = -1}$$

$\Rightarrow$ put $z=2$ in eq (1)

$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$

$1 = A\left(\frac{1}{2}\right)4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$

$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$

$1 = \frac{A}{4} + 0 + 0$

$\boxed{A = 4}$

put $Z = 3$ in eq (1)

$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$

$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

$1 = \frac{4A}{9} + \frac{2B}{9} + \frac{1C}{9}$

$1 = \frac{4}{9}(4) + \frac{2B}{9} - \frac{1}{9}$

$1 + \frac{1}{9} - \frac{16}{9} = \frac{2B}{9}$

$-\frac{6}{9} \times \frac{9}{2} = B$

$\boxed{-3 = B}$

Hence $x(n) = \left(4(2)^{n} - 3 - n\right] u(n)$.

Question No (2)

=> Part (B)

Determine the Partial Fraction expansion of the following Proper Function.

$$x(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Solution :- Eliminate the -ve power by ⊗ing both numerator and denominator by z^2

$$x(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$$z = (z-0.5)A + (z-1)B \rightarrow (1) \quad \because \text{LCM}$$

Now set $z = p_1 = 1$ in eq (1) we eliminate the term involving A.

$$1 = (1-0.5)A$$

$$\boxed{A = 2}$$

Return to eq (1) $z = p_2 = 0.5$ than elementory the term involving A, So we have

$$0.5 = (0.5-1)B$$

$$\boxed{B = -1}$$

Question No (3)

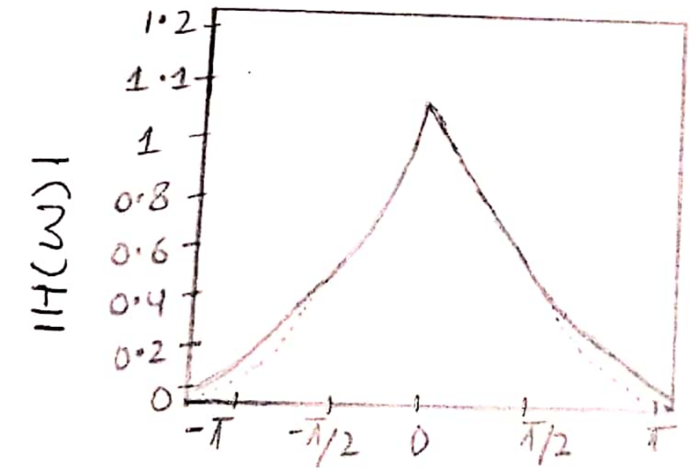
=> part (A)

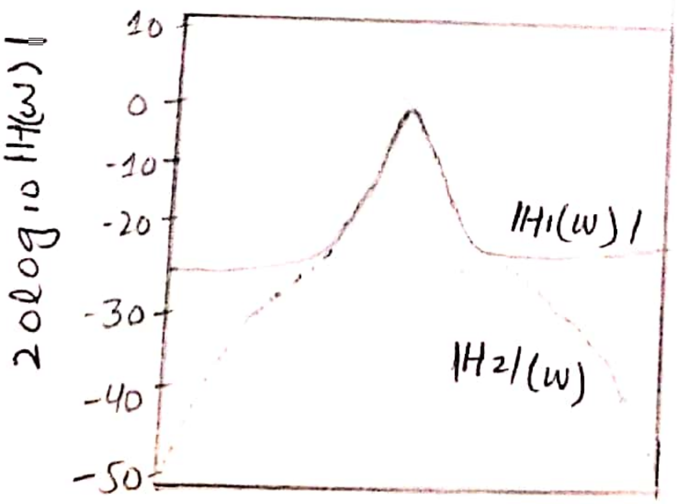
A two Pole low pass filter has the System response $H(z) = \frac{b_0}{(1 - Pz^{-1})^2}$

