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SECTION	A
Subject	Differential equation
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Question No 1 part (i)

Answer 1:

Solution:

$$w = \sin(x+ct) + \cos(2x+2ct)$$

Given $$\frac{\partial^2 w}{\partial t^2} = \frac{c^2\, \partial^2 w}{\partial x^2} \rightarrow (1)$$

Now $$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$= \frac{\partial}{\partial t}(\sin(x+ct) + \frac{\partial}{\partial t}(\cos(2x+2ct)$$

$$\frac{\partial W}{\partial t} = c\cos(x+ct) - 2c\sin(2x+ct)$$

Now $$\frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+ct)\right]$$

$$\boxed{\frac{\partial^2 W}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)}$$

Now $$\frac{\partial W}{\partial x} = \frac{\partial}{\partial x} = \left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$\frac{\partial W}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 W}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 W}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

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(1) $\Rightarrow$

$$-c^2\sin(x+ct)-4c^2\cos(2x+2ct)=c^2\left[-\sin(x+ct)-4\cos(2x+2ct)\right]$$

$$-\cancel{c^2\sin(x+ct)}-\cancel{4c^2\cos(2x+2ct)}=-\cancel{c\sin(2x+ct)}-\cancel{4c^2\cos(x+2ct)}$$

$0 = 0 \rightarrow$ satisfied

Question No 1

part (ii)

ANSWER:-

Sol:-

$$W = \tan(2x + ct)$$

Now $\frac{\partial W}{\partial t} = c\sec^2(2x+ct)$

and $\frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$

$= c^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$

Now $\frac{\partial W}{\partial x} = 2\sec^2(2x+ct)$

$\frac{\partial^2 W}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$

① $\Rightarrow$ $\cancel{4c^2\sec^2(2x+ct)\tan(2x+ct)} = \cancel{4c^2\sec^2(2x+ct)\tan(2x+ct)}$

$0 = 0$ satisfied.

Question No 2

ANSWER:-

Solution:-

Given function is

$$F(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

We have to find the Fouries Co-effient, a_0, a_n and b_n

Now.
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$= \boxed{-\frac{\pi}{2} + \pi = \pi/2} \rightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos x)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin x}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]^{0}$$

$$\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{-\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2}; & \text{if } n \text{ is odd} \\ 0; & \text{if } n \text{ is even} \end{cases}} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\, dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\, dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\, dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$(3) \leftarrow b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

The required fourier Series is

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\cdot\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

Question No 3

ANSWER:-

Solution:-

$$y'' - 4y' + 13y = 8\sin 3x, \quad y(0) = 1$$
and $y'(0) = 2$

Associated homogenous eq of ① is

$$y'' - 4y' + 13y = 0 \rightarrow ②$$

Change ② into Auxiliary equation

put $y = m$ in ②

$$m^2 - 4m + 13 = 0$$

use Quadratic equation formula

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

$$m = 4 \pm \sqrt{36i}$$

$$m = \frac{4 \pm 6i}{2}$$

$$\boxed{\begin{aligned} m_1 &= 2 + 3i \\ m_2 &= 2 - 3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_1 \sin 3x) \rightarrow ④$$

Let $y_p = A\cos 3x + B\sin x \rightarrow$ (*)

Diff writte respect to "x"

$y'_p = -3A\sin 3x + 3B\cos 3x$

Again Diff w.r. to "x"

$y''_p = -9A\cos 3x - 9B\sin 3x$

put in (1)

$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x)$

$+ 13(A\cos 3x + B\sin 3x) = 8\sin 3x$

$9A\cos 3x - 12B\cos 3x + 13A\cos 3x$

$-9B\sin 3x + 12A\sin 3x + 13B\sin 3x = 8\sin 3x$

$(-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

$(4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing Co-efficient is

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \rightarrow$ (a)

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$\Rightarrow \boxed{A = 3B} \rightarrow$ (b)

put (b) in equ (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \rightarrow$ (c)

put (c) in equ (b)

$\Rightarrow \boxed{A = 3/5} \rightarrow$ (d) put (c) and (d) → in (*)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$$

The G sol is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + 3/5\cos 3x + 1/5\sin 3x$$

Now we need to find the value of c_1 and c_2 for this

put $x=0$ & $y=1$ in (C)

$$1 = (c_1(1) + c_2(0)) + 3/5(1) + 1/5(0)$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - 3/5$$

$$c_2 = 2/5 \rightarrow (**)$$

Diff: (C) w.r.to "x"

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - 6/15\sin 3x + 3/5\cos 3x \rightarrow (D)$$

put $y' = 2$, $x=0$ in (D)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - 6/5\sin 3x + 3/5\cos 3x$$

put $y' = 2$ $x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - 6/5\sin 3(0) + 3/5\cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + 3/5$$

$$2 = 2c_1 + 3c_2 + 3/5$$

put $c_1 = 2/5$

$2 = 4/5 + 3c_2 + 3/5$

$2 = \frac{7}{5} + 3c_2$

$3c_2 = 2 - 7/5$

$3c_2 = 3/5$

$\boxed{c_2 = \frac{3}{15}} \rightarrow$ ✱✱✱

put ✱✱ and ✱✱✱ in (c)

$$y = e^{3x}\left(2/5 \cos 3x + \frac{3}{15} \sin 3x\right) + 3/5 \cos 3x + \frac{3}{5} \sin 3x$$

$$\boxed{y = \frac{2}{5} e^{2x} \cos 3x + \frac{3}{15} e^{2x} \sin 3x + \frac{3}{5} \cos 3x + 1/5 \sin 3x}$$

Required General solution.

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Question No 4

ANSWER:

Solution:

$$(D^2 - DD')z \neq \cos x \cos 2y \rightarrow (a)$$

- put A.E $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i.e $D = m$, $D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

therefore $C.F = f_1(y) + f_2(y+x)$

From equ (a)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

As $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$CF = f_1(y-x) + x f_2(y-x)$

$$PI = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x) + \sin(x-y)\right]$$

By General Method

$m = -1$; $y - x = c$

$$= \frac{1}{D+D'} = \int\left[(2c + \sin(-c))\right] dx$$

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$\Rightarrow C x^2 - \frac{x^2}{2} \sin c$

Replacing c by $y-x$

$\therefore = x^2(y-x) - \frac{x^2}{2} \sin(y-x) = x^2y - x^3 + \frac{x^2}{2} \sin(x-y)$.

Hence the requried solution is

$z = C.F + P.I = f(y-x) + x f_2(y-x) + x^2y$
$- x^3 + \frac{1}{2} x^2 \sin(x-y)$