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Page (1)

Question # 1

$x_1 - 9x_2 + x_3 = 0$
$2x - 8x_3 = 0$
$5x_1 - 5x_3 = 10$

solution

Consistent meen that system has a solution

$AX = D$

$$A = \begin{vmatrix} 1 & -9 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

Augumented Matrix [AD]

$$AD = \begin{bmatrix} 1 & -9 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 5 & 0 & -5 & 10 \end{bmatrix} R3-5R1$$

R3-5R1
$5 - 5(1) = 0$
$0 - 5(-9) = 45$
$-5 - 5(1) = -10$
$10 - 5(0) = 10$

$$= \begin{bmatrix} 1 & -9 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 0 & 45 & -10 & 10 \end{bmatrix} R3/5$$

Name :- Midrar khan ID :- 12990

Page (2)

$$= \begin{bmatrix} 1 & -9 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 0 & 9 & -2 & 2 \end{bmatrix} \quad R_2/2$$

$$= \begin{bmatrix} 1 & -9 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 9 & -2 & 2 \end{bmatrix} \quad \begin{matrix} R_3 - 9R_2 \\ 0-9(0)=0 \\ 9-9(1)=0 \\ -2-9(-4)=-34 \\ 2-9(0)=2 \end{matrix}$$

$$\begin{bmatrix} 1 & -9 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 34 & 2 \end{bmatrix} \rightarrow eq\ (A)$$

Rank (A) = No of non-zero rows
= 3

Rank (AB) = 3

system will have unique solution

from eq (A) we can also prove it by determining the values of $x_1\, x_2$ & x_3

$x_1 - 9x_2 + x_3 = 0$

$x_2 - 4x_3 = 0$

$34x_3 = 2$

$x_3 = 1/17$

$x_2 = -4(x_3) = -4(1/17)$

$x_2 = -4/A$

$x_1 = 9x_2 - x_3 = 9\left(\frac{-4}{19}\right) \frac{-1}{17} = \frac{-37}{19}$

x_1, x_2 and x_3 has unique value

Name :- Midrar khan ID :- 12990

Pefe (3)

Q No 3 Solve the following Systems of linear equations by Gauss-Jordan Method

$$2x+2y+4z=18$$
$$x+3y+2z=13$$
$$3x+2y-3z=14$$

Sol:-
$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_1=\frac{1}{2}R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\xrightarrow[R_3=R_3-3R_1]{R_2=R_2-R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \xrightarrow{R_2=\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix}$$

$$R_3=R_3+2R_2 \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow[R_3=-\frac{1}{3}R_3]{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$R_1=R_1-2R_3 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \left.\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}\right\} \text{Solution}$$

so $z = 3$

Name :- Midrar khan   ID :- 12990

Pafe (4)

Question no # 4
show that is Diagosiable

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

solution

Matrix A is diagnosible if

$$A = CDC^{-1}$$

$$\det(A - \lambda I_3) = 0$$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$\Rightarrow 4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} -2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix} -2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 4-\lambda\,[(3-\lambda)(1-\lambda)-8) - 2(-5(1-\lambda)+4) - 2(-20+2(3-\lambda)) = 0$$

$$\Rightarrow 4-\lambda[3-3\lambda\ \lambda+\lambda^2-8] - 2[-5+5\lambda+4] - 2[-20+6-2\lambda] = 0$$

$$\Rightarrow 4-\lambda[\lambda^2-4\lambda-5] - 2[5\lambda-1] - 2[-14-2\lambda] = 0$$

$$\Rightarrow 4\lambda^2 + 16\lambda - 20 - \lambda^3 + 4\lambda^2 + 5\lambda - 10\lambda + 2 + 28 + 4\lambda = 0$$

$$\Rightarrow -\lambda^3 + 8\lambda^2 + 15\lambda + 10 = 0$$

Name :- Midrar khan ID :- 12990

Pafe (5)

$\lambda = 9.65$
$\lambda = -0.89$
$\lambda = -0.829$

for $\lambda = -0.89$

$$A - \lambda I_3 = \begin{bmatrix} -5.65 & 2 & -2 \\ -5 & -6.65 & 2 \\ -2 & 4 & -8.65 \end{bmatrix}$$

for $\lambda = -0.82$

$$A - \lambda I_3 = \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 3.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

In end or by solving 2 eigen spaces or 2 basic vector in total

So Matrix A is not diagonasible.

1) = 0

Question # 5

$3x_1 + 5x_2 - 4x_3 = 0$
$-3x_1 - 25x_2 + 4x_3 = 0$
$6x_1 + x_2 - 8x_3 = 0$

Sol System has non-trival solution if & only if its determinant is non-zero

$Ax = D$

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix}$$

$$\det A = 3\begin{vmatrix} -25 & 4 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -3 & 4 \\ 6 & -8 \end{vmatrix} - 4\begin{vmatrix} -3 & -25 \\ 6 & 1 \end{vmatrix}$$

$= 3(200-4) - 5(24-24) - 4(-3+150)$

$= 588 - 5(0) - 588$

$= 0$

As determinant comes out zero, then this system has either no non-trival solutions or an infinite number of solutions

Name:- Midrar khan ID:-12990

Pafe (7)

Question # 6

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Maximum Possible Rank for Matrix ~~A~~

$|A|$ ~~is~~ $= 0$

$$|A| = \begin{vmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{vmatrix}$$

OR

Rank = No of non-zero rows

$$A = \begin{vmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{vmatrix}$$

R3 − R1

$1-1=0$

$3-3=0$

$4-4=0$

$0-3=-3$

$$= \begin{vmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 & 4 & 3 \\ 1 & 9 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{vmatrix}$$

R2/3

R2 − R1

$1-1=0$

$3-3=0$

$4-4=0$

$3-3=0$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

Rank = 2