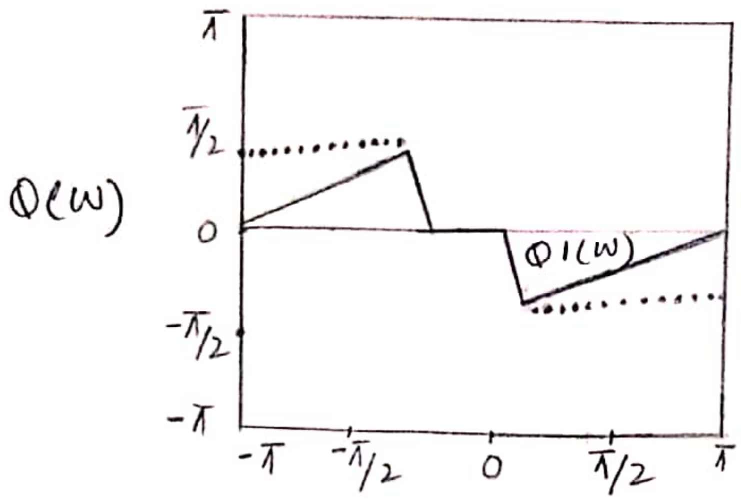
=> Determine the values of b_0 and P Such that the frequency response $H(\omega)$ Satisfies the condition $H(0) = 1$ and

$$|H(\frac{\pi}{4})|^2 = \frac{1}{2}.$$

Solution :- Linear Time Invarient System as frequency Selective filters.







Magnitude of and Phase response of (1) a single pole filter and (2) a one pole one zero filter.

$$H_1(z) = (1-a)(1-a z^{-1})$$

$$H_2(z) = ((11-a)/2]\,[1+z^{-1}/(1-a z^{-1})]$$

$$\boxed{a = 0.9}$$

Question No (3)

=> Part B

Design a two pole band pass filter that has the center of the $w = \pi/2$ zero in the frequency response characteristic at $w = 0$ and $w = \pi$ and its magnitude response is

$$\frac{1}{\sqrt{}} \text{ at } w = \frac{4\pi}{9}$$

Solution :- The filter must have poles at $P_{12} = re$

And zero at $z = 1$ And $z = -1$

Consequently the same system function

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G \frac{z^2 - 1}{z^2 + r^2}$$

=> The gain factor is determined evaluating the frequency response

⇒ $H(w)$ of the filter at $w = \pi/2$

$$H = (\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w)$ at $w = 4\pi/9$ we have

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

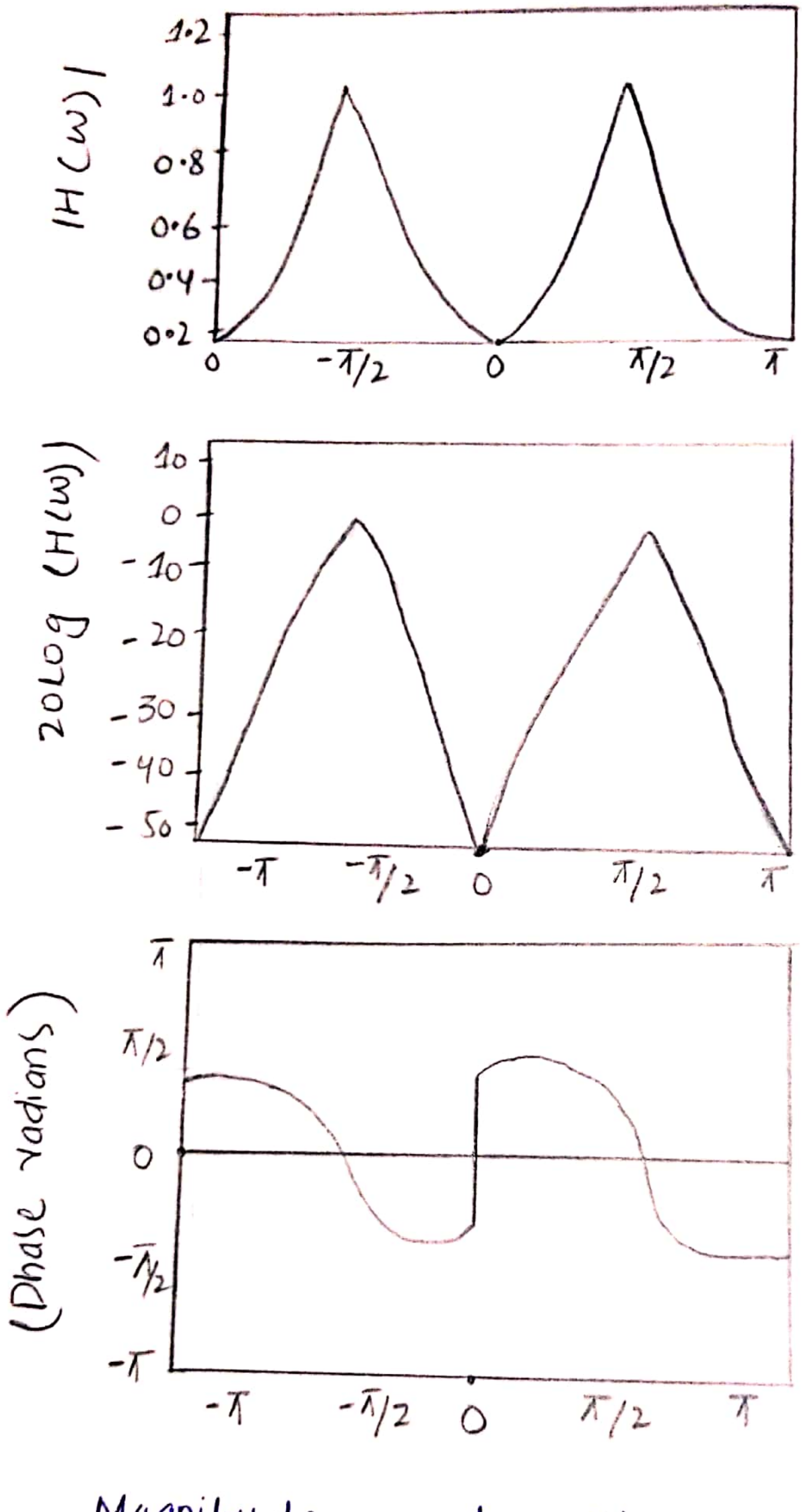
⇒ **Equivalently**

$$1.94\,(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation there for the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

its frequency response is illustrated.



Magnitude and phase response of a simple bandpass filter is

$$H(z) = 0.15 \left[(1-z)^{-2} / (1+0.7 z^{-2}) \right].$$

Question No (4)

=> Part (A)

A finit duration Sequence of Length L is given as $x(n) = \begin{cases} 1; & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$

Determine the N point DFT of the Sequence for $N \geq L$

Solution :- The Fourier transformer of this sequence is

$$X(w) = \sum^{L-1} x(n)\, e^{-jwn}$$

$$X(w) = \sum_{n=0}^{L-1} e^{-jwn} = \frac{1 - e^{-jwL}}{1 - e^{-jw}}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)} \, e^{-jw(L-1)/2}$$

The magnitude end phase of X (w) are illustrated for L=10

The N point DFT of x (n) is Simply X (w) evaluated at the set of N equally spaced frequency.

$\Rightarrow \quad \omega_K = 2\pi K/N$

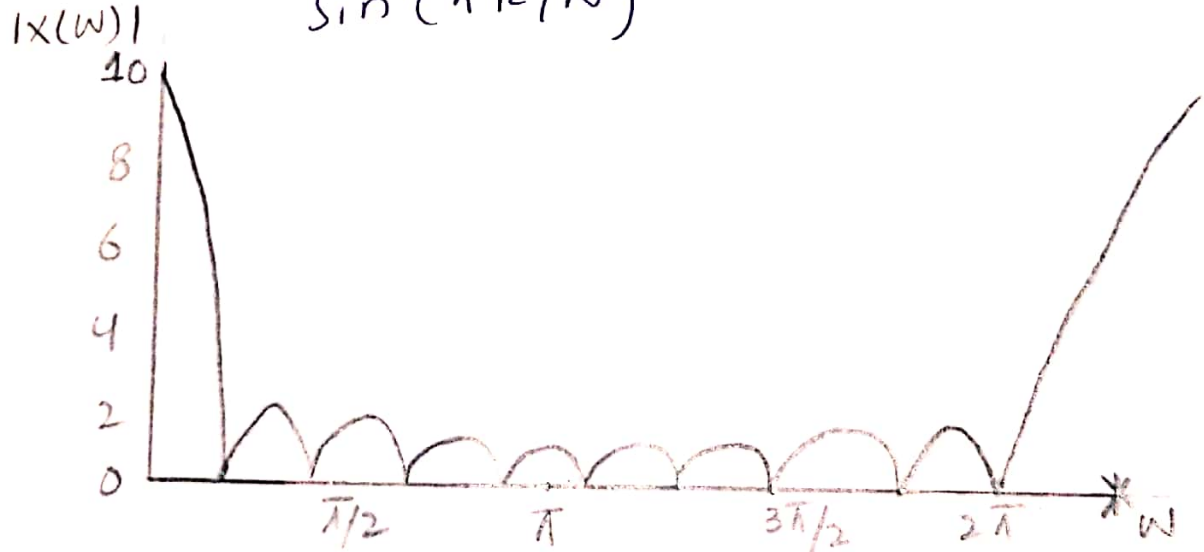
$K = 0, 1, 2, \ldots\ldots N-1$

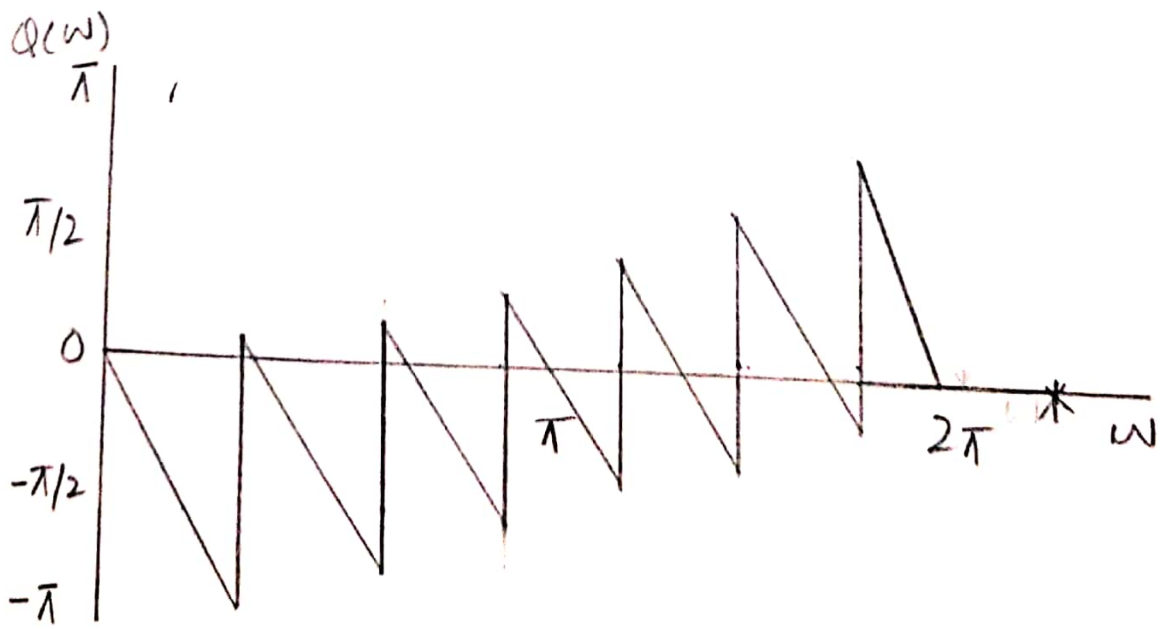
Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}}$$

$K = 0, 1, \ldots\ldots N-1$

$$X(K) = \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)N}$$





If N is Selected such that N = L

Than the Discrete Fourier transformer

becomes

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$

Thus their is only one non zero value in the DFT this is apparent from observation of X (W) since X (W) = 0 at the frequencies $W_K = 2\pi K/L$

$K \neq 0$;

The reader should verify that x(n) can be recoverd from X (K) by performing an L- point IDFT

provides a plot of the N - point DFT in magnitude phase for L = 10 N = 50 and N = 100 Now the special charactristic of the sequence are more clearly evident.

Question No (4)

=> Part (B)

Determine the N point DFT of this Sequence for $N \geq L$

Compute the DFT of the Four point Sequence

$x(n) = (0\,1\,2\,3)$.

Solution :- The first Step is to determine the matrix W_4 By exploiting the perodicity property of W_4

$W_N^{k+N/2} = -W_N^k$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^{2} & W_4^2 & W_4^2 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^7 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^2 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

=> Than They

$$y_4 = W_4 \lambda_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of λ_4 may be determined by Conjugating the element in W_4 to obtain W_4^t and than applying the formula.

$\longleftrightarrow$

Thank